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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2008 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

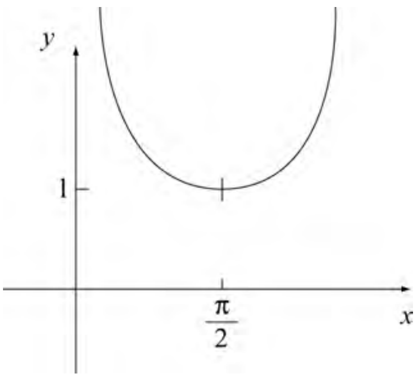
MPC3

Q	Solution	Marks	Total	Comments
1(a)(i)	$y=\left(2x^2-5x+1\right)^{20}$			
	$\frac{\mathrm{d}y}{\mathrm{d}x}=20\left(2x^2-5x+1\right)^{19}(4x-5)$ OE	M1 A1	2	chain rule $20(\quad)^{19}\mathrm{f}(x)$ with no further incorrect working
(ii)	$y=x\cos x$			
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-x\sin x+\cos x$	M1 A1	2	product rule $\pm x\sin x\pm\cos x$ CSO
(b)	$y=\frac{x^3}{x-2}$			
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x-2)3x^2-x^3\times 1}{(x-2)^2}$	M1 A1		quotient rule $\frac{\pm vu'\pm uv'}{(x-2)^2}$ condone missing brackets
	$=\frac{3x^3-6x^2-x^3}{(x-2)^2}$			
	$=\frac{2x^2(x-3)}{(x-2)^2}$	A1	3	CSO
	Total		**7**	
2(a)	$\cot x=2\Rightarrow\tan x=0.5$	M1		
	$x=0.46,\ 3.61$	A1	2	AWRT; no others within range
(b)	$\operatorname{cosec}^2 x=\frac{3\cot x+4}{2}$			
	$2\left(1+\cot^2 x\right)=3\cot x+4$	M1		Correct use of $\operatorname{cosec}^2 x=1+\cot^2 x$
	$\left(2\cot^2 x-3\cot x+2-4=0\right)$			
	$2\cot^2 x-3\cot x-2=0$	A1	2	AG; correct with no slips from line with no fractions
(c)	$(2\cot x+1)(\cot x-2)(=0)$	M1		Attempt to solve
	$\cot x=-\frac{1}{2},\ 2$	A1		
	$\tan x=-2,\ 0.5$			
	$x=0.46,\ 3.61,\ 2.03,\ 5.18$	B1 B1	4	2 correct Allow 3.6(0) 4 correct (with no extras in range) AWRT **SC** Degrees $\left.\begin{array}{l}26.57,\ 206.57\\116.57,\ 296.57\end{array}\right\}$ B1 for 2 correct
	Total		**8**	

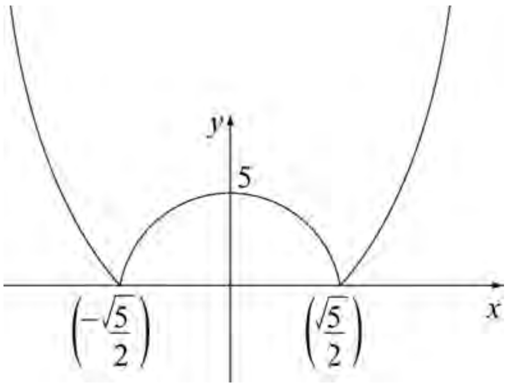
MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$x+(1+3x)^{\frac{1}{4}}=0$ $\mathrm{f}\left(-0.32\right)=0.1$ $\mathrm{f}\left(-0.33\right)=-0.01$	M1		AWRT; allow + ve, –ve
	Change of sign $\therefore\ -0.33< x<-0.32$	A1	2	
(b)	$x=-(1+3x)^{\frac{1}{4}}$			
	$x^4=1+3x$	M1		Attempt to isolate x^4
	$\dfrac{x^4-1}{3}=x$	A1	2	AG
(c)	$x_1=-0.3$			
	$\left(x_2=-0.331\right)$ AWRT	M1		
	$\left(x_3=-0.329\right)$ AWRT	A1		
	$x_4=-0.329$	A1	3	
	Total		**7**	
4(a)	all $\left(\text{real}\right)$ values	B1	1	No x in answer, unless f (x)
(b)(i)	$\mathrm{fg}\left(x\right)=\left(\dfrac{1}{x-3}\right)^3$	B1	1	ISW
(ii)	$\left(\dfrac{1}{x-3}\right)^3=64$			
	$\dfrac{1}{x-3}=4$	M1		$\sqrt[3]{\ }$
	$x-3=\dfrac{1}{4}$	M1		Invert
	$x=3\dfrac{1}{4}$	A1	3	
(c)(i)	$y=\dfrac{1}{x-3}$			
	$x=\dfrac{1}{y-3}$	M1		Swap x and y
	$x\left(y-3\right)=1$			
	$xy-3x=1$	M1		attempt to isolate
	$y=\dfrac{1+3x}{x}=\mathrm{g}^{-1}\left(x\right)$ or $\dfrac{1}{x}+3$	A1	3	
(ii)	$\left(\text{real values}\right)\ \left(\mathrm{g}^{-1}\left(x\right)\right)\neq 3$	B1	1	
	Total		**9**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$y = 2x^2 - 8x + 3$ $\left(\frac{dy}{dx} =\right) 4x - 8$	B1	1	
(ii)	$\int_4^6 \frac{x-2}{2x^2 - 8x + 3}\,dx$ $= \frac{1}{4}\left[\ln\left\|2x^2 - 8x + 3\right\|\right]_4^6$	M1A1		M1 for $k\ln\left(2x^2 - 8x + 3\right)$; allow $k\ln u$
	$= \frac{1}{4}[\ln 27 - \ln 3]$	m1		Correct substitution into $k\ln\left(2x^2 - 8x + 3\right)$ or 3, 27 into $k\ln u$
	$= \frac{1}{4}\ln 9$ $= \frac{1}{2}\ln 3$	A1	4	
(b)	$\int x\sqrt{(3x-1)}\,dx$ $u = 3x - 1 \qquad du = 3dx$	B1		OE
	$\int \; = \left(\frac{1}{9}\right)\int\left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right)(du)$	M1		$\int$ 2 terms in u with rational indices
	$= \left(\frac{1}{9}\right)\left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{u^{\frac{3}{2}}}{\frac{3}{2}}(+c)\right]$	A1F		Must be 2 terms with correct indices $\left(\text{only ft for } x = \frac{u-1}{3}\right)$
	$= \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + \boldsymbol{c}$	A1	4	CSO OE
	Total		**9**	
6(a)		M1 A1	 2	Correct shape Vertex
(b)	<table><tr><th>x</th><th>y</th></tr><tr><td>0.15</td><td>6.692</td></tr><tr><td>0.25</td><td>4.042</td></tr><tr><td>0.35</td><td>2.916</td></tr><tr><td>0.45</td><td>2.299</td></tr></table>	M1 B1		Correct x values ≥ 3 correct y values
	$\int \; \simeq 0.1 \times \sum y \qquad \left(\sum y = 15.949\right)$	B1		correct h used correctly
	$= 1.59$	A1	4	
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	Stretch (I)			
	Scale factor $\frac{1}{2}$ (II)	M1		I + (II or III)
	parallel to x-axis (III)	A1		All correct
	(Or scale factor 4 parallel to y-axis)			
	Translation	M1		
	$\begin{bmatrix} 0 \\ -5 \end{bmatrix}$ OE	A1	4	
	Alternatives			
	translate $\begin{pmatrix} 0 \\ -\frac{5}{4} \end{pmatrix}$, stretch sf 4 || y-axis			Mark translation first. Mark stretch as above, but relative to their translation.
	translate $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$, stretch sf $\frac{1}{2}$ || x-axis			
(b)		M1		Modulus graph symmetrical about y-axis
		A1		left of $-\frac{\sqrt{5}}{2}$ and right of $\frac{\sqrt{5}}{2}$
		A1	3	(0, 5), cusps drawn and no straight lines between cusps
(c)(i)	$4x^2 - 5 = 4$			
	$4x^2 = 9$			
	$x = \pm\frac{3}{2}$ OE	B1		
	$4x^2 - 5 = -4$	M1		$16x^4 - 40x^2 + 9 = 0$
	$4x^2 = 1$			
	$x = \pm\frac{1}{2}$	A1	3	
(ii)	$x \le -\frac{3}{2}$, $x \ge \frac{3}{2}$	B1F		2 correct statements
	$-\frac{1}{2} \le x$, $x \le \frac{1}{2}$	B1F	2	4 correct statements
				SC c(ii) 1 mark penalty for strict inequalities
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$e^{-2x} = 3$			
	$-2x = \ln 3$	M1		
	$x = -\frac{1}{2}\ln 3$	A1	2	OE ISW
(b)	$\int x e^{-2x}\ dx$			
	$u = x \quad \frac{dv}{dx} = e^{-2x}$			
	$\frac{du}{dx} = 1 \quad v = -\frac{1}{2}e^{-2x}$	M1		differentiating and integrating
	$\int = -\frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}\,(dx)$	m1		correct subs of their values into parts formula
		A1		
	$= -\frac{1}{2}\ xe^{-2x} - \frac{1}{4}e^{-2x} + c$	A1	4	No further incorrect working
(c)(i)	$y = e^{-2x} + 6x$			
	$\frac{dy}{dx} = -\ 2e^{-2x} + 6 = 0$	M1		$ke^{-2x} + 6 = 0$
	$\frac{dy}{dx} = 0 \Rightarrow -2\left(e^{-2x} - 3\right) = 0$			
	$x = -\frac{1}{2}\ln 3$	A1		OE
	$y = 3 + 6\left(-\frac{1}{2}\ln 3\right)$	M1		Correct substitute of their valid x
	$= 3 - 3\ln 3$	A1	4	OE ISW
(ii)	$\frac{d^2y}{dx^2} = 4e^{-2x} \begin{cases} = 12 \\ > 0 \end{cases}$	M1		Other methods need justification Allow error in $\frac{d^2y}{dx^2}$ or x-value, but not both
	$\therefore$ minimum	A1	2	
(iii)	$(V) = \pi\int_0^1 y^2 dx = (\pi)\int_{(0)}^{(1)} \left(e^{-2x} + 6x\right)^2 (dx)$	M1		Either
	$= (\pi)\int_{(0)}^{(1)} \left(e^{-4x} + 12xe^{-2x} + 36x^2\right) dx$	B1		Correct expansion
	$= (\pi)\left[-\frac{1}{4}e^{-4x} - 6xe^{-2x} - 3e^{-2x} + 12x^3\right]_{(0)}^{(1)}$	A1 A1		3 correct terms; '–6','–3' correct or 12 × their (b) All correct
	$= \pi\left[\left(-\frac{1}{4}e^{-4} - 9e^{-2} + 12\right) - \left(-\frac{1}{4} - 3\right)\right]$			
	$= \pi\left[15\frac{1}{4} - 9e^{-2} - \frac{1}{4}e^{-4}\right]$			
	$= 44.1$	B1	5	AWRT
	Total		**17**	
	TOTAL		**75**	