Version 1,0

General Certificate of Education (A-level)
June 2012

Mathematics MPC3

(Specification 6360)

Pure Core 3

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

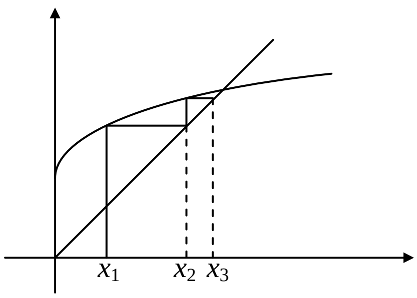
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

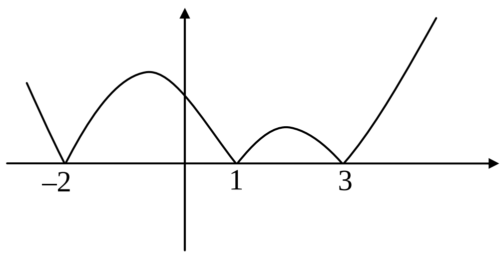
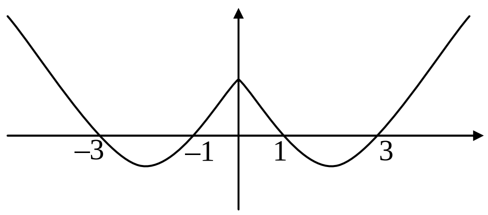
Q	Solution	Marks	Total	Comments
1	x : 0.5, 0.7, 0.9, 1.1 y : 3.9163, 1.8748, 0.9520, 0.3773	B1 M1		All 4 correct x values (and no extras used) 3+ y decimal values rounded or truncated to 2 dp or better (in table or in formula) (PI by correct answer)
	$\int = 0.2 \times \sum y$	m1		Correct substitution of their 4 y values (of which 3 are correct), either listed or totalled
	$(= 0.2 \times 7.12\ldots)$ $= 1.424$	A1	4	CAO
	Total		**4**	

Q	Solution	Marks	Total	Comments
2(a)	$\mathrm{f}(x) = 4\ln x - \sqrt{x}$			Or reverse
	$\left.\begin{array}{l}\mathrm{f}(0.5) = -3.5\\ \mathrm{f}(1.5) = 0.4\end{array}\right\}$ must have both values correct	M1		Allow $\mathrm{f}(0.5) < 0$ and $\mathrm{f}(1.5) > 0$ only if f(x) defined
	Change of sign $\therefore\ 0.5 < \alpha < 1\cdot5$	A1	2	f(x) must be defined and all working correct, including both statement and interval (either may be written in words or symbols) **OR** comparing 2 sides: $\left.\begin{array}{ll}4\ln 0.5 = -2.8 & \sqrt{0.5} = 0.7\\ 4\ln 1.5 = 1.6 & \sqrt{1.5} = 1.2\end{array}\right\}$ (M1) At 0.5, LHS < RHS; at 1.5, LHS > RHS $\therefore 0.5 < \alpha < 1.5$ (A1)
(b)	$\ln x = \dfrac{\sqrt{x}}{4}$ or $x^4 = \mathrm{e}^{\sqrt{x}}$			Must be seen
	$x = \mathrm{e}^{\frac{\sqrt{x}}{4}}$	B1	1	AG; no errors seen
(c)	$x_2 = 1.193$	B1		
	$x_3 = 1.314$	B1	2	If B0B0 scored but either value seen correct to 2 or 4 dp, score SC1
(d)		M1		Vertical line from x_1 to curve (condone omission from x-axis to $y = x$) and then horizontal to $y = x$
		A1	2	2[nd] vertical and horizontal lines, and x_2, x_3 (not the values) must be labelled on x-axis
	Total		**7**	

Q	Solution	Marks	Total	Comments
3(a)	$\left(\dfrac{dy}{dx}=\right)\quad x^3\times\dfrac{1}{x}+3x^2\ln x$	M1		$px^3\times\dfrac{1}{x}+qx^2\ln x$ where p and q are integers
		A1	2	$p=1, q=3$
(b)(i)	$\left(\dfrac{dy}{dx}=\right)\quad e^2+3e^2\ln e \quad (=4e^2)$	M1		Substituting e for x in their $\dfrac{dy}{dx}$, but must have scored M1 in (a)
	$y=e^3\ln e \quad (=e^3)$	B1		
	$y-e^3=4e^2(x-e)$	A1	3	OE **but** must have evaluated ln e (twice) for this mark (must be in exact form, but condone numerical evaluation after correct equation)
(ii)	$-e^3=4e^2(x-e)$ or $4e^2x=3e^3$ OE	M1		Correctly substituting $y = 0$ into a correct tangent equation in (b)(i)
	$x=\dfrac{3}{4}e$	A1	2	CSO; ignore subsequent decimal evaluation
	Total		**7**	
4(a)	$\int xe^{6x}\,dx$			
	$u=x \quad \dfrac{dv}{(dx)}=e^{6x}$	M1		All 4 terms in this form, $k=\dfrac{1}{6}$, 1 or 6
	$\dfrac{du}{(dx)}=1 \quad v=ke^{6x}$	A1		$k=\dfrac{1}{6}$
	$\dfrac{1}{6}xe^{6x}-\int\dfrac{1}{6}e^{6x}\,(dx)$	A1F		Correct substitution of their terms into parts formula
	$=\dfrac{1}{6}xe^{6x}-\dfrac{1}{36}e^{6x}\,(+c)$ OE	A1	4	No ISW for incorrect simplification
(b)	$(V=)\quad \pi\int_0^1 xe^{6x}\,dx$	B1		Must include π, limits and dx
	$=(\pi)\left[\left(\dfrac{1}{6}e^6-\dfrac{1}{36}e^6\right)-\left(-\dfrac{1}{36}\right)\right]$	M1		Correct substitution of 0 and 1 into their answer in (a), must be of the form $axe^{6x}-be^{6x}$, where $a>0$, $b>0$ and F(1) – F(0) seen
	$=\pi\left[\dfrac{5}{36}e^6+\dfrac{1}{36}\right]$	A1	3	CAO; ISW
	Total		**7**	

Q	Solution	Marks	Total	Comments
5(a)	$\mathrm{f}(x) \geq 0$	M1 A1	2	$\mathrm{f}(x) > 0,\ \mathrm{f} \geq 0,\ x \geq 0,\ y > 0,\ \text{range} \geq 0$ Condone $y \geq 0$
(b)(i)	$\mathrm{fg}(x) = \sqrt{2\left(\frac{10}{x}\right) - 5}$ $\left(= \sqrt{\frac{20}{x} - 5}\right)$ OE	B1	1	No ISW
(ii)	$\sqrt{\frac{20}{x} - 5} = 5$ $\frac{20}{x} = 5^2 + 5$ $x = \frac{2}{3}$	M1 A1	2	Correctly squaring their fg(x) and correctly isolating their x term No ISW
(c)(i)	$y = \sqrt{2x - 5}$ $\left(\mathrm{f}^{-1}(x) =\right) \frac{x^2 + 5}{2}$	M1 M1 A1	3	Swap x and y / Correctly squaring } either order
(ii)	$x^2 = 9$ or if $\sqrt{9}$ or 3 seen $x = 3$ and $x = -3$ rejected	M1 A1	2	Candidate must have scored full marks in (c)(i) (ie no follow through) Must see both
	Total		**10**	

Q	Solution	Marks	Total	Comments
6	$u = x^4 + 2$			
	$\dfrac{du}{dx} = 4x^3$	B1		or $du = 4x^3\ dx$
	$\displaystyle\int \frac{x^7}{(x^4+2)^2}\ dx$			
	$= \displaystyle\int \frac{k(u-2)}{u^2}\ du$ or $\displaystyle\int \frac{k(u-2)^{\frac{7}{4}}}{u^2} \frac{du}{(u-2)^{\frac{3}{4}}}$	M1		Either expression all in terms of u including replacing dx, but condone omission of du
	$= \left(\dfrac{1}{4}\right) \displaystyle\int \frac{1}{u} - \frac{2}{u^2}\ du$	m1		$k\int au^{-1} + bu^{-2}\,du$, where k, a, b are constants
	$= \left(\dfrac{1}{4}\right)\left[\ln u + \dfrac{2}{u}\right]$	A1		Must have seen du on an earlier line where every term is a term in u
	$\left(\int = \left(\frac{1}{4}\right)\left[\ln u + \frac{2}{u}\right]_2^3\right)$			$\left(\left(\frac{1}{4}\right)\left[\ln\left(x^4+2\right) + \frac{2}{\left(x^4+2\right)}\right]_0^1\right)$
	$= \left(\dfrac{1}{4}\right)\left[\left(\ln 3 + \dfrac{2}{3}\right) - \left(\ln 2 + 1\right)\right]$	m1		Dependent on previous A1 Correct change of limits, correct substitution and F(3) – F(2) or correct replacement of u, correct substitution and F(1) – F(0)
	$= \dfrac{1}{4}\ln\left(\dfrac{3}{2}\right) - \dfrac{1}{12}$	A1	6	OE in exact form
	Total		**6**	

Q	Solution	Marks	Total	Comments
7(a)		M1		Modulus graph, 4 sections touching x-axis at $-2, 1, 3$
		A1		Correct $x > 3$, $x < -2$
		A1	3	Correct $-2 \le x \le 3$ with maximum at 2 lower than maximum at -1 and correct cusps at $x = -2$, $x = 1$ and $x = 3$ The maximums need to be at $x = -1$ and 2 (approx)
(b)		M1		Symmetrical about y-axis, from original curve for $0 < x < 1$ and $x > 3$
		A1	2	Correct graph including cusp at $x = 0$
(c)	Translate $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$	E1 B1		
	Stretch (I) sf $\frac{1}{2}$ (II) $//$ y-axis (III) either order	M1		I and (either II or III)
		A1	4	I + II + III
(d)	$x = -2$	B1		
	$y = 5$	B1	2	Each value may be stated or shown as coordinates
	Total		**11**	

Q	Solution	Marks	Total	Comments
8(a)	LHS = $\dfrac{(1-\cos\theta)+(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$	M1		Combining fractions
	$= \dfrac{2}{1-\cos^2\theta}$	A1		Correctly simplified
	$= \dfrac{2}{\sin^2\theta}$	m1		Use of $\sin^2\theta+\cos^2\theta=1$
	$2\,\text{cosec}^2\theta=32$ $\text{cosec}^2\theta=16$	A1	4	AG; no errors seen **OR** $1-\cos\theta+1+\cos\theta=32(1+\cos\theta)(1-\cos\theta)$ (M1) $2=32(1-\cos^2\theta)$ (A1) $2=32\sin^2\theta$ (m1) $\text{cosec}^2\theta=16$ (A1)
(b)	$\text{cosec}\, y=(\pm)\sqrt{16}$ or better (PI by further working)	M1		or $\sin y=(\pm)\sqrt{\dfrac{1}{16}}$ or better
	(y =) 0.253, (2.889,) (3.394,) (6.031,) (–0.253)	B1		Sight of any of these correct to 3dp or better
	(y =) 0.25, 2.89, 3.39 (or better)	A1		Must see these 3 answers, with or without either/both of −0.25 or 6.03 Ignore answers outside interval −0.25 to 6.03 but extras in this interval scores A0
	x = 0.43, 1.74, 2(.00), 0.17	B1 B1	5	3 correct (must be 2 dp) All 4 correct (must be 2 dp) and no extras in interval (ignore answers outside interval)
	Total		**9**	

Q	Solution	Marks	Total	Comments
9(a)	$\left(\frac{\mathrm{d}x}{\mathrm{d}y} = \right) \quad \frac{\cos y \times \cos y - \sin y \times -\sin y}{\cos^2 y}$	M1		Condone incorrect signs, poor notation, omission of $\frac{\mathrm{d}x}{\mathrm{d}y}$ or using $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$= \frac{\cos^2 y + \sin^2 y}{\cos^2 y}$	A1		RHS correct with terms squared, including correct notation Must see this line
	$= \frac{1}{\cos^2 y}$ or $\left(= 1 + \tan^2 y\right)$			Must see one of these
	$\frac{\mathrm{d}x}{\mathrm{d}y} = \sec^2 y$	A1 CSO	3	AG; all correct including correct use of $\frac{\mathrm{d}x}{\mathrm{d}y}$ throughout
(b)	$\sec^2 y = 1 + (x-1)^2$	M1		Correct use of $\sec^2 y = 1 + \tan^2 y$ and in terms of x
	$= 1 + x^2 - 2x + 1$ OE			
	$= x^2 - 2x + 2$	A1	2	AG; must see "$\sec^2 y =$", $(x-1)^2$ expanded and no errors seen
(c)	$\frac{\mathrm{d}x}{\mathrm{d}y} = x^2 - 2x + 2$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sec^2 y}$			Must be seen
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x^2 - 2x + 2}$	B1	1	AG and no errors seen

Q	Solution	Marks	Total	Comments
9 cont (d)(i)	$y = \tan^{-1}(x-1) - \ln x$			
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) \frac{1}{x^2-2x+2} - \frac{1}{x}$	M1		Must be correct
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x} = 0\right)$			
	$\pm x^2 + bx + c \ (=0)$	m1		Expression in this form (generous), where b and $c \neq 0$
	$x^2 - 3x + 2 = 0$	A1		Must see correct equation = 0
	$x = 1,\ 2$	A1	4	Both answers must be seen The two A marks are independent
(ii)		M1		$y'' = p\left(x^2-2x+2\right)^{-2}(2x-2) \pm qx^{-2}$ where p and q are constants
	$y'' = -\left(x^2-2x+2\right)^{-2}(2x-2) + x^{-2}$	A1	2	$p = -1$, $q = 1$ including correct brackets
(iii)	$x = 1,\ y'' = 1$	M1		Must have scored full marks in (d)(i) and (ii)
	At $x=1,\ y''>0\ \therefore$ min When $x = 1$, $y = 0$ hence on x-axis	A1	2	Must see $y'' > 0$ or in words Both statements fully correct
	Total		**14**	
	TOTAL		**75**	