C3 Jan 2008

① $\dfrac{2x^4 - 3x^2 + x + 1}{x^2 - 1} = 2x^2 - 1 + \dfrac{x}{(x^2-1)}$

$$\begin{array}{r} 2x^2 - 1 \phantom{+x+1} \\ x^2 + 0x - 1 \,\big)\, \overline{2x^4 + 0x^3 - 3x^2 + x + 1} \\ \underline{2x^4 + 0x^3 - 2x^2 \phantom{+x+1}} \\ -x^2 + x + 1 \\ \underline{-x^2 + 0x + 1} \\ x \end{array}$$

$\underline{a = 2}$, $\underline{b = 0}$, $\underline{c = -1}$, $\underline{d = 1}$, $\underline{e = 0}$

② $y = e^{2x}\tan x$

a) Turning points at $\frac{dy}{dx} = 0$:

$u = e^{2x}$ $\quad v = \tan x$

$u' = 2e^{2x}$ $\quad v' = \sec^2 x$

$\frac{dy}{dx} = e^{2x}\sec^2 x + 2e^{2x}\tan x$
$= e^{2x}(\sec^2 x + 2\tan x)$

$e^{2x}(\sec^2 x + 2\tan x) = 0 \Rightarrow \sec^2 x + 2\tan x = 0$

$1 + \tan^2 x + 2\tan x = 0$

$(\tan x + 1)^2 = 0$

$\Rightarrow \underline{\tan x = -1}$ QED

b) When $x = 0$,

$\frac{dy}{dx} = e^0(\sec^2 0 + 2\tan 0) = 1 \times (1 + 0) = 1$

$y = e^0 \tan 0 = 1 \times 0 = 0.$

$y - 0 = 1(x - 0) \Rightarrow \underline{y = x}$

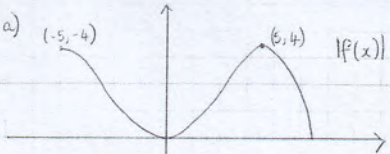
③ $f(x) = \ln(x+2) - x + 1 \qquad (x > -2).$

a) $f(2) = \ln 4 - 1 > 0$
$f(3) = \ln 5 - 2 < 0$ } change of sign $\Rightarrow$ root in interval.

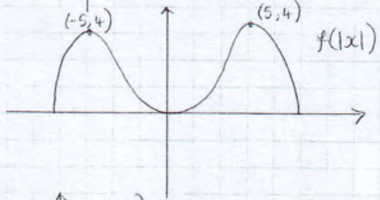
b) $x_0 = 2.5$
$x_1 = 2.50408$
$x_2 = 2.50498$
$x_3 = 2.50518$

c) $f(2.5045) = 0.00058 > 0$
$f(2.5055) = -0.00020 < 0$ } change of sign $\Rightarrow$ root $= 2.505$ to 3dp.

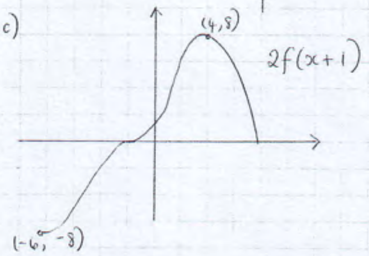
④ a)




b)




c)




⑤ $R = 1000e^{-ct}$

a) When $t = 0$, $R = 1000e^0 = \underline{1000}$

b) $t = 5730$:

$1000e^{-5730c} = 500$

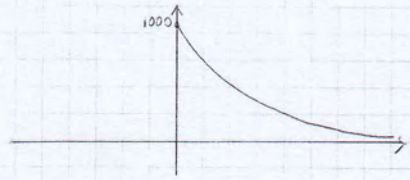
$e^{-5730c} = 0.5$

$c = \dfrac{\ln 0.5}{-5730} = -0.000121^{0}$

c) $t = 22920$:

$1000e^{-c \times 22920} = \underline{6.25}$

d)




⑥

a) $\cos(A+B) = \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x$

$\Rightarrow \cos 3x = (2\cos^2 x - 1)\cos x - 2\sin x \cos x \sin x$

$= 2\cos^3 x - \cos x - 2\cos x \sin^2 x$

$= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x)$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$= \underline{4\cos^3 x - 3\cos x}.$

