

C3 Jan07 - Solutions

$$\begin{aligned}
 1 \text{ a) } \sin 3\theta &= \sin(2\theta + \theta) = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta \\
 &= 2\sin \theta \cos \theta \cos \theta + (\cos^2 \theta - \sin^2 \theta) \sin \theta \\
 &= 2\sin \theta \cos^2 \theta + \sin \theta \cos^2 \theta - \sin^3 \theta \\
 &= 3\sin \theta \cos^2 \theta - \sin^3 \theta \\
 &= 3\sin \theta (1 - \sin^2 \theta) - \sin^3 \theta \\
 &= 3\sin \theta - 3\sin^3 \theta - \sin^3 \theta \\
 &= \underline{3\sin \theta - 4\sin^3 \theta} \quad \#
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \sin \theta = \frac{\sqrt{3}}{4} &\Rightarrow \sin 3\theta = \frac{3\sqrt{3}}{4} - 4\left(\frac{\sqrt{3}}{4}\right)^3 \\
 &= \frac{3\sqrt{3}}{4} - 4 \times \frac{3\sqrt{3}}{64} \\
 &= \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \underline{\underline{\frac{9\sqrt{3}}{16}}}
 \end{aligned}$$

$$\begin{aligned}
 2 \text{ a) } f(x) &= 1 - \frac{3}{(x+2)} + \frac{3}{(x+2)^2} \\
 &= \frac{(x+2)^2 - 3(x+2) + 3}{(x+2)^2} \\
 &= \frac{x^2 + 4x + 4 - 3x - 6 + 3}{(x+2)^2}
 \end{aligned}$$

$$\begin{aligned}
 y - \frac{\pi}{4} &= -\sqrt{2}x + 2 \\
 \underline{\underline{y}} &= \underline{\underline{-\sqrt{2}x + 2 + \frac{\pi}{4}}}
 \end{aligned}$$

$$4. \text{ i) } y = x(9+x^2)^{-1}$$

turning points when $\frac{dy}{dx} = 0$.

$$\begin{aligned}
 \frac{dy}{dx} &= (9+x^2)^{-1} + x \times -(9+x^2)^{-2} \times 2x \\
 &= (9+x^2)^{-1} - 2x^2(9+x^2)^{-2}
 \end{aligned}$$

$$\therefore \frac{1}{9+x^2} - \frac{2x^2}{(9+x^2)^2} = 0$$

$$\frac{9+x^2 - 2x^2}{(9+x^2)^2} = 0$$

$$9 - x^2 = 0$$

$$x = \pm 3$$

$$\text{When } x = 3 \quad y = \frac{3}{9+(-3)^2} = \frac{3}{18} = \frac{1}{6}$$

\therefore Coords $\underline{\underline{(3, \frac{1}{6})}}$ and $\underline{\underline{(-3, -\frac{1}{6})}}$

(2)

$$= \frac{x^2+x+1}{(x+2)^2} \quad \#$$

$$\begin{aligned}
 \text{b) } x^2+x+1 &= \left(x+\frac{1}{2}\right)^2 - \frac{1}{4} + 1 \\
 &= \left(x+\frac{1}{2}\right)^2 + \frac{3}{4} > 0 \quad \#
 \end{aligned}$$

c) Numerator > 0 from b)
Denominator $> 0 \Rightarrow f(x) > 0$. #

$$3 \text{ a) } x = 2\sin y$$

$$\text{When } y = \frac{\pi}{4} \quad x = 2\sin \frac{\pi}{4} = 2 \times \frac{\sqrt{2}}{2} = \underline{\underline{\sqrt{2}}} \quad \#$$

$$\text{b) } \frac{dx}{dy} = 2\cos y \Rightarrow \frac{dy}{dx} = \frac{1}{2\cos y}$$

$$\text{When } y = \frac{\pi}{4} \quad \frac{dy}{dx} = \frac{1}{2\cos \frac{\pi}{4}} = \frac{1}{2 \times \frac{\sqrt{2}}{2}} = \frac{1}{\sqrt{2}} \quad \#$$

$$\text{c) Grad of tangent at P} = \frac{1}{\sqrt{2}}$$

$$\therefore \text{Grad of normal} = -\sqrt{2}$$

$$\therefore \text{eqn. of normal } y - \frac{\pi}{4} = -\sqrt{2}(x - \sqrt{2})$$

