

**General Certificate of Education (A-level)  
January 2012**

**Mathematics**

**MPC3**

**(Specification 6360)**

**Pure Core 3**

**Final**

***Mark Scheme***

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### Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct $x$ marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

## MPC3

Q	Solution	Marks	Total	Comments																
1(a)	<table style="border-collapse: collapse; margin-left: auto; margin-right: auto;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>x</math></td> <td style="padding: 5px;"><math>y</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">0</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>\frac{1}{2}</math></td> <td style="padding: 5px;">2</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">1</td> <td style="padding: 5px;">4</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>1\frac{1}{2}</math></td> <td style="padding: 5px;">8</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">2</td> <td style="padding: 5px;">16</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>2\frac{1}{2}</math></td> <td style="padding: 5px;">32</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">3</td> <td style="padding: 5px;">64</td> </tr> </table>	$x$	$y$	0	1	$\frac{1}{2}$	2	1	4	$1\frac{1}{2}$	8	2	16	$2\frac{1}{2}$	32	3	64	B1		all 7 $x$ values correct (and no extra) (PI by 7 correct $y$ values)
	$x$	$y$																		
0	1																			
$\frac{1}{2}$	2																			
1	4																			
$1\frac{1}{2}$	8																			
2	16																			
$2\frac{1}{2}$	32																			
3	64																			
		B1		5 or more correct $y$ values, exact $\left(4^{\frac{1}{2}}, 4^1, \dots\right)$ or evaluated (in table or in formula)																
	$A = \frac{1}{3} \times \frac{1}{2} [65 + 4 \times 42 + 2 \times 20]$ $= \frac{91}{2} \text{ or } 45.5 \text{ or } \frac{273}{6}$	M1		correct substitution of their 7 $y$ -values into Simpson's rule																
		A1	4	CAO																
(b)(i)	$f(x) = 4^x + 2x - 8$ or $g(x) = 8 - 2x - 4^x$ $f(1.2) = -0.3$ or $g(1.2) = 0.3$ $f(1.3) = 0.7$ or $g(1.3) = -0.7$	M1		attempt at evaluating $f(1.2)$ and $f(1.3)$																
	AWR $\pm 0.3$ and $\pm 0.7$ condone $f(1.2) < 0$ , $f(1.3) > 0$ if $f$ is defined  change of sign $\therefore 1.2 < \alpha < 1.3$ ( $f(x)$ must be defined and all working correct)			<b>alternative method</b> $4^{1.2} = 5.3, 8 - 2 \times 1.2 = 5.6$ $4^{1.3} = 6.1, 8 - 2 \times 1.3 = 5.4$																
		A1	2	at 1.2 LHS < RHS at 1.3 LHS > RHS $\therefore 1.2 < \alpha < 1.3$																
(ii)	$(x_2 = ) 1.243$	B1																		
	$(x_3 = ) 1.232$	B1	2	these values only																
	<b>Total</b>		<b>8</b>																	

## MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$f(1) = 21$ $f(16) = 1$ $1 \leq f(x) \leq 21$	M1	2	sight of 1 and 21
		A1		allow $f(x)$ replaced by $f, y$
(b)(i)	$y = \frac{63}{4x-1}$ $x = \frac{63}{4y-1}$ $x(4y-1) = 63$ or better $f^{-1}(x) = \frac{1}{4} \left( \frac{63}{x} + 1 \right)$	M1	3	reverse $x, y$ one correct step } Either order condone $y =$
		M1		
		A1		
(ii)	$\frac{1}{4} \left( \frac{63}{x} + 1 \right) = 1$ $\frac{63}{x} + 1 = 4$ , or better $(x =) 21$	M1	2	one correct step from their (b)(i) = 1, or $x = f(1)$ note: 21 scores 2/2
		A1		
(c)(i)	$(fg(x) =) \frac{63}{4x^2-1}$	B1	1	
(ii)	$\frac{63}{4x^2-1} = 1$ $4x^2-1 = 63$ or better $x^2 = 16$ OE $x = -4$ <b>ONLY</b>	M1	3	one correct step from their (c)(i) = 1 eg $(2x+8)(2x-8) = 0$ , or $x = \pm 4$
		A1		
		A1		
<b>Total</b>			<b>11</b>	

## MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\left(\frac{dy}{dx} = \right) 12x^2 - 6$	B1	1	do not ISW
(b)	$\int_2^3 \frac{2x^2 - 1}{4x^3 - 6x + 1} dx$ $= \left[ \frac{1}{6} \ln(4x^3 - 6x + 1) \right]_{(2)}^{(3)}$ $= \frac{1}{6} \ln(4 \times 3^3 - 6 \times 3 + 1)$ $- \frac{1}{6} \ln(4 \times 2^3 - 6 \times 2 + 1)$ $= \frac{1}{6} \ln 91 - \frac{1}{6} \ln 21$ $= \frac{1}{6} \ln \frac{91}{21} \quad \text{or} \quad \left( = \frac{1}{6} \ln \frac{13}{3} \right)$	M1 A1 m1 A1F A1	5	$k \ln(4x^3 - 6x + 1)$ , $k$ is a constant $k = \frac{1}{6}$ correct substitution in F(3) – F(2). condone poor use or lack of brackets. $k \ln 91 - k \ln 21$ only follow through on their $k$ or if using the substitution $u = 4x^3 - 6x + 1$ $\int = k \int \frac{du}{u}$ M1 $= \frac{1}{6} \ln u$ A1 then, either change limits to 21 and 91 m1 then A1F A1 as scheme or changing back to 'x', then m1 A1F A1 as scheme
<b>Total</b>			<b>6</b>	
4(a)	$\sec^2 \theta - 1 = \dots$ $\sec^2 \theta + 3 \sec \theta - 10 (= 0)$ $(\sec \theta + 5)(\sec \theta - 2) = 0$ $\sec \theta = -5, 2$ $\left( \cos \theta = -\frac{1}{5}, \frac{1}{2} \right)$ $60^\circ, 300^\circ, 101.5^\circ, 258.5^\circ$ (AWRT)	B1 M1 m1 A1	6	correct use of $\sec^2 \theta = 1 + \tan^2 \theta$ quadratic expression in $\sec \theta$ with all terms on one side attempt at factors of their quadratic, $(\sec \theta \pm 5)(\sec \theta \pm 2)$ , or correct use of quadratic formula
(b)	$4x - 10 = 60^\circ, 101.5^\circ, 258.5^\circ, 300^\circ$ $4x = 70^\circ, 111.5^\circ, 268.5^\circ, 310^\circ$ $x = 17.5^\circ, 27.9^\circ, 67.1^\circ, 77.5^\circ$ (AWRT)	M1 A1F A1	3	$4x - 10 =$ any of their (60), all their answers from (a), BUT must have scored B1 CAO, ignore answers outside interval
<b>Total</b>			<b>9</b>	

## MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	stretch SF 4 in y-direction translate $\begin{pmatrix} e \\ 0 \end{pmatrix}$	$\left. \begin{array}{l} \text{I} \\ \text{II} \\ \text{III} \end{array} \right\} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{either order}$	M1A1  E1 B1	I + (II or III)  4 accept 'e in positive x-direction'
(b)		M1  A1  A1	3	mod graph, in 2 connected sections, both in the first quadrant, touching x-axis  curve touches x-axis at $1 + e$ (or 3.7 or better), and labelled (ignore scale)  correct curvature, including at their $1 + e$ , approx. asymptote at $x = e$
(c)(i)	$ 4\ln(x - e)  = 4$ $4\ln(x - e) = 4$ $4\ln(x - e) = -4$ or better $(x =) 2e$ do not ISW $(x =) e + e^{-1}$ or $(x =) e + \frac{1}{e}$ do not ISW	M1  A1  A1	3	must see 2 equations, condone omission of brackets  accept values of AWRT 5.42, 5.43, 5.44  accept values of AWRT 3.08, 3.09 if M0 then $x = 2e$ with or without working scores SC1
(ii)	$x \geq 2e$  $e < x \leq e + \frac{1}{e}$	B1  B2	3	accept values of AWRT 5.42, 5.43, 5.44  accept values of AWRT 2.72, 3.08, 3.09  if B2 not earned, then SC1 for any of $e \leq x \leq e + \frac{1}{e}$ , $e < x < e + \frac{1}{e}$ , $e \leq x < e + \frac{1}{e}$
<b>Total</b>			<b>13</b>	

## MPC3 (cont)

Q	Solution	Marks	Total	Comments		
6(a)	$\left(\frac{dx}{d\theta} = \right) \frac{(\sin \theta \times 0) - 1 \times \cos \theta}{\sin^2 \theta}$ $= -\frac{\cos \theta}{\sin^2 \theta} \quad \text{or} \quad = -\frac{\cos \theta}{\sin \theta \sin \theta}$ $= -\operatorname{cosec} \theta \cot \theta$	M1	3	quotient rule $\frac{\pm \sin \theta \times k \pm 1 \times \cos \theta}{\sin^2 \theta}$ where $k = 0$ or $1$ must see the '0' either in the quotient or in eg $\frac{du}{d\theta} = 0$ etc or equivalent		
		A1		CSO, AG must see one of the previous expressions		
		A1				
		(b)		$x = \operatorname{cosec} \theta$	B1	OE, eg $dx = -\operatorname{cosec} \theta \cot \theta d\theta$
		$\frac{dx}{d\theta} = -\operatorname{cosec} \theta \cot \theta$		B1	at any stage of solution	
		Replacing $\sqrt{(\operatorname{cosec}^2 \theta - 1)}$ by $\sqrt{\cot^2 \theta}$ , or better		B1		
		$\int = \int \frac{-\operatorname{cosec} \theta \cot \theta}{\operatorname{cosec}^2 \theta \sqrt{(\operatorname{cosec}^2 \theta - 1)}} d\theta$		M1	all in terms of $\theta$ , and including their attempt at $dx$ , but condone omission of $d\theta$	
				A1	fully correct and must include $d\theta$ (at some stage in solution)	
		$\int \frac{-\operatorname{cosec} \theta \cot \theta}{\operatorname{cosec}^2 \theta \cot \theta} (d\theta)$		A1	OE eg $\int -\sin \theta (d\theta)$	
		$= \int \frac{-1}{\operatorname{cosec} \theta} (d\theta)$		A1		
$= \cos \theta$	A1					
$x = 2, \theta = 0.524$ AWRT	B1	correct change of limits				
$x = \sqrt{2}, \theta = 0.785$ AWRT	B1					
			or $(\pm) \cos \theta = (\pm) \left[ \sqrt{\left(1 - \frac{1}{x^2}\right)} \right]_{\sqrt{2}}^2$ OE			
0.8660 – 0.7071	m1	c's $F(0.52) - F(0.79)$ substitution into $\pm \cos \theta$ only				
		or $\left(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right)$				
= 0.159	A1	9				
	<b>Total</b>		<b>12</b>			

## MPC3 (cont)

Q	Solution	Marks	Total	Comments	
7(a)	$\left[ \frac{dy}{dx} = \right] p e^{-\frac{1}{4}x} x^2 + q x e^{-\frac{1}{4}x}$	M1		$p, q$ constants	
		A1		$p = -\frac{1}{4}$ and $q = 2$	
	$\left[ \Rightarrow e^{-\frac{1}{4}x} \left( -\frac{1}{4}x^2 + 2x \right) = 0 \right]$				
	$e^{-\frac{1}{4}x} \neq 0$	E1		or $e^{-\frac{1}{4}x} = 0$ impossible OE (may be seen later)	
	$\left( e^{-\frac{1}{4}x} \right) (ax^2 + bx) = 0$	m1		or $e^{-\frac{1}{4}x} x(ax + b) = 0$	
	$x = 0, 8$ $x = 0, y = 0$ $x = 8, y = 64e^{-2}$	A1 A1 B1		condone $y = 8^2 e^{-\frac{8}{4}}$ etc ignore further numerical evaluation	
(b)(i)	$\int x^2 e^{-\frac{1}{4}x} dx$	$u = x^2$	$\frac{dv}{dx} = e^{-\frac{1}{4}x}$		
		$\frac{du}{dx} = 2x$	$v = ke^{-\frac{1}{4}x}$	M1	
		$k = -4$		A1	
		$-4x^2 e^{-\frac{1}{4}x} - \int -4e^{-\frac{1}{4}x} \times 2x(dx)$ , or better		A1F	correct substitution of their terms
	$u = mx$	$\frac{dv}{dx} = ne^{-\frac{1}{4}x}$			
	$\frac{du}{dx} = m$	$v = -4ne^{-\frac{1}{4}x}$	m1	both differentiation and integration <b>must</b> be correct	
	$\int = -4x^2 e^{-\frac{1}{4}x} + 8 \left( -4xe^{-\frac{1}{4}x} + \int 4e^{-\frac{1}{4}x} dx \right)$				
	$= \left[ -4x^2 e^{-\frac{1}{4}x} - 32xe^{-\frac{1}{4}x} - 128e^{-\frac{1}{4}x} \right]_{(0)}^{(4)}$		A1		
	$= -e^{-1} [64 + 256] - [-128]$	m1 (dep on M1 only)		correct substitution and attempt at subtraction in $ax^2 e^{-\frac{1}{4}x} + bxe^{-\frac{1}{4}x} + ce^{-\frac{1}{4}x}$ (may be in 3 stages)	
	$= 128 - \frac{320}{e}$	A1	7	or $128 - 320e^{-1}$ ignore further numerical evaluation	
(ii)	$v = \pi \int_{(0)}^{(4)} 9x^2 e^{-\frac{1}{4}x} (dx)$	M1		condone omission of brackets, limits	
	$= 9\pi \left( 128 - \frac{320}{e} \right)$	A1F	2	$9\pi \times$ (their exact b(i))	
	<b>Total</b>		<b>16</b>		
	<b>TOTAL</b>		<b>75</b>		