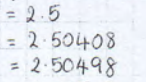
b) i) $\dfrac{\cos x}{1 + \sin x} + \dfrac{1 + \sin x}{\cos x} = \dfrac{\cos^2 x + (1 + \sin x)^2}{\cos x(1 + \sin x)}$

$= \dfrac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x(1 + \sin x)}$

$= \dfrac{2(1 + \sin x)}{\cos x(1 + \sin x)} = 2\sec x$ QED

ii) $2\sec x = 4$

$\sec x = 2 \Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \underline{\frac{\pi}{3}}, \underline{\frac{5\pi}{3}}$




⑦ $y = 3\sin 2x + 4\cos 2x$

a) $\frac{dy}{dx} = 6\cos 2x - 8\sin 2x$

When $x = 0$, $\frac{dy}{dx} = 6\cos 0 - 8\sin 0 = 6.$

$m_{normal} = -\frac{1}{6}$

$y - 4 = -\frac{1}{6}(x - 0)$

$\underline{y = -\frac{1}{6}x + 4}$

b) $R\sin(2x + \alpha) = R\sin 2x\cos\alpha + R\cos 2x \sin\alpha$

$= 3\sin 2x + 4\cos 2x$

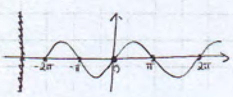
$\Rightarrow R\cos\alpha = 3$
$R\sin\alpha = 4$ } $R = 5$, $\tan\alpha = \frac{4}{3} \Rightarrow \alpha = 0.927$

$\therefore \underline{y = 5\sin(2x + 0.927)}$

c) On $x$-axis, $y = 0 \Rightarrow \sin(2x + 0.927) = 0$ $\qquad -\pi \le x \le \pi$

$2x + 0.927 = 0, \pi, 2\pi, -\pi, -2\pi$ $\qquad -5.356 \le 2x + 0.927 < 7.211$

$x = \underline{-3.61}, \underline{-2.03}, \underline{-0.46}, \underline{1.11}, \underline{5.36}$




⑧ $f(x) = 1 - 2x^3$

$g(x) = \frac{3}{x} - 4$

a) $x = 1 - 2y^3$

$2y^3 = 1 - x$

$y^3 = \frac{1-x}{2}$

$y = \sqrt[3]{\frac{1-x}{2}} \Rightarrow \underline{f^{-1}(x) = \sqrt[3]{\frac{1-x}{2}}}$

b) $gf(x) = \dfrac{3}{1-2x^3} - 4$

$= \dfrac{3}{1-2x^3} - \dfrac{4(1-2x^3)}{1-2x^3} = \dfrac{3 - 4 + 8x^3}{1-2x^3} = \underline{\dfrac{8x^3 - 1}{1-2x^3}}$

c) $gf(x) = 0$

$\Rightarrow 8x^3 - 1 = 0$

$8x^3 = 1$

$x^3 = \frac{1}{8} \Rightarrow \underline{x = \frac{1}{2}}$

d) S.P. at $\frac{dy}{dx} = 0$

$u = 8x^3 - 1$ $\quad v = 1 - 2x^3$ $\quad v^2 = (1 - 2x^3)^2$

$u' = 24x^2$ $\quad v' = -6x^2$

$\dfrac{dy}{dx} = \dfrac{24x^2(1 - 2x^3) + 6x^2(8x^3 - 1)}{(1-2x^3)^2}$

$= \dfrac{24x^2 - 48x^5 + 48x^5 - 6x^2}{(1-2x^3)^2}$

$= \dfrac{18x^2}{(1-2x^3)^2}$

At $\frac{dy}{dx} = 0$, $\dfrac{18x^2}{(1-2x^3)^2} = 0 \Rightarrow 18x^2 = 0$

$\Rightarrow x = 0$

When $x = 0$, $y = \dfrac{8 \times 0^3 - 1}{1 - 2 \times 0^3} = \dfrac{-1}{1} = -1$

S.P. $= \underline{(0, -1)}$