General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Friday 23 May 2008 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Find $\frac{dy}{dx}$ when:

(a) $y = (3x + 1)^5$; *(2 marks)*

(b) $y = \ln(3x + 1)$; *(2 marks)*

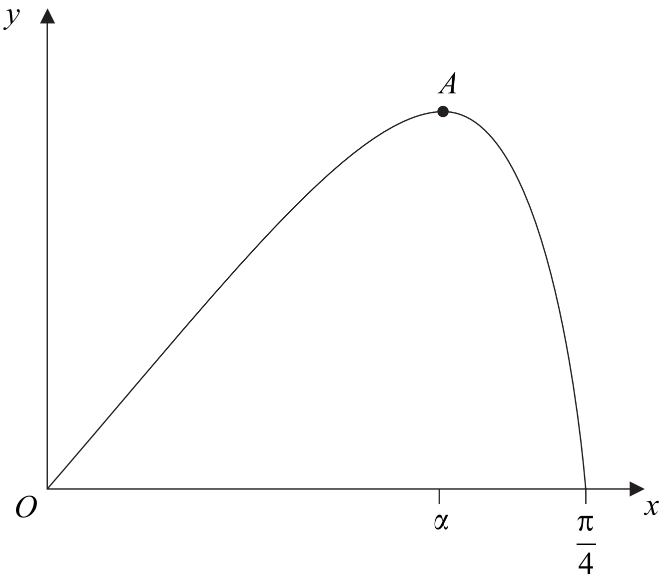
(c) $y = (3x + 1)^5 \ln(3x + 1)$. *(3 marks)*

2 (a) Solve the equation $\sec x = 3$, giving the values of x in radians to two decimal places in the interval $0 \leqslant x < 2\pi$. *(3 marks)*

(b) Show that the equation $\tan^2 x = 2\sec x + 2$ can be written as $\sec^2 x - 2\sec x - 3 = 0$. *(2 marks)*

(c) Solve the equation $\tan^2 x = 2\sec x + 2$, giving the values of x in radians to two decimal places in the interval $0 \leqslant x < 2\pi$. *(4 marks)*

3 A curve is defined for $0 \leqslant x \leqslant \frac{\pi}{4}$ by the equation $y = x\cos 2x$, and is sketched below.



(a) Find $\frac{dy}{dx}$. *(2 marks)*

(b) The point A, where $x = \alpha$, on the curve is a stationary point.

(i) Show that $1 - 2\alpha\tan 2\alpha = 0$. *(2 marks)*

(ii) Show that $0.4 < \alpha < 0.5$. *(2 marks)*

(iii) Show that the equation $1 - 2x\tan 2x = 0$ can be rearranged to become $x = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x}\right)$. *(1 mark)*

(iv) Use the iteration $x_{n+1} = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x_n}\right)$ with $x_1 = 0.4$ to find x_3, giving your answer to two significant figures. *(2 marks)*

(c) Use integration by parts to find $\int_0^{0.5} x\cos 2x \, dx$, giving your answer to three significant figures. *(5 marks)*

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4 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = x^2, \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{2x-3}, \qquad \text{for real values of } x,\ x \neq \frac{3}{2}$$

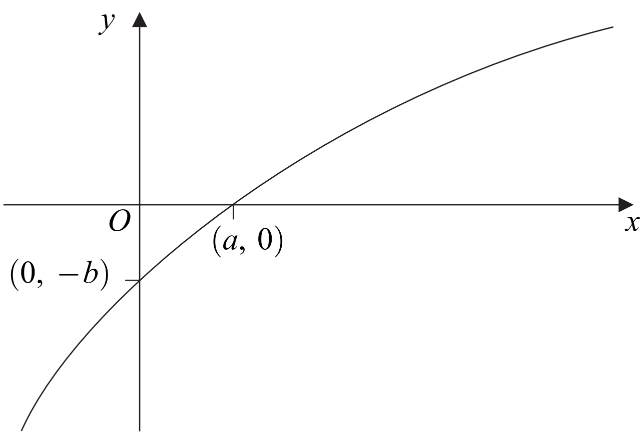
(a) State the range of f. *(1 mark)*

(b) (i) The inverse of g is g^{-1}. Find $\mathrm{g}^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

(c) Solve the equation $\mathrm{fg}(x) = 9$. *(3 marks)*

5 (a) The diagram shows part of the curve with equation $y = \mathrm{f}(x)$. The curve crosses the x-axis at the point $(a, 0)$ and the y-axis at the point $(0, -b)$.



On separate diagrams, sketch the curves with the following equations. On each diagram, indicate, in terms of a or b, the coordinates of the points where the curve crosses the coordinate axes.

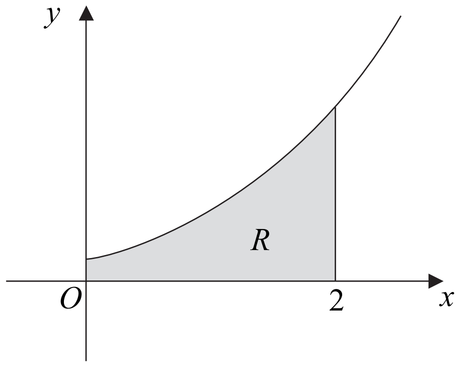
(i) $y = |\,\mathrm{f}(x)\,|$. *(2 marks)*

(ii) $y = 2\mathrm{f}(x)$. *(2 marks)*

(b) (i) Describe a sequence of geometrical transformations that maps the graph of $y = \ln x$ onto the graph of $y = 4\ln(x+1) - 2$. *(6 marks)*

(ii) Find the exact values of the coordinates of the points where the graph of $y = 4\ln(x+1) - 2$ crosses the coordinate axes. *(4 marks)*

6 The diagram shows the curve with equation $y = (e^{3x} + 1)^{\frac{1}{2}}$ for $x \geqslant 0$.



(a) Find the gradient of the curve $y = (e^{3x} + 1)^{\frac{1}{2}}$ at the point where $x = \ln 2$. *(5 marks)*

(b) Use the mid-ordinate rule with four strips to find an estimate for $\int_0^2 (e^{3x} + 1)^{\frac{1}{2}}\,dx$, giving your answer to three significant figures. *(4 marks)*

(c) The shaded region R is bounded by the curve, the lines $x = 0$, $x = 2$ and the x-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° about the x-axis. *(4 marks)*

7 (a) Given that $y = \dfrac{\sin\theta}{\cos\theta}$, use the quotient rule to show that $\dfrac{dy}{d\theta} = \sec^2\theta$. *(3 marks)*

(b) Given that $x = \sin\theta$, show that $\dfrac{x}{\sqrt{1 - x^2}} = \tan\theta$. *(2 marks)*

(c) Use the substitution $x = \sin\theta$ to find $\displaystyle\int \frac{1}{(1 - x^2)^{\frac{3}{2}}}\,dx$, giving your answer in terms of x. *(5 marks)*

END OF QUESTIONS

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