(4)

$$\text{ii) } y = (1+e^{2x})^{3/2}$$

$$\frac{dy}{dx} = \frac{3}{2}(1+e^{2x})^{1/2} \times 2e^{2x}$$

$$\frac{dy}{dx} = 3e^{2x}(1+e^{2x})^{1/2}$$

$$\left. \frac{dy}{dx} \right|_{x=\frac{1}{2}\ln 3} = 3e^{\ln 3} (1+e^{\ln 3})^{1/2} = 9(1+3)^{1/2} = \underline{\underline{18}}$$

$$5 \text{ a) } y = \sqrt{3}\cos x + \sin x$$

$$R\sin(x+\alpha) = R\sin x \cos \alpha + R\cos x \sin \alpha$$

$$\therefore R\cos \alpha = 1 \quad R\sin \alpha = \sqrt{3}$$

$$\tan \alpha = \sqrt{3}$$

$$\alpha = \frac{\pi}{3}$$

$$\therefore R = \frac{1}{\cos \frac{\pi}{3}} = 2$$

$$\therefore \underline{\underline{y = 2\sin(x + \frac{\pi}{3})}}$$

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b) $1 = 2\sin(x + \pi/3) \quad 0 < x < 2\pi$
 $\sin(x + \pi/3) = \frac{1}{2}$
 $x + \pi/3 = \sin^{-1} \frac{1}{2}$
 $x + \pi/3 = \pi/6 \quad \pi/3 \leq x + \pi/3 \leq 7\pi/3$
 $x + \pi/3 = \pi/6, 5\pi/6, 13\pi/6$
 $x = -\pi/6, 3\pi/6, 11\pi/6$
 $x = \pi/2, 11\pi/6$

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c)
 meet x axis when $y=0 \quad 2 - \frac{1}{2}e^x = 0$
 $e^x = 4$
 $x = \ln 4$

d) $x_1 = -\frac{1}{2}e^{-0.3} = -0.3704$ (4dp)
 $x_2 = -\frac{1}{2}e^{-0.370409} = -0.3452$ (4dp)
 e) $x_3 = -0.3540$
 $x_4 = -0.3509$
 $x_5 = -0.3520$
 $x_6 = -0.3516$
 $x_7 = -0.3517 \quad \therefore x = -0.352$ (3dp)

6. a) $y = \ln(4 - 2x)$
 $e^y = 4 - 2x$
 $2x = 4 - e^y$
 $x = 2 - \frac{1}{2}e^y \quad \therefore F^{-1}x \rightarrow 2 - \frac{1}{2}e^x \quad x \in \mathbb{R}$
 b) $x < 2$

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7. a) $f(-2) = (-2)^4 - 4(-2) - 8 = 16$
 $f(-1) = (-1)^4 - 4(-1) - 8 = -3$
 \therefore root exists between -1 and -2. #

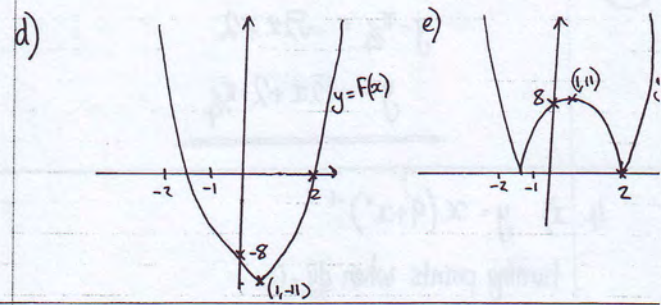
b) turning point when $\frac{dy}{dx} = 0$
 $\frac{dy}{dx} = 4x^3 - 4 = 0$
 $x^3 - 1 = 0$
 $x^3 = 1 \Rightarrow x = 1 \Rightarrow y = 1^4 - 4(1) - 8 = -11$

c) Coords (1, -11)

$$\begin{array}{r} x^3 + 2x^2 + 4x + 4 \\ x-2 \overline{) x^4 \quad -4x-8} \\ \underline{x^4 - 2x^3} \quad \quad \quad \\ 2x^3 - 4x - 8 \\ \underline{2x^3 - 4x^2} \quad \quad \quad \\ 4x^2 - 4x - 8 \\ \underline{4x^2 - 8x} \quad \quad \quad \\ 4x - 8 \\ \underline{4x - 8} \\ 0 \end{array}$$

$\therefore a=2, b=4, c=4$

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8. i) $\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x$
 LHS = $\sec^2 x - \operatorname{cosec}^2 x$
 $= \tan^2 x + 1 - (\cot^2 x + 1)$
 $= \tan^2 x + 1 - \cot^2 x - 1 = \tan^2 x - \cot^2 x \quad \#$

ii) $y = \arccos x$

a) $x = \cos y$
 $x = \sin(\pi/2 - y)$
 $\pi/2 - y = \arcsin x$

b) $\arccos x + \arcsin x$
 $= y + \pi/2 - y$
 $= \pi/2$