

C2 TRIGONOMETRY

Answers - Worksheet A

$$1 \quad \frac{AC}{\sin 118} = \frac{16}{\sin 26}$$

$$AC = \frac{16 \times \sin 118}{\sin 26}$$

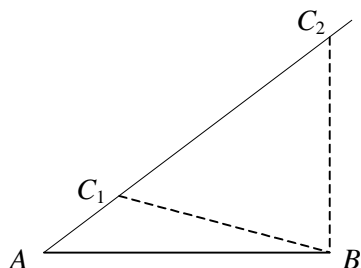
$$= 32.2 \text{ cm}$$

$$2 \quad \frac{\sin \angle PRQ}{8.2} = \frac{\sin 57}{11.4}$$

$$\sin \angle PRQ = \frac{8.2 \times \sin 57}{11.4} = 0.6033$$

$$\angle PRQ = 37.1^\circ$$

3



$$\frac{\sin \angle ACB}{16.2} = \frac{\sin 37}{12.3}$$

$$\sin \angle ACB = \frac{16.2 \times \sin 37}{12.3} = 0.7926$$

$$\angle ACB = 52.4 \text{ or } 180 - 52.4 = 52.4 \text{ or } 127.6$$

$$\angle ABC = 180 - (37 + \angle ACB) = 90.568 \text{ or } 15.432$$

$$\frac{AC}{\sin \angle ABC} = \frac{12.3}{\sin 37}$$

$$AC = \frac{12.3 \times \sin \angle ABC}{\sin 37} = 20.4 \text{ or } 5.4$$

$\therefore \angle ACB = 52.4^\circ, AC = 20.4 \text{ cm}$ or $\angle ACB = 127.6^\circ, AC = 5.4 \text{ cm}$ (all 1dp)

$$4 \quad XZ^2 = 7.8^2 + 15.3^2$$

$$- (2 \times 7.8 \times 15.3 \times \cos 31.5^\circ)$$

$$= 91.422$$

$$XZ = 9.56 \text{ cm (3sf)}$$

$$5 \quad 18^2 = 13^2 + 17^2 - (2 \times 13 \times 17 \times \cos \angle ACB)$$

$$\cos \angle ACB = \frac{13^2 + 17^2 - 18^2}{2 \times 13 \times 17}$$

$$= 0.3032$$

$$\angle ACB = 72.4^\circ \text{ (1dp)}$$

$$6 \quad \text{a } \alpha = 180 - (40 + 32) = 108 \quad \text{b } x^2 = 2.7^2 + 3.8^2$$

$$\frac{x}{\sin 108} = \frac{23.1}{\sin 40}$$

$$x = \frac{23.1 \times \sin 108}{\sin 40}$$

$$x = 34.2 \text{ cm (3sf)}$$

$$- (2 \times 2.7 \times 3.8 \times \cos 83)$$

$$x^2 = 19.229$$

$$x = 4.39 \text{ m (3sf)}$$

$$\text{c } \frac{\sin \alpha}{7.6} = \frac{\sin 61}{10.5}$$

$$\sin \alpha = \frac{7.6 \times \sin 61}{10.5} = 0.6331$$

$$\alpha = 39.276$$

$$\beta = 180 - (61 + 39.276) = 79.724$$

$$\frac{x}{\sin 79.724} = \frac{10.5}{\sin 61}$$

$$x = \frac{10.5 \times \sin 79.724}{\sin 61}$$

$$x = 11.8 \text{ cm (3sf)}$$

$$7 \quad \text{a } \frac{\sin \alpha}{67} = \frac{\sin 96.5}{92}$$

$$\sin \alpha = \frac{67 \times \sin 96.5}{92}$$

$$\sin \alpha = 0.7236$$

$$\alpha = 46.351$$

$$\theta = 180 - 96.5 - \alpha$$

$$\theta = 37.1^\circ \text{ (1dp)}$$

$$\text{b } 1.9^2 = 0.8^2 + 1.7^2$$

$$- (2 \times 0.8 \times 1.7 \times \cos \theta)$$

$$\cos \theta = \frac{0.8^2 + 1.7^2 - 1.9^2}{2 \times 0.8 \times 1.7}$$

$$\cos \theta = -0.02941$$

$$\theta = 91.7^\circ \text{ (1dp)}$$

$$\text{c } l^2 = 7.4^2 + 8.7^2$$

$$- (2 \times 7.4 \times 8.7 \times \cos 43.7)$$

$$l^2 = 37.3608, l = 6.1123$$

$$\frac{\sin \theta}{7.4} = \frac{\sin 43.7}{6.1123}$$

$$\sin \theta = \frac{7.4 \times \sin 43.7}{6.1123} = 0.8364$$

$$\theta = 56.8^\circ \text{ (1dp)}$$

8 a area

$$= \frac{1}{2} \times 2.1 \times 3.4 \times \sin 66$$

$$= 3.26 \text{ m}^2 \text{ (3sf)}$$

b area

$$= \frac{1}{2} \times 35 \times 68 \times \sin 116$$

$$= 1070 \text{ cm}^2 \text{ (3sf)}$$

c $\frac{\sin \alpha}{5.8} = \frac{\sin 72.4}{6.5}$

$$\sin \alpha = \frac{5.8 \times \sin 72.4}{6.5} = 0.8505$$

$$\alpha = 58.270$$

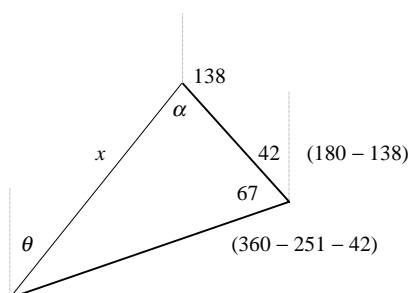
$$\beta = 180 - (72.4 + \alpha) = 49.330$$

area

$$= \frac{1}{2} \times 5.8 \times 6.5 \times \sin 49.330$$

$$= 14.3 \text{ cm}^2 \text{ (3sf)}$$

9



a $x^2 = 4.2^2 + 7.8^2 - (2 \times 4.2 \times 7.8 \times \cos 67)$

$$x^2 = 52.879$$

$$x = 7.27 \text{ miles (3sf)}$$

b $\frac{\sin \alpha}{7.8} = \frac{\sin 67}{7.2718}$

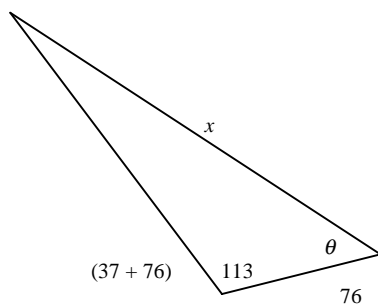
$$\sin \alpha = \frac{7.8 \times \sin 67}{7.2718} = 0.9874$$

$$\alpha = 80.882$$

$$\theta = 138 + \alpha - 180 = 38.882$$

$$\text{bearing} = 039^\circ \text{ (nearest degree)}$$

10



$$x^2 = 3.2^2 + 6.9^2 - (2 \times 3.2 \times 6.9 \times \cos 113)$$

$$x^2 = 75.105$$

$$x = 8.67 \text{ km (3sf)}$$

$$\frac{\sin \theta}{6.9} = \frac{\sin 113}{8.666}$$

$$\sin \theta = \frac{6.9 \times \sin 113}{8.666} = 0.7329$$

$$\theta = 47.130$$

$$\text{bearing} = 180 + 76 + \theta = 303^\circ \text{ (nearest degree)}$$

11 $9.7^2 = 10.4^2 + 11.0^2 - (2 \times 10.4 \times 11.0 \times \cos \angle BAC)$

$$\cos \angle BAC = \frac{10.4^2 + 11.0^2 - 9.7^2}{2 \times 10.4 \times 11.0} = 0.5903$$

$$\angle BAC = 53.819$$

$$\text{area} = \frac{1}{2} \times 10.4 \times 11.0 \times \sin 53.819 = 46.2 \text{ cm}^2$$

12 $\frac{1}{2} \times 22.5 \times YZ \times \sin 34 = 100$

$$YZ = \frac{200}{22.5 \times \sin 34} = 15.896$$

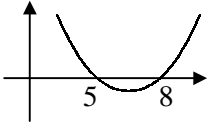
$$XZ^2 = 22.5^2 + 15.896^2 - (2 \times 22.5 \times 15.896 \times \cos 34)$$

$$= 165.906$$

$$XZ = 12.9 \text{ cm (3sf)}$$

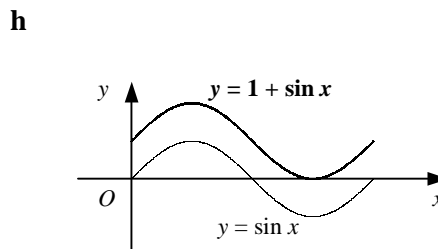
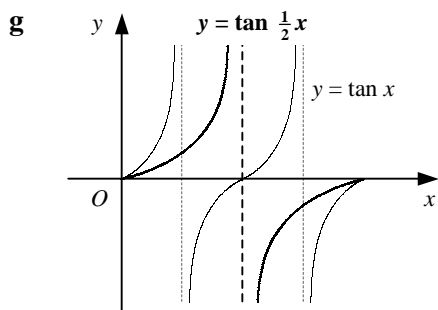
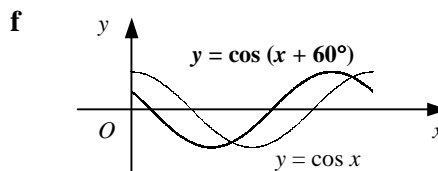
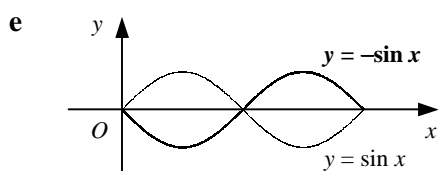
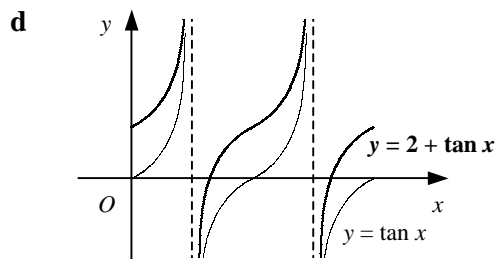
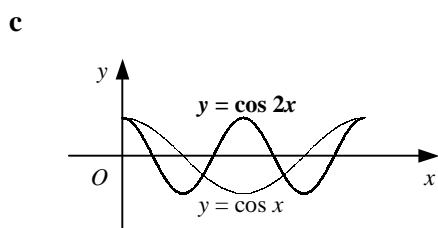
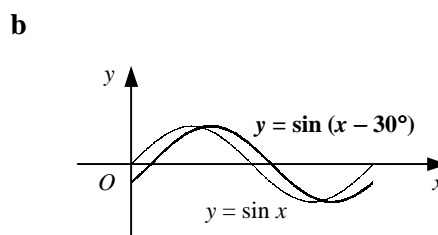
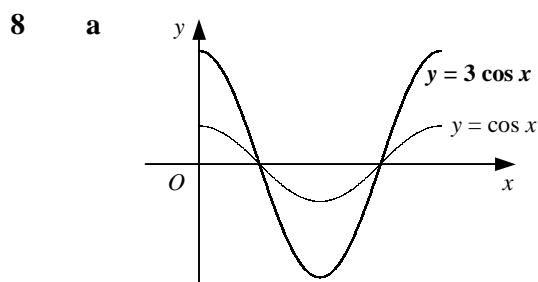
C2 TRIGONOMETRY

Answers - Worksheet C

- 1** $(2 \times 12.6) + 12.6\theta = 31.7$
 $\theta = 6.5 \div 12.6 = 0.5159^\circ$
 $A = \frac{1}{2} \times (12.6)^2 \times 0.5159 = 40.95 \text{ cm}^2$
- 2** **a** $\frac{1}{2} \times (7.3)^2 \times \theta = 38.4$
 $\theta = 38.4 \div 26.645 = 1.44^\circ$ (3sf)
b chord $AB = 2 \times 7.3 \sin(\frac{1}{2}\theta) = 9.633$
arc $AB = 7.3\theta = 10.521$
 $P = 9.633 + 10.521 = 20.2 \text{ cm}$ (3sf)
- 3** **a** $\frac{1}{2}r^2\theta = 40 \therefore \theta = \frac{80}{r^2}$
 $P = 2r + r\theta = 2r + (r \times \frac{80}{r^2})$
 $= (2r + \frac{80}{r}) \text{ cm}$
b $2r + \frac{80}{r} < 26$
 $2r^2 + 80 < 26r$
 $r^2 - 13r + 40 < 0$
 $(r-5)(r-8) < 0$
 $5 < r < 8$
- 
- 4** **a** $AB^2 = 10^2 = 100, AC^2 + BC^2 = 6^2 + 8^2 = 100$
 $AB^2 = AC^2 + BC^2$
 $\therefore \angle ACB = 90^\circ$ (converse of Pythagoras')
 \therefore triangle ABC is right-angled
b $\tan(\angle ABC) = \frac{AC}{BC} = \frac{3}{4} \therefore \angle ABC = 0.64^\circ$
c $\angle BAC = \frac{\pi}{2} - 0.6435 = 0.9273$
area of sectors:
centre $A = \frac{1}{2} \times 4^2 \times 0.9273 = 7.4184$
centre $B = \frac{1}{2} \times 6^2 \times 0.6435 = 11.5830$
centre $C = \frac{1}{4} \times \pi \times 2^2 = 3.1416$
area of triangle $ABC = \frac{1}{2} \times AC \times BC = 24$
shaded area
 $= 24 - (7.4184 + 11.5830 + 3.1416)$
 $= 1.86 \text{ cm}^2$
- 5** **a** let centre of circle be O
let midpoint of AB be M
 $AM^2 = OA^2 - OM^2 = 5^2 - 3^2 = 16$
 $AM = 4 \therefore AB = 8 \text{ cm}$
b $\cos(\angle AOM) = \frac{3}{5}$
 $\angle AOB = 2 \times \angle AOM = 1.8546^\circ$
arc $AB = 5 \times 1.8546 = 9.2730$
 $P = 2 \times (6 + 14 - 8 + 9.2730) = 42.5 \text{ cm}$
c area of segment
 $= \frac{1}{2} \times 5^2 \times 1.8546 - \frac{1}{2} \times 5^2 \times \sin 1.8546^\circ$
 $= 23.182 - 12 = 11.182$
area of logo $= (6 \times 14) + (2 \times 11.182)$
 $= 106 \text{ cm}^2$ (3sf)
- 6** **a** $OC = (r+2) \text{ cm}$
 $A_1 = [\frac{1}{2} \times 8^2 \times \theta] - [\frac{1}{2} \times (r+2)^2 \times \theta]$
 $= \frac{1}{2} \theta [64 - (r^2 + 4r + 4)]$
 $= \frac{1}{2} \theta (60 - 4r - r^2) \text{ cm}^2$
b $A_2 = \frac{1}{2} r^2 \theta$
 $\therefore \frac{1}{2} \theta (60 - 4r - r^2) = 7 \times \frac{1}{2} r^2 \theta$
 $60 - 4r - r^2 = 7r^2$
 $2r^2 + r - 15 = 0$
 $(2r-5)(r+3) = 0$
 $r > 0 \therefore r = 2.5$
- 7** let length of wire $= 3l$
area of $A = \frac{1}{2} \times l^2 \times \sin \frac{\pi}{3} = 0.43301l^2$
angle at centre of $B = l \div l = 1^\circ$
area of $B = \frac{1}{2} \times l^2 \times 1 = 0.5l^2$
% change $= \frac{0.5l^2 - 0.43301l^2}{0.43301l^2} \times 100\%$
 $= 15.5\%$, increase

C2 TRIGONOMETRY
Answers - Worksheet D

- 1 **a** 0.755 **b** -0.354 **c** 0.530 **d** -0.255
- 2 **a** = $\frac{1}{2}$ **b** = $\frac{1}{\sqrt{2}}$ **c** = 1 **d** = $\frac{\sqrt{3}}{2}$
e = 1 **f** = $\frac{1}{\sqrt{3}}$ **g** = $-\cos 60^\circ = -\frac{1}{2}$ **h** = $\sin 45^\circ = \frac{1}{\sqrt{2}}$
i = $\tan 30^\circ = \frac{1}{\sqrt{3}}$ **j** = $-\cos 45^\circ = -\frac{1}{\sqrt{2}}$ **k** = $-\sin 60^\circ = -\frac{\sqrt{3}}{2}$ **l** = $-\tan 60^\circ = -\sqrt{3}$
m = $\cos 30^\circ = \frac{\sqrt{3}}{2}$ **n** = $-\tan 30^\circ = -\frac{1}{\sqrt{3}}$ **o** = $\cos 60^\circ = \frac{1}{2}$ **p** = $\sin 45^\circ = \frac{1}{\sqrt{2}}$
q = $-\tan 45^\circ = -1$ **r** = $\sin 60^\circ = \frac{\sqrt{3}}{2}$ **s** = $\tan 30^\circ = \frac{1}{\sqrt{3}}$ **t** = $-\cos 30^\circ = -\frac{\sqrt{3}}{2}$
- 3 **a** 0.913 **b** -0.851 **c** 0.042 **d** 0.252
- 4 **a** = $\frac{1}{2}$ **b** = 0 **c** = $\frac{1}{\sqrt{2}}$ **d** = $\sqrt{3}$
e = $\frac{1}{2}$ **f** = $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ **g** = $-\tan \frac{\pi}{4} = -1$ **h** = $-\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$
i = $-\tan \frac{\pi}{3} = -\sqrt{3}$ **j** = $-\cos \frac{\pi}{4} = -\frac{1}{\sqrt{2}}$ **k** = $-\sin \frac{\pi}{6} = -\frac{1}{2}$ **l** = $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$
m = $\sin 0 = 0$ **n** = $-\tan \frac{\pi}{4} = -1$ **o** = $-\cos \frac{\pi}{3} = -\frac{1}{2}$ **p** = $-\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$
- 5 **a** (0, 0), (180, 0), (360, 0), (540, 0), (720, 0)
b (90, 1), (270, -1), (450, 1), (630, -1)
- 6 **a** (0, 0), (180, 0), (360, 0), (540, 0), (720, 0)
b $x = 90, x = 270, x = 450, x = 630$
- 7 **a** stretch by a factor of 3 in the y -direction about the x -axis
b stretch by a factor of $\frac{1}{4}$ in the x -direction about the y -axis
c translation by 60 units in the negative x -direction
d reflection in the y -axis



9 a $(-90^\circ, -2), (90^\circ, 2)$

b $(-180^\circ, 1), (0, 3), (180^\circ, 1)$

c $(-150^\circ, -1), (-90^\circ, 1), (-30^\circ, -1),$
 $(30^\circ, 1), (90^\circ, -1), (150^\circ, 1)$

d $(-135^\circ, -1), (45^\circ, 1)$

10 a 360°

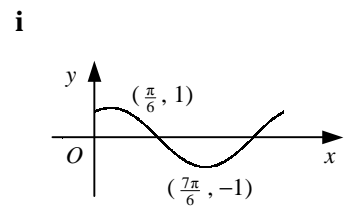
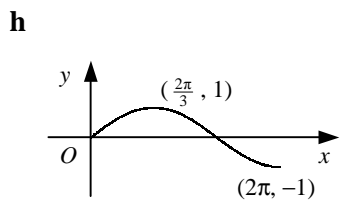
