



1. Use the derivatives of  $\sin x$  and  $\cos x$  to prove that the derivative of  $\tan x$  is  $\sec^2 x$ . (4)
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2. The function  $f$  is given by  $f: x \mapsto 2 + \frac{3}{x+2}$ ,  $x \in \mathbb{R}$ ,  $x \neq -2$ .

(a) Express  $2 + \frac{3}{x+2}$  as a single fraction. (1)

(b) Find an expression for  $f^{-1}(x)$ . (3)

(c) Write down the domain of  $f^{-1}$ . (1)

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3. (a) Express as a fraction in its simplest form

$$\frac{2}{x-3} + \frac{13}{x^2 + 4x - 21}. \quad (3)$$

- (b) Hence solve

$$\frac{2}{x-3} + \frac{13}{x^2 + 4x - 21} = 1. \quad (3)$$


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4. (a) Simplify  $\frac{x^2 + 4x + 3}{x^2 + x}$ . (2)

(b) Find the value of  $x$  for which  $\log_2(x^2 + 4x + 3) - \log_2(x^2 + x) = 4$ . (4)

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5. (i) Prove, by counter-example, that the statement

$$\text{“sec}(A + B) \equiv \text{sec } A + \text{sec } B, \text{ for all } A \text{ and } B\text{”}$$

is false

(2)

- (ii) Prove that

$$\tan \theta + \cot \theta \equiv 2 \operatorname{cosec} 2\theta, \quad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}.$$

(5)

6. (a) Prove that

$$\frac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan \theta, \quad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}.$$

(3)

- (b) Solve, giving exact answers in terms of  $\pi$ ,

$$2(1 - \cos 2\theta) = \tan \theta, \quad 0 < \theta < \pi.$$

(6)

7. Given that  $y = \log_a x$ ,  $x > 0$ , where  $a$  is a positive constant,

- (a) (i) express  $x$  in terms of  $a$  and  $y$ ,

(1)

- (ii) deduce that  $\ln x = y \ln a$ .

(1)

- (b) Show that  $\frac{dy}{dx} = \frac{1}{x \ln a}$ .

(2)

The curve  $C$  has equation  $y = \log_{10} x$ ,  $x > 0$ . The point  $A$  on  $C$  has  $x$ -coordinate 10. Using the result in part (b),

- (c) find an equation for the tangent to  $C$  at  $A$ .

(4)

The tangent to  $C$  at  $A$  crosses the  $x$ -axis at the point  $B$ .

- (d) Find the exact  $x$ -coordinate of  $B$ .

(2)

8. The curve with equation  $y = \ln 3x$  crosses the  $x$ -axis at the point  $P(p, 0)$ .

(a) Sketch the graph of  $y = \ln 3x$ , showing the exact value of  $p$ . (2)

The normal to the curve at the point  $Q$ , with  $x$ -coordinate  $q$ , passes through the origin.

(b) Show that  $x = q$  is a solution of the equation  $x^2 + \ln 3x = 0$ . (4)

(c) Show that the equation in part (b) can be rearranged in the form  $x = \frac{1}{3}e^{-x^2}$ . (2)

(d) Use the iteration formula  $x_{n+1} = \frac{1}{3}e^{-x_n^2}$ , with  $x_0 = \frac{1}{3}$ , to find  $x_1, x_2, x_3$  and  $x_4$ . Hence write down, to 3 decimal places, an approximation for  $q$ . (3)

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9.

**Figure 3**

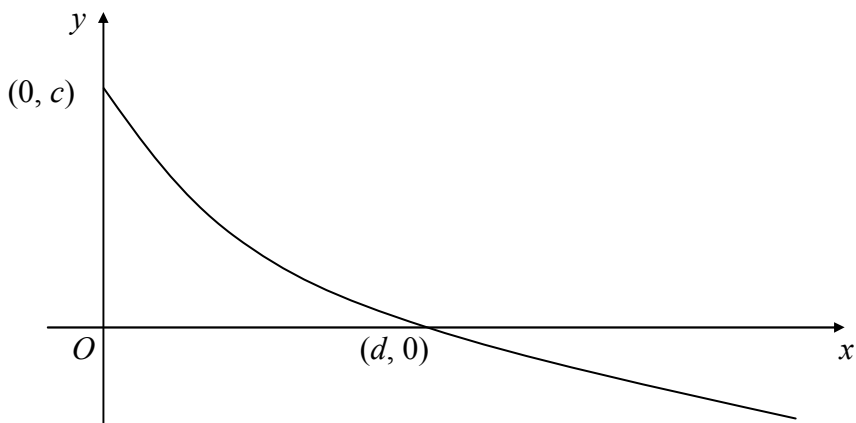


Figure 3 shows a sketch of the curve with equation  $y = f(x)$ ,  $x \geq 0$ . The curve meets the coordinate axes at the points  $(0, c)$  and  $(d, 0)$ .

In separate diagrams sketch the curve with equation

(a)  $y = f^{-1}(x)$ , (2)

(b)  $y = 3f(2x)$ . (3)

Indicate clearly on each sketch the coordinates, in terms of  $c$  or  $d$ , of any point where the curve meets the coordinate axes.

Given that  $f$  is defined by

$$f : x \mapsto 3(2^{-x}) - 1, \quad x \in \mathbb{R}, \quad x \geq 0,$$

(c) state

(i) the value of  $c$ ,

(ii) the range of  $f$ . (3)

(d) Find the value of  $d$ , giving your answer to 3 decimal places. (3)

The function  $g$  is defined by

$$g : x \mapsto \log_2 x, \quad x \in \mathbb{R}, \quad x \geq 1.$$

(e) Find  $fg(x)$ , giving your answer in its simplest form. (3)

**END**