General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Monday 11 June 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Question 4 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

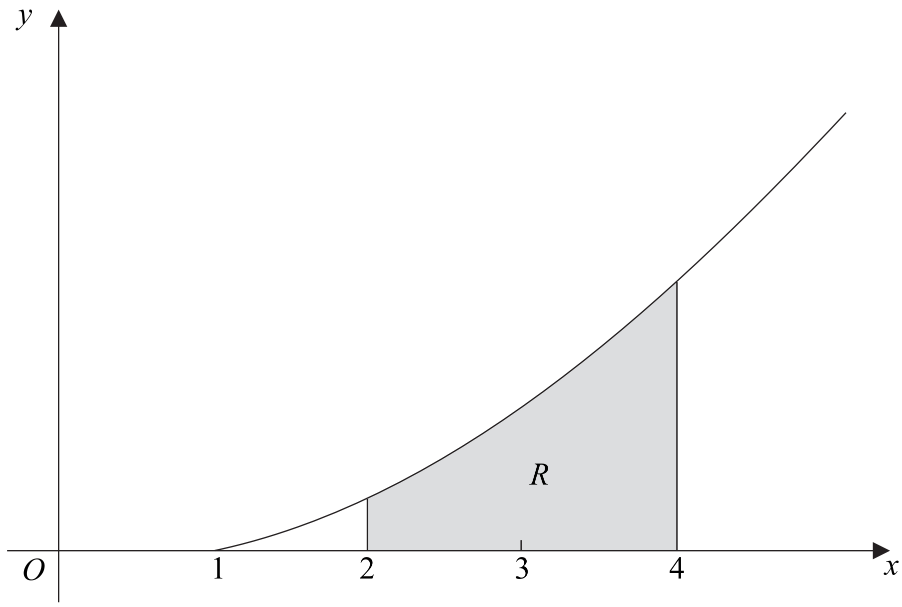
1 (a) Differentiate $\ln x$ with respect to x. *(1 mark)*

(b) Given that $y = (x+1)\ln x$, find $\dfrac{dy}{dx}$. *(2 marks)*

(c) Find an equation of the normal to the curve $y = (x+1)\ln x$ at the point where $x = 1$. *(4 marks)*

2 (a) Differentiate $(x-1)^4$ with respect to x. *(1 mark)*

(b) The diagram shows the curve with equation $y = 2\sqrt{(x-1)^3}$ for $x \geqslant 1$.



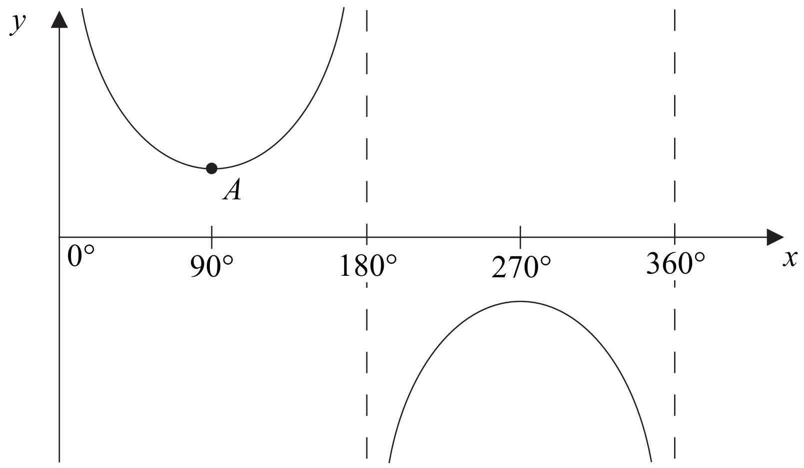
The shaded region R is bounded by the curve $y = 2\sqrt{(x-1)^3}$, the lines $x = 2$ and $x = 4$, and the x-axis.

Find the exact value of the volume of the solid formed when the region R is rotated through 360° about the x-axis. *(4 marks)*

(c) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sqrt{x^3}$ onto the graph of $y = 2\sqrt{(x-1)^3}$. *(4 marks)*

3 (a) Solve the equation $\operatorname{cosec} x = 2$, giving all values of x in the interval $0° < x < 360°$.
(2 marks)

(b) The diagram shows the graph of $y = \operatorname{cosec} x$ for $0° < x < 360°$.



(i) The point A on the curve is where $x = 90°$. State the y-coordinate of A.
(1 mark)

(ii) Sketch the graph of $y = |\operatorname{cosec} x|$ for $0° < x < 360°$. *(2 marks)*

(c) Solve the equation $|\operatorname{cosec} x| = 2$, giving all values of x in the interval $0° < x < 360°$.
(2 marks)

Turn over for the next question

4 [**Figure 1**, printed on the insert, is provided for use in this question.]

(a) Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to $\int_1^2 3^x \, dx$, giving your answer to three significant figures. *(4 marks)*

(b) The curve $y = 3^x$ intersects the line $y = x + 3$ at the point where $x = \alpha$.

(i) Show that α lies between 0.5 and 1.5. *(2 marks)*

(ii) Show that the equation $3^x = x + 3$ can be rearranged into the form

$$x = \frac{\ln(x+3)}{\ln 3}$$ *(2 marks)*

(iii) Use the iteration $x_{n+1} = \frac{\ln(x_n + 3)}{\ln 3}$ with $x_1 = 0.5$ to find x_3 to two significant figures. *(2 marks)*

(iv) The sketch on **Figure 1** shows part of the graphs of $y = \frac{\ln(x+3)}{\ln 3}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

5 The functions f and g are defined with their respective domains by

$$f(x) = \sqrt{x-2} \quad \text{for } x \geqslant 2$$

$$g(x) = \frac{1}{x} \quad \text{for real values of } x, \ x \neq 0$$

(a) State the range of f. *(2 marks)*

(b) (i) Find $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = 1$. *(3 marks)*

(c) The inverse of f is f^{-1}. Find $f^{-1}(x)$. *(3 marks)*

6 (a) Use integration by parts to find $\int x\mathrm{e}^{5x}\,\mathrm{d}x$. *(4 marks)*

(b) (i) Use the substitution $u = \sqrt{x}$ to show that

$$\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,\mathrm{d}x = \int \frac{2}{1+u}\,\mathrm{d}u$$ *(2 marks)*

(ii) Find the exact value of $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})}\,\mathrm{d}x$. *(3 marks)*

7 (a) A curve has equation $y = (x^2 - 3)\mathrm{e}^x$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

(ii) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. *(2 marks)*

(b) (i) Find the x-coordinate of each of the stationary points of the curve. *(4 marks)*

(ii) Using your answer to part (a)(ii), determine the nature of each of the stationary points. *(2 marks)*

8 (a) Write down $\int \sec^2 x\,\mathrm{d}x$. *(1 mark)*

(b) Given that $y = \frac{\cos x}{\sin x}$, use the quotient rule to show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\mathrm{cosec}^2 x$. *(4 marks)*

(c) Prove the identity $(\tan x + \cot x)^2 = \sec^2 x + \mathrm{cosec}^2 x$. *(3 marks)*

(d) Hence find $\int_{0.5}^{1} (\tan x + \cot x)^2\,\mathrm{d}x$, giving your answer to two significant figures. *(4 marks)*

END OF QUESTIONS

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Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
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AQA
ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MPC3**
Unit Pure Core 3

Insert

Insert for use in **Question 4**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 4)