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General Certificate of Education (A-level)
June 2013

Mathematics **MPC3**

(Specification 6360)

Pure Core 3

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

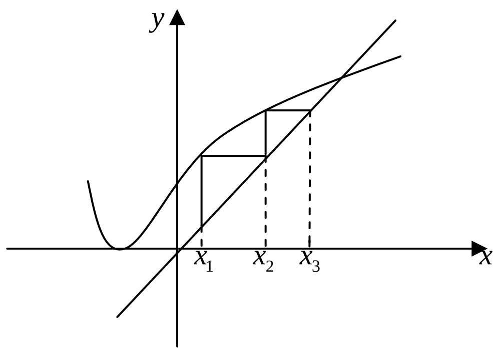
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

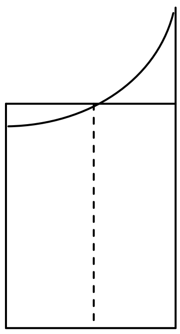
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

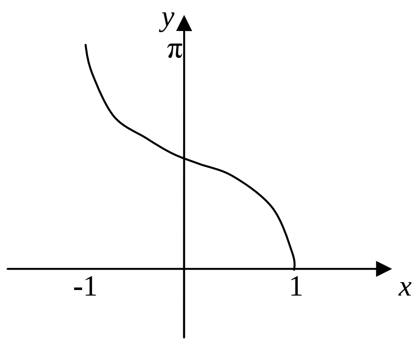
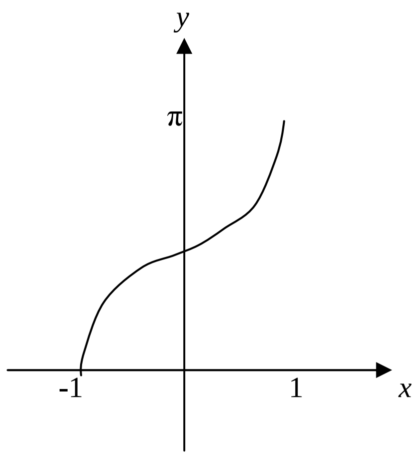
Otherwise we require evidence of a correct method for any marks to be awarded.

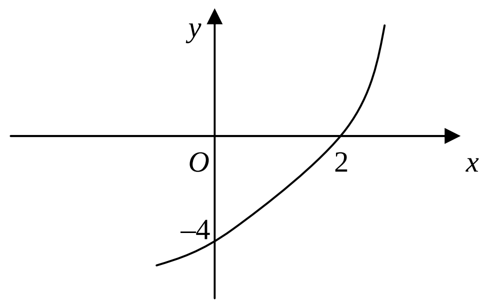
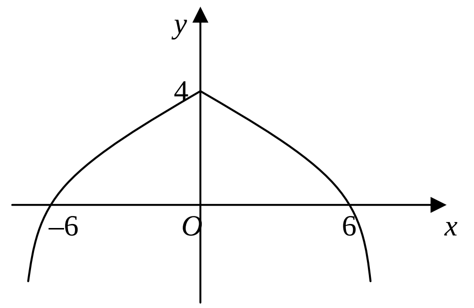
Q	Solution	Marks	Total	Comments
1(a)	$(2x-3=x)$			
	$x=3$	B1		
	$2x-3=-x$	M1		or $-(2x-3)=x$ or $-2x+3=x$
	$x=1$	A1	3	
(b)	$(\|2x-3\|\geq\|x\|)$			No ISW in part(b), ***mark their final line as their answer***.
	$x\leq 1$	B1		Or $1\geq x$
	$x\geq 3$	B1	2	Or $3\leq x$ Or "$x\leq 1$ *or* $x\geq 3$" for B1 B1
	Total		**5**	
2(a)	$(y=x^4\tan 2x)$ $\left(\frac{dy}{dx}=\right)$ $4x^3\tan 2x+x^4 2\sec^2 2x$	M1		$4x^3\tan 2x+Ax^4\sec^2 kx$ OE where A is a non-zero constant.
		A1		A1 for k = 2 may have $(\sec 2x)^2$ or $\frac{1}{\cos^2 2x}$
		A1	3	A1 all correct ISW if attempt to simplify is incorrect.
(b)	$\left(\frac{dy}{dx}=\right)\frac{\pm 2x(x-1)\pm 1(x^2)}{(x-1)^2}$	M1 A1		Use of the quotient rule $\frac{2x(x-1)-1(x^2)}{(x-1)^2}$
	$\left(=\frac{x^2-2x}{(x-1)^2}\right)$			**Simplification not required**
	$\left(\frac{dy}{dx}=\right)\frac{3}{4}$ or 0.75 OE	A1	3	Obtained from correct $\frac{dy}{dx}$
	Total		**6**	

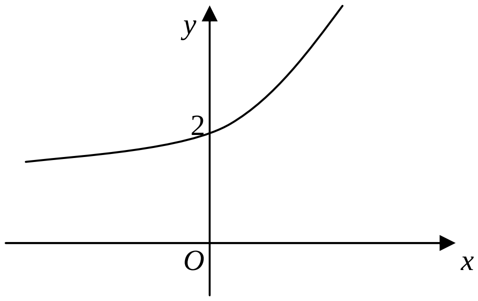
Q	Solution	Marks	Total	Comments
3(a)	f(3) $= -0.2(18)$ f(4) $= 0.01(83)$	M1		$\mathrm{f}(x) = \mathrm{e}^{-x} - 2 + \sqrt{x}$ Both values correct
	Change of sign $\Rightarrow$ $3 < \alpha < 4$	A1	2	Must have **both statement and interval** in words or symbols.
(b)	$\left(x_{n+1} = (2 - \mathrm{e}^{-x_n})^2 \quad x_1 = 3.5\right)$ $\left(x_2 = 3.8801...\right)$ $x_2 = 3.880$	B1		Do **not** accept 3.88
	$\left(x_3 = 3.9178...\right)$ $x_3 = 3.918$	B1	2	Do **not** accept 3.917
(c)		M1		Staircase to curve from x_1 including at least two stairs between curve and line $y = x$.
		A1	2	x_2 and x_3 marked on the x-axis . Do **not** accept marking on the Curve or on the line.
	Total		**6**	

Q	Solution	Marks	Total	Comments
4	$\left(8\sec x - 2\sec^2 x = \tan^2 x - 2\right)$			
	$8\sec x - 2\sec^2 x = \sec^2 x - 1 - 2$	M1		Using $\tan^2 x = \sec^2 x - 1$ *and NOT replacing* $\sec^2 x$ with $1 + \tan^2 x$.
	$3\sec^2 x - 8\sec x - 3 (= 0)$	A1		
	$(3\sec x + 1)(\sec x - 3)(= 0)$	m1		Correct factors or correct use of quadratic equation formula or completing the square for 'their' equation. $\sec x - \frac{8}{6} = \pm\sqrt{\frac{64}{36} + 1}$
	Or $\sec x = \frac{8 \pm \sqrt{(-8)^2 - 4(3)(-3)}}{2(3)}$			
	$\sec x = 3, \; -\frac{1}{3}$ (or -0.33)	A1		Both correct.
	$\sec x = \frac{1}{\cos x}$	B1		PI
	$\left(\cos x = \frac{1}{3} \text{ or } 0.33\right)$			$\left(\sec x = -\frac{1}{3} \text{ is impossible}\right)$
	$x = 1.23, \; 5.05$	A1		One correct. Must have earned A1 for correct quadratic, but independent of the second A1.
		A1	7	Both correct and no extras in $0 < x < 2\pi$. CAO
	Total		**7**	

Q	Solution	Marks	Total	Comments
5(a)	x_i: $0.4(\frac{2}{5})$, $1.2(\frac{6}{5})$, $2(\frac{10}{5})$, $2.8(\frac{14}{5})$, $3.6(\frac{18}{5})$ y_i: 5.20231, 5.35985, 5.91608, 6.99657, 8.58231	B1		All 5 *x*-values correct, PI by 5 correct *y*-values.
		B1		At least 4 correct *y*-values rounded or truncated to at least 4 s.f. or in surd form $\sqrt{27+(0.4)^3}, \sqrt{27+(1.2)^3}$, etc. or $\sqrt{27.064}$, $\sqrt{28.728}$, etc. or sight of 32.057...
	$\int_0^4 \sqrt{27+x^3} \approx 0.8\sum_1^5 y_i$	M1		**Correct use of mid-ordinate rule** using 0.8 with candidate's 5 *y*-values. Dependent on first B1
	$(=0.8\times 32.057...)$			
	$=25.6$	A1	4	CAO (must be exactly this) **and no error seen**
(b)				Could be gained without answering part (a)
		B1		Diagram showing curve through the midpoint of the top of rectangle. May have one or more rectangles.
	"Smaller" OE	E1	2	Dependent on B1
	Total		**6**	

Q	Solution	Marks	Total	Comments
6(a)		B1		Correct sketch of $\cos^{-1} x$.
	$(-1,\ \pi)$ and $(1,\ 0)$	B1	2	Stated
(b)		B1		Correct sketch of $\pi - \cos^{-1} x$ Must touch negative *x*-axis.
	$(-1,\ 0)$ and $(1,\ \pi)$	B1	2	Stated
	Total		**4**	

Q	Solution	Marks	Total	Comments
7(a)		M1		Reflection in the *x*-axis.
		A1	2	Intersection with the *x*-axis and *y*-axis marked 2 and –4. Accept (2, 0) and (0, –4) instead of marking on the axes.
(b)		M1		Reflection of $0< x <6$ part in the *y*-axis giving two connected sections
		A1		Correct curve beyond ± 6, correct curvature and correct cusp at $x=0$ (generous)
		A1	3	±6 and 4 marked correctly Accept (6, 0), (0, 4) and (-6,0) instead of marking on the axes.
(c)	Reflection	M1		
	in the *y*-axis	A1		
	(followed by) 1) stretch 2) parallel to the *x*-axis 3) by factor 2	M1		1 and either 2 or 3
		A1	4	1, 2 and 3
	OR Vice versa			
	Total		**9**	

Q	Solution	Marks	Total	Comments
8(a)(i)	$f(x)=\ln(2x-3)$ $2x-3=e^y$ $2y-3=e^x$	M1 M1		Either order: M1 for antilog M1 for replacing f(x) or y with x
	$\left(f^{-1}(x)=\right)\ \frac{1}{2}(e^x+3)$ OE	A1	3	Correct expression in x
(ii)	$f^{-1}(x)>\frac{3}{2}$	B1	1	Do **not** condone $f^{-1}(x)\ge\frac{3}{2}$, $y>\frac{3}{2}$, $x>\frac{3}{2}$ range $>\frac{3}{2}$, $f^{-1}>\frac{3}{2}$
(iii)		M1 A1	2	Correct shape crossing y-axis and above x-axis 2 marked on the y-axis
(b)(i)	$\left(gf(x)=\right)\ e^{2\ln(2x-3)}-4$	M1		Correct composition
	$=e^{\ln(2x-3)^2}-4$	m1		PI by correct expression
	$=(2x-3)^2-4$	A1	3	
(ii)	$\left(fg(x)=\right)\ \ln(2(e^{2x}-4)-3)$	M1		OE correct composition
	$\ln(2e^{2x}-11)=\ln 5$ $2e^{2x}-11=5$ OE	A1		Correct antilog of correct equation
	$e^{2x}=8$ $2x=\ln 8$ $x=\frac{1}{2}\ln 8$	A1	3	OE **exact solution**, e.g. $\ln\sqrt{8}$ or $\frac{3}{2}\ln 2$ or $\ln 2^{\frac{3}{2}}$
	Total		**12**	

Q	Solution	Marks	Total	Comments
9				$V = \pi\int x^2 \mathrm{d}y$ $16x^2 - (y-8)^2 = 32$
	$x^2 = \frac{1}{16}(y-8)^2 + 2$	B1		OE
	$V = (\pi)\int_{(0)}^{(16)}\left(\frac{1}{16}(y-8)^2 + 2\right)(\mathrm{d}y)$	M1		Accept 'their' x^2 in terms of y Condone missing limits and π **wherever bracketed**
	$V = (\pi)\left[\frac{1}{16}\times\frac{1}{3}(y-8)^3 + 2y\right]_{(0)}^{(16)}$	A1		OE, for correct integration of correct integrand
	$V = (\pi)\left[\frac{1}{16}\times\frac{1}{3}(16-8)^3 + 2(16) - \frac{1}{16}\times\frac{1}{3}(-8)^3\right]$	A1		OE, correct use of correct limits in correct expression, PI by correct answer.
	$V = \frac{160}{3}\pi$	A1	5	OE exact value, eg $\pi\, 53\frac{1}{3}$ or $\pi\, 53.\dot{3}$ or $\frac{2560}{48}\pi$
	Total		**5**	

Q	Solution	Marks	Total	Comments
10(a)(i)	$u=\ln x \quad \frac{dv}{dx}=1$ $\frac{du}{dx}=\frac{1}{x} \quad v=x$	M1 A1		$\frac{d\ln x}{dx}$ & $\int dx$ attempted All correct
	$\left(\int \ln x\,dx=\right) \quad x\ln x-\int x\times\frac{1}{x}(dx)$	m1		Correct substitution of their terms into parts
	$=x\ln x-x+C$	A1	4	All correct (**constant needed**)
(ii)	$u=(\ln x)^2 \quad \frac{dv}{dx}=1$ $\frac{du}{dx}=(2\ln x)\frac{1}{x} \quad v=x$	M1 A1		$\frac{d(\ln x)^2}{dx}$ & $\int dx$ attempted All correct
	$\left(\int (\ln x)^2 dx=\right) \quad x(\ln x)^2-\int x\times\frac{2}{x}\ln x\,(dx)$	m1		OE correct substitution of their terms into parts
	$=x(\ln x)^2-2(x\ln x-x)+C$ OE	A1	4	All correct (constant needed) including correct use of brackets. **Do not penalise missing constant if already penalised in part (i)** ISW
(b)	$\frac{du}{dx}=\frac{1}{2\sqrt{x}}$ or $\frac{1}{2}x^{-\frac{1}{2}}$ $(dx=2u\,du)$	B1		$u=\sqrt{x}$
	$\int_{(1)}^{(4)}\frac{1}{x+\sqrt{x}}dx=\int_{(1)}^{(2)}\frac{1}{u^2+u}2u\ (du)$	M1		All in terms of u including attempt at replacing dx (not simply writing du), condone missing limits and du
		A1		Integrand correct unsimplified
	$=2\int_{(1)}^{(2)}\frac{1}{u+1}\ (du)$	A1		
	$=2\ln(u+1)\Big\vert_{(1)}^{(2)}$	A1F		FT *their* $\int\frac{k}{u+1}(du)$
	$=2\ln(2+1)-2\ln(1+1)$ or $2\ln(\sqrt{4}+1)-2\ln(\sqrt{1}+1)$	A1F		correct use of correct limits on $k\ln(u+1)$ or $k\ln\left(\sqrt{x}+1\right)$
	$=2\ln\frac{3}{2}$ *or* $\ln\frac{9}{4}$ *or* $2\ln 3-2\ln 2$	A1	7	OE ISW
	Total		**15**	
	TOTAL		**75**	