Version

**General Certificate of Education (A-level)**
**January 2013**

# Mathematics MPC3

## (Specification 6360)

## Pure Core 3

# Final

# *Mark Scheme*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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## Key to mark scheme abbreviations

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| –*x* EE | deduct *x* marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

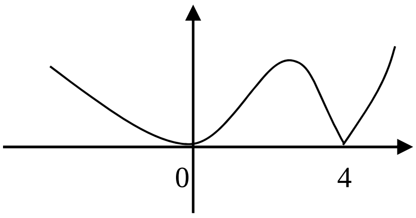
**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MPC3**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)** | $f(2) = -3$<br>$f(3) = 10$ | M1 | | $f(x) = x^3 - 6x + 1$ must have both values correct<br>allow $f(2) < 0$ and $f(3) > 0$<br>only if $f(x)$ is defined and no errors seen |
| | change of sign $\Rightarrow 2 < \alpha < 3$ | A1 | 2 | must have **both statement and interval** which may be written in words/symbols |
| **(b)** | $x^3 = 6x - 1$<br>or $x^2 - 6 + \frac{1}{x} = 0$<br>or $x^2 - 6 = -\frac{1}{x}$ | | | must see one of these lines and no errors |
| | $x^2 = 6 - \frac{1}{x}$ | B1 | 1 | AG |
| **(c)** | $x_2 = \sqrt{6 - \frac{1}{2.5}} = 2.366(432)$ | B1 | | at least 4sf needed<br>PI by correct $x_3$ |
| | $x_3 = 2.362$ | B1 | 2 | SC1 if B0B0 scored and $x_3$ = 2.3617 |
| | **Total** | | **5** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 2(a) | $y(0)=0$ | | | |
| | $y(1)=\frac{1}{3}=0.\dot{3}$ | | | |
| | $y(2)=\frac{1}{3}=0.\dot{3}$ | B1 | | all 5 $x$-values PI by 5 correct $y$-values |
| | $y(3)=\frac{3}{11}=0.\dot{2}\dot{7}$ | | | |
| | $y(4)=\frac{4}{18}=0.\dot{2}$ | B1 | | at least 4 $y$-values exact or rounded or truncated to at least 4sf |
| | $\frac{1}{3}\times1\left(0+0.\dot{2}+4\left[0.\dot{3}+0.\dot{2}\dot{7}\right]+2\left[0.\dot{3}\right]\right)$ | M1 | | correct use of Simpson's rule using $\frac{1}{3}$ and 4 and 2 correctly with candidate's 5 $y$-values |
| | = 1.104 | A1 | 4 | CAO (must be exactly this value) |
| (b) | $\int_0^4 \frac{x}{x^2+2}\,dx=\frac{1}{2}\left[\ln\left(x^2+2\right)\right]$ | M1<br>A1 | | for $k\ln\left(x^2+2\right)$<br>all correct; limits not needed |
| | $=\frac{1}{2}(\ln 18-\ln 2)$ | A1F | | For $k$ (ln18 – ln2) |
| | $=\frac{1}{2}\ln 9$ | A1F | | combining candidate's logarithms correctly (must be seen) |
| | $=\ln 3$ | A1 | 5 | CAO (must be exactly this)<br>NMS scores 0/5 |
| | **Total** | | **9** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 3(a) | $\left(\frac{dy}{dx}=\right)3e^{3x}+\frac{1}{x}$ | B1<br>B1 | 2 | B1 for one term correct<br>B1 all correct |
| (b)(i) | $\left(\frac{du}{dx}=\right)\frac{\pm\cos x(1+\cos x)\pm\sin x(\sin x)}{(1+\cos x)^2}$ | M1 | | clear attempt at quotient/product rule<br>condone poor use of brackets |
| | $\frac{\cos x(1+\cos x)-\sin x(-\sin x)}{(1+\cos x)^2}$ | A1 | | any correct form seen |
| | $=\frac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2}$<br>$=\frac{\cos x+1}{(1+\cos x)^2}$ | | | |
| | $=\frac{1}{1+\cos x}$ | A1cso | 3 | AG be convinced<br>correct use of brackets and correct notation used throughout<br>(eg A0 if $\cos x^2$ etc seen) |
| (ii) | $\left(\frac{dy}{dx}=\right)\frac{1+\cos x}{\sin x}\times\frac{1}{1+\cos x}$ OE | M1 | | correct use of chain rule |
| | $=\frac{1}{\sin x}$ | | | |
| | $=\operatorname{cosec} x$ | A1 | 2 | AG, must see $=\frac{1}{\sin x}$ and no errors seen; condone incorrect use of brackets only if penalised in part (b)(i) |
| | **Total** | | **7** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a)** |   | M1 | | reflection in the $x$-axis for the negative $\mathrm{f}(x)$ and remainder as given on sketch |
| | | A1 | 2 | correct curvatures, correct cusp at $x = 4$<br>condone straight lines for $x < 0$ and $x > 4$<br>4 marked on $x$-axis |
| **(b)** | **Either** | | | |
| | 1. Stretch | M1 | | 1 and either 2 or 3 |
| | 2. \|\| $x$-axis | | | |
| | 3. by factor 0.5 | A1 | | 1, 2 and 3 |
| | (followed by) translation | E1 | | |
| | $\begin{bmatrix}0.5\\0\end{bmatrix}$ | B1 | 4 | |
| | **or** | | | |
| | translation | (E1) | | |
| | $\begin{bmatrix}1\\0\end{bmatrix}$ | (B1) | | |
| | (followed by) 1. Stretch | (M1) | | 1 and either 2 or 3 |
| | 2. \|\| $x$-axis | | | |
| | 3. by factor 0.5 | (A1) | | 1, 2 and 3 |
| | **Total** | | **6** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5(a)** | | M1 | | $f(x) > -\frac{4}{3}$, $f \geq -\frac{4}{3}$, range $\geq -\frac{4}{3}$ |
| | $f(x) \geq -\frac{4}{3}$ | A1 | 2 | |
| **(b)(i)** | $x \geq -\frac{4}{3}$ | B1F | 1 | correct or FT from **(a)** |
| **(ii)** | $x^2 = 3y + 4$ | | | |
| | $x = (\pm)\sqrt{3y+4}$ | M1 | | either order – M1 for correctly changing the subject or reversing operations; M1 for replacing *y* with *x* |
| | $\left(f^{-1}(x) =\right)(-)\sqrt{3x+4}$ | M1 | | |
| | $\left(f^{-1}(x) =\right) -\sqrt{3x+4}$ | A1 | 3 | (dependent on both M1 marks) correct sign |
| **(c)(i)** | $3x - 1 = 1$ | M1 | | Or $3x - 1 = e^0$ or $3x - 1 = \pm 1$ |
| | $\frac{2}{3}$ OE | A1 | 2 | CAO, NMS $\frac{2}{3}$ OE scores 2/2 |
| **(ii)** | g has **NO** inverse because two values of *x* map to one value (of *y*) **or** it is many-one **or** it is not one-one **or** 'it is two-one' | B1 | 1 | must indicate no inverse with valid reason; do not accept contradictory reasons |
| **(iii)** | $\ln\left\lvert 3 \times \frac{x^2 - 4}{3} - 1 \right\rvert$ | M1 | | |
| | $\ln\lvert x^2 - 5 \rvert$ | A1 | 2 | NMS scores 0/2, condone *k* = – 5 after correct expression seen |
| **(iv)** | $\ln\lvert x^2 - 5 \rvert = 0$ | | | |
| | $\lvert x^2 - 5 \rvert = 1$ | | | |
| | $x^2 - 5 = 1$ (or – 1 or $e^0$ or $-e^0$ seen) | M1 | | $x^2 - k = 1$ etc, for candidate's positive integer, *k* |
| | $x^2 = 6, 4$ or candidate's $k + 1$ or $k - 1$ | | | |
| | $x = \sqrt{6}$, 2 | A1F | | exact values PI by correct answers |
| | $x = -\sqrt{6}, -2$ | A1F | | |
| | $(x \leq 0 \Rightarrow)$ $x = -\sqrt{6}, -2$ | A1 | 4 | CAO, rejecting the positive |
| | **Total** | | **15** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 6(a) | $\frac{\sec^2 x}{(\sec x+1)(\sec x-1)} = \frac{\sec^2 x}{\sec^2 x - 1}$ | | | |
| | $\sec^2 x = 1+\tan^2 x$ used | M1 | | M1 for correct use of $\sec^2 x = 1+\tan^2 x$ at least once or $\left(\text{cosec}^2 x = 1+\cot^2 x\right)$ |
| | $= \frac{\sec^2 x}{\tan^2 x}$ or $\frac{1+\tan^2 x}{\tan^2 x}$ | | | $\left(= \frac{1}{\cos^2 x \tan^2 x}\right)$ |
| | $= \frac{1}{\sin^2 x}$ or $\cot^2 x + 1$ | A1 | | Shown, with no errors |
| | $= \text{cosec}^2 x$ | A1 | 3 | AG (No errors, omissions or poor notations seen) |
| (b) | $\text{cosec}^2 x = \text{cosec}\, x + 3$<br>$\text{cosec}^2 x - \text{cosec}\, x - 3 = 0$ | B1 | | must have = 0 |
| | $\text{cosec}\, x = \frac{1 \pm \sqrt{13}}{2}$ or (2.3... **and** – 1.3...) | M1 | | correct solution of the quadratic, or by completing the square $\left(\text{cosec}\, x = \pm\sqrt{\frac{13}{4}} + \frac{1}{2}\right)$<br>PI by values for sin $x$ |
| | $\sin x = \frac{2}{1 \pm \sqrt{13}}$ | B1F | | B1F for $\text{cosec}\, x = \frac{1}{\sin x}$ seen or implied |
| | $= 0.434$ **and** $-0.768$ $(\text{or } -0.767)$ | A1 | | PI |
| | $x = 26°, 154°, -50°, -130°$ | B1<br>B1 | 6 | B1 for any three values correct AWRT<br>B1 for all four values correct AWRT and no extras in the interval $-180° < x < 180°$ |
| (c) | $2\theta - 60° = x$ | M1 | | where $x$ is a written value from candidate's **(b)** in degrees<br>PI by their answer |
| | $\theta = 43°, 5°$ | A1 | 2 | CSO<br>Ignore solutions outside interval $0° < \theta < 90°$ |
| | **Total** | | **11** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 7(a) | $y = 4x\cos 2x$ | | | |
| | $\left(\frac{dy}{dx} =\right) 4\cos 2x - 4x(2)\sin 2x$ | M1 | | anything reducible to $A\cos 2x + Bx\sin 2x$ where $A$ and $B$ are non-zero integers |
| | | A1 | | OE, all correct |
| | gradient of the tangent $A\cos\frac{2\pi}{4} + B\times\frac{\pi}{4}\sin\frac{2\pi}{4}$ | m1 | | substituting $\frac{\pi}{4}$ into candidate's derived function |
| | $= -2\pi$ | A1 | | must have $-2\pi$ using correct $\frac{dy}{dx}$ |
| | an equation of the tangent is $y = -2\pi\left(x - \frac{\pi}{4}\right)$ | A1 | 5 | OE, dependent on previous A1 |
| (b) | | | | $\left(\int_0^{\frac{\pi}{4}} 4x\cos 2x\,dx\right)$ |
| | $u = Ax \quad \frac{dv}{dx} = \cos 2x$ | M1 | | all 4 terms in this form seen or used |
| | $\frac{du}{dx} = A \quad v = B\sin 2x$ | A1 | | $A$ = 4 and $B = \frac{1}{2}$ **or** $A$ = 1 and $B$ = 2, etc |
| | $= \left[4x\frac{1}{2}\sin 2x\right]_{(0)}^{\left(\frac{\pi}{4}\right)} - \int_{(0)}^{\left(\frac{\pi}{4}\right)} 4\times\frac{1}{2}\sin 2x(dx)$ | m1 | | correct substitution of candidate's terms into integration by parts formula condone missing limits |
| | $= \left[4x\frac{1}{2}\sin 2x\right]_{(0)}^{\left(\frac{\pi}{4}\right)} - \left[-\cos 2x\right]_{(0)}^{\left(\frac{\pi}{4}\right)}$ | A1F | | candidate's second integration completed correctly<br>FT on one error including coefficient<br>condone missing limits |
| | $= \frac{\pi}{2} - 1$ | A1 | 5 | OE, exact value |
| | **Total** | | **10** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **8(a)** | $\int e^{1-2x}dx = ke^{1-2x}$ or $e\left(ke^{-2x}\right)$ | M1 | | where $k$ is a rational number |
| | $\int_0^{\ln 2} e^{1-2x}dx = -\frac{1}{2}e^{1-2x}\Big\|_0^{\ln 2}$ or $e\left[-\frac{1}{2}e^{-2x}\right]_0^{\ln 2}$ | A1 | | correct integration<br>condone missing limits |
| | $= -\frac{1}{2}e^{1-2\ln 2} - -\frac{1}{2}e^{1-2(0)}$ | A1 | | correct (no decimals) |
| | $= -\frac{1}{2}\left(\frac{1}{4}e\right) + \frac{1}{2}e$ | | | eliminating ln |
| | $= \frac{3}{8}e$ | A1 | 4 | AG, be convinced |
| **(b)** | $u = \tan x$ | | | |
| | $\frac{du}{dx} = \sec^2 x$ | M1 | | PI below, condone $du = \sec^2 x\,dx$ |
| | Replacing $dx$ by $\frac{1}{\sec^2 x}(du)$ in integral | A1 | | or $\frac{1}{1+u^2}(du)$ |
| | $\sec^2 x = 1 + u^2$ | B1 | | PI below |
| | $x = 0 \Rightarrow u = 0$<br>$x = \frac{\pi}{4} \Rightarrow u = 1$ | B1 | | this could be gained by changing $u$ to $\tan x$ after the integration and using $x = 0$ and $x = \frac{\pi}{4}$ |
| | $\int_0^{\frac{\pi}{4}} \sec^4 x\sqrt{\tan x}\,dx$ | | | |
| | $= \int\left(1+u^2\right)\sqrt{u}\,(du)$ or $\int\left(1+u^2\right)^2\sqrt{u}\,\frac{(du)}{1+u^2}$ | M1 | | all in terms of $u$ including replacing $dx$<br>all correct, condone omission of $du$ |
| | $= \int\left(u^{\frac{5}{2}} + u^{\frac{1}{2}}\right)(du)$ | A1 | | must be in this form |
| | $= \frac{2}{7}u^{\frac{7}{2}} + \frac{2}{3}u^{\frac{3}{2}}$ | A1 | | accept correct unsimplified form |
| | $= \frac{20}{21}$ | A1 | 8 | CAO |
| | **Total** | | **12** | |
| | **TOTAL** | | **75** | |