A-LEVEL
MATHEMATICS
Pure Core 3 – MPC3
Mark scheme

6360
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Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1	$y(0)=0$ $y\left(\frac{\pi}{4}\right)=0.6266(57068)$ $y\left(\frac{\pi}{2}\right)=1.253(314137)$ $y\left(\frac{3\pi}{4}\right)=1.085(401882)$ $y(\pi)=0$	B1 B1		All 5 x-values*, PI by 5 correct y-values All 5 y-values exact** or correct to at least 4SF (rounded or truncated)
	$\frac{1}{3}\times\frac{\pi}{4}\{0+0+4[0.6267+1.0854]+2[1.2533]\}$	M1		Correct use of Simpson's rule using $\frac{1}{3}\times\frac{\pi}{4}$ (or 0.26...) and 4 and 2 correctly with their 5 y-values from any x-values
	= 2.449	A1	4	CAO (must be exactly this value)
	Total		**4**	

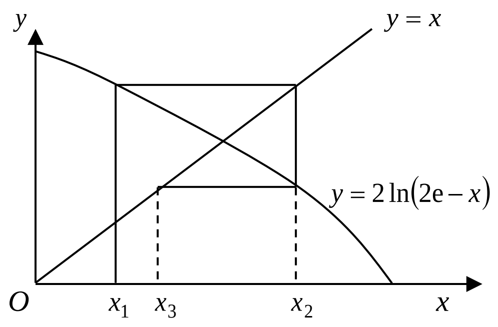
* Accept decimals 0.78(5398...), 1.5(7079...), 2.3(5619...), 3.1(4159...)

** $y\left(\frac{\pi}{4}\right)=\left(\frac{\pi}{4}\right)^{\frac{1}{2}}\sin\frac{\pi}{4}$, etc.

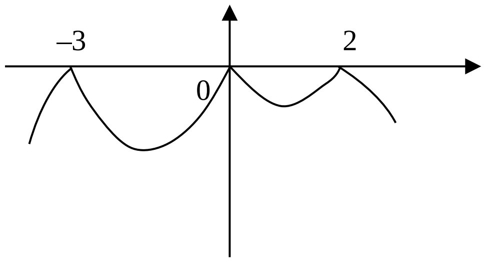
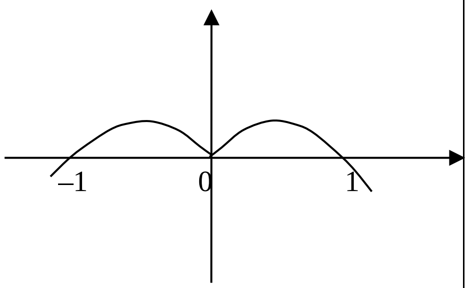
The minimum evidence for M1 is the 3 correct non-zero values of y in any form **and** sight of 2.4490(97...), but condone omission of the two zeros.

If a candidate's calculator setting is in degrees, they may earn the first B1 for 0, $\frac{\pi}{4}$, etc, and then B0, but M1 is available .

NMS: An answer of 2.449 without anything else gains 0/4.

Q	Solution	Mark	Total	Comment
2(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\quad -2\times\frac{1}{(2\mathrm{e}-x)}$ *or* $\frac{-2(2\mathrm{e}-x)}{(2\mathrm{e}-x)^2}$	M1		M1 for $\frac{k}{(2\mathrm{e}-x)}$ or $k(2\mathrm{e}-x)^{-1}, k\in\mathbb{R}$
		A1	2	OE, all correct
(b)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\quad -2\times\frac{1}{(2\mathrm{e}-\mathrm{e})}\quad\left(=-\frac{2}{\mathrm{e}}\right)$	M1		Substituting e for x in **their** $\frac{\mathrm{d}y}{\mathrm{d}x}$ PI
	Gradient of the normal $=-\frac{2\mathrm{e}-\mathrm{e}}{-2}\quad\left(=\frac{\mathrm{e}}{2}\right)$	m1		Must have also earned M1 in part (a)
	$y=2\ln(2\mathrm{e}-\mathrm{e})\quad(=2)$	B1		
	$y-2=\frac{\mathrm{e}}{2}(x-\mathrm{e})$ or $y=\frac{\mathrm{e}}{2}x-\frac{\mathrm{e}^2}{2}+2$	A1	4	OE, but must have simplified the gradient and replaced ln(2e–e) with 1
(c)(i)	$[\mathrm{f}(x)=]\ 2\ln(2\mathrm{e}-x)-x$ *or* $[\mathrm{g}(x)=]\ x-2\ln(2\mathrm{e}-x)$ $\mathrm{f}(1)=1.98$ or $\mathrm{g}(1)=-1.98$ $\mathrm{f}(3)=-1.22$ or $\mathrm{g}(3)=1.22$	M1		Must have both values correct rounded or truncated to 1 sf. Allow f(1) > 0 and f(3) < 0 only if f(x) is defined. **OR** evaluating **both** sides of $2\ln(2\mathrm{e}-x)=x$: $2\ln(2\mathrm{e}-1)=2.98\quad x=1$ $2\ln(2\mathrm{e}-3)=1.78\quad x=3$ (M1) $2.98>1$ **and** $1.78<3\ \Rightarrow\ 1<\alpha<3$ (A1)
	Change of sign $\Rightarrow\ 1<\alpha<3$	A1	2	All working must be correct together with correct statement
(ii)	$x_2=2.980\quad(2.97976...)$	B1		Not 2.98
	$x_3=1.798\quad(1.7977...)$	B1	2	If B0, B0 scored but both values given correct to 3 sf or more than 3 dp, then SC1.
(iii)		M1		Vertical line from x_1 to curve(condone omission from x - axis to $y=x$) and then horizontal line from the curve to $y=x$ *
		A1	2	Second vertical and horizontal lines * and x_2 , x_3(or the values) must be labelled on x - axis **
	Total		**12**	
c(iii)	* On diagram, the solid lines may be dotted and the dotted lines need not be shown. ** Condone correct values (unrounded or 3 dp) marked on the x-axis instead of x_2 and x_3.			

Q	Solution	Mark	Total	Comment
3(a)(i)	$k\,x\left(x^2+1\right)^{\frac{3}{2}}$ or $k\,x\,u^{\frac{3}{2}}$	M1		Attempted use of the chain rule
	$\frac{5}{2}\times 2x\left(x^2+1\right)^{\frac{3}{2}}$	A1	2	OE, all correct
(ii)	$2e^{2x}$	B1		Differentiating e^{2x} correctly
	$\left(\frac{dy}{dx}=\right)\quad 2e^{2x}\left(x^2+1\right)^{\frac{5}{2}}+e^{2x}\left(\text{their part (a)(i)}\right)$	M1		OE
	$\left(\frac{dy}{dx}=\right)\quad 2e^{2x}\left(x^2+1\right)^{\frac{5}{2}}+e^{2x}\frac{5}{2}\times 2x\left(x^2+1\right)^{\frac{3}{2}}$			
	$\left(\text{When } x=0\ \right)\quad \frac{dy}{dx}=2$	A1	3	Substituting $x = 0$ and CSO
(b)	$\frac{4\left(x^2+1\right)-2x\left(4x-3\right)}{\left(x^2+1\right)^2}$	M1		M1 for $\frac{\pm4\left(x^2+1\right)\pm 2x\left(4x-3\right)}{\left(x^2+1\right)^2}$
		A1		All correct
	$-4x^2+6x+4\ (=0)$ *or* $2x^2-3x-2\ (=0)$	m1		Forming a quadratic equation with all terms on one side $ax^2+bx+c\,(=0)$ $b\neq 0,\ c\neq 0$
	$(2x+1)(x-2)\ (=0)$	A1		OE correct factors or using the formula as far as $x=\frac{3\pm\sqrt{25}}{4}$ or completing the square as far as $x-\frac{3}{4}=\pm\sqrt{\frac{25}{16}}$
	$x=2\ ,\quad x=-\frac{1}{2}$	A1	5	CAO, simplified answers
	Total		**10**	

Q	Solution	Mark	Total	Comment
4(a)		M1		Reflection in the x-axis for the positive f(x) and the remainder as given in the sketch.
		A1		Correct $-3 < x < 2$ with minimum at $x < 0$ lower than minimum at $x > 0$ and correct cusps at $x = -3, 0, 2$.
		A1	3	Correct branches for $x > 2$ and $x < -3$, including the curvature of both branches and 2 and –3 marked *
(b)		M1		Symmetrical about the y-axis using only the original curve for $x>0$
		A1	2	–1 and 1 labelled on the x-axis and correct cusp at $x=0$
(c)(i)*	Stretch (I) s.f. $\frac{1}{2}$ (II) // x-axis (III)	M1		(I) and either (II) or (III)
		A1		(I) and (II) and (III)
	(followed by) Translation	E1		Not 'shift', 'move', etc.
	$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$	B1	4	Or in words Not (–1, 0)
(ii)	(1, –3) Or $x = 1$, $y = -3$	B1 B1	2	B1 for each coordinate
	Total		**11**	

(a) * The two A1 marks are independent. Condone straight lines for the branches for $x>2$ and $x<-3$ but not curves which are concave upward.

***(c)(i)** Alternative:

Translation E1

$\begin{bmatrix} -2 \\ 0 \end{bmatrix}$ B1

(followed by)

Stretch (I)

s.f. $\frac{1}{2}$ (II)

// $x-$axis (III)

M1 : (I) and either (II) or (III) A1: (I), (II) and (III)

Q	Solution	Mark	Total	Comment
5(a)		M1		$f(x) > -4$, *or* $****** \geq -4$,
	$f(x) \geq -4$	A1	2	
(b)	$y = (x-3)^2 - 4$			
	$x - 3 = (\pm)\sqrt{y+4}$	M1		
	$x = 3 \pm \sqrt{y+4}$	A1		condone $x = 3 + \sqrt{y+4}$
	$y = 3 \pm \sqrt{x+4}$	B1		interchanging *x* and *y* at any stage
	$\left(f^{-1}(x) =\right) \quad 3 + \sqrt{x+4}$	A1	4	negative clearly rejected. must have $\pm$ earlier.
(c)(i)	$\left(gf(x) =\right) \quad \lvert x^2 - 6x + 5 - 6 \rvert$ or $\lvert x^2 - 6x - 1 \rvert$	B1	1	
(ii)	'*their* $x^2 - 6x + 5 - 6$' $= 6$	M1		and attempt to solve 3 term quadratic
	'*their* $x^2 - 6x + 5 - 6$' $= -6$	M1		and attempt to solve 3 term quadratic
	$x = 7$, $x = -1$, $x = 5$, $x = 1$	A1		all four solutions seen and correct
	$x = 5, \quad x = 7$	E1	4	values 1 and –1 clearly rejected
	Total		**11**	

(a) $f(x) > -4$, $f \geq -4$, ≥ -4 , $x \geq -4$, range ≥ -4 , $y \geq -4$ score M1 only
$y > -4$, etc scores M0 (two errors)

(b) Alternative

$y = x^2 - 6x + 5$

$x^2 - 6x + (5 - y) = 0$

$x = \dfrac{6 \pm \sqrt{36 - 4(5-y)}}{2}$ correctly solving M1

$x = \dfrac{6 \pm \sqrt{16 + 4y}}{2}$ A1

B1 for swapping *x* and *y* and A1 for $\dfrac{6 + \sqrt{16 + 4x}}{2}$ having rejected minus sign

Q	Solution	Mark	Total	Comment
6(a)	$\int x^2 \sin 2x \, dx$			
	$u = x^2 \quad \frac{du}{dx} = 2x$	M1		$\frac{du}{dx} = kx, k = 1 \text{ or } 2$ and $v = p\cos 2x$ $p = \pm 1, \pm 2, \pm 0.5$
	$\frac{dv}{dx} = \sin 2x \quad v = -\frac{1}{2}\cos 2x$	A1		All correct
	$\left(\int x^2 \sin 2x \, dx =\right)$ $-\frac{1}{2}x^2 \cos 2x + \int x\cos 2x \,(dx)$	A1F		Correct substitution of **their** terms into parts formula
	$u = x \quad \frac{du}{dx} = 1$ $\frac{dv}{dx} = \cos 2x \quad v = \frac{1}{2}\sin 2x$	m1		Correct follow through unsimplified from their first integral above
	$\left(\int x^2 \sin 2x \, dx = \right)$ $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x - \frac{1}{2}\int \sin 2x (dx)$	A1		Correct
	$= -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{2}\times\frac{1}{2}\cos 2x + C$	A1		OE, **must** have constant of integration
			6	
(b)	$(V =) \quad \pi\int_0^{\frac{\pi}{2}} x^2 \sin 2x \, dx$	B1		Fully correct including dx and limits
	$= (\pi)\left[-\frac{1}{2}\left(\frac{\pi}{2}\right)^2 \cos\pi + \frac{1}{2}\left(\frac{\pi}{2}\right)\sin\pi + \frac{1}{4}\cos\pi - \frac{1}{4}\right]$	M1		Attempt at $F\left(\frac{\pi}{2}\right) - F(0)$ FT their expression from part (a)
	$= \pi\left(\frac{\pi^2}{8} - \frac{1}{2}\right)$	A1		OE in exact form with $\cos\pi$ and $\sin\pi$ evaluated
			3	
	Total		**9**	

Q	Solution	Mark	Total	Comment
7	$\frac{\mathrm{d}u}{\mathrm{d}x} = -3x^2$ or $\mathrm{d}u = -3x^2\ \mathrm{d}x$ and substituting for dx and x in terms of u	M1		Condone $\frac{\mathrm{d}u}{\mathrm{d}x} = 3x^2$ or $\mathrm{d}u = 3x^2\mathrm{d}x$ for M1
	$\int \frac{-(3-u)}{3u}\,\mathrm{d}u$	A1		OE correct unsimplified integral in terms of u only with du seen on this line or later
	$= \int\left(\frac{1}{3} - \frac{1}{u}\right)(\mathrm{d}u)$	A1		PI by the next line
	$= \left[\frac{u}{3} - \ln u\right]_{(3)}^{(2)}$	A1F		FT on their $\int\left(a + \frac{b}{u}\right)\,\mathrm{d}u$
	$= \left[\frac{2}{3} - \ln 2 - \left(\frac{3}{3} - \ln 3\right)\right]$	m1		Correct use of correct limits in u for expression of form au +$b$ln$u$ or in terms of x
	$-\ln 2 + \ln 3 - \frac{1}{3}$ *or* $\ln\frac{3}{2} - \frac{1}{3}$	A1	6	OE exact value
	Total		**6**	

Q	Solution	Mark	Total	Comment
8(a)	$\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}=\frac{(1-\sin x)^2+\cos^2 x}{\cos x(1-\sin x)}$	M1		Combining fractions correctly
	$=\frac{1-2\sin x+\sin^2 x+\cos^2 x}{\cos x(1-\sin x)}$			
	$=\frac{1-2\sin x+1}{\cos x(1-\sin x)}$	m1		Using $\sin^2 x+\cos^2 x=1$
	$=\frac{2-2\sin x}{\cos x(1-\sin x)}$ *or* $\frac{2(1-\sin x)}{\cos x(1-\sin x)}$	A1		Must have factorised denominator
	$=\frac{2}{\cos x}$			
	$=2\sec x$	A1	4	AG, both expressions seen
(b)	$\tan^2 x-2=2\sec x$			
	$\sec^2 x-1-2=2\sec x$			Using $\tan^2 x=\sec^2 x-1$, OE
	$\sec^2 x-2\sec x-3\,(=0)$	B1		Or $3\cos^2 x+2\cos x-1\,(=0)$
	$(\sec x-3)(\sec x+1)\quad(=0)$	M1		Correctly factorising their expression or substituting into formula
	$\sec x=3$ *or* -1	A1		Or $\cos x=\frac{1}{3}$ *or* -1
	$\sec x=3 \Rightarrow x=71°,\ 289°$	B1 B1		no extras inside the interval $0\le x<360°$,−1 EE
	$\sec x=-1 \Rightarrow x=180°$	B1	6	
(c)	$2\theta-30°=70.5°,\ 180°,\ 289.5°$	M1		For RHS accept any *x*-value from part (b) PI
	$\theta=50°,\ 105°,\ 160°$	A1	2	Allow $51°,\ 105°,\ 160°$
	Total		**12**	
	TOTAL		**75**	

(b) $x=70°$ and $290°$ scores B0 B0
AWRT $x=71°$ and $289°$ both not given to the nearest degree earns **SC1**.

(c) Condone correct answers not given to the nearest degree **if already penalised in part (b)**,
AWRT $\theta=50°$ *or* 51°, 105°, 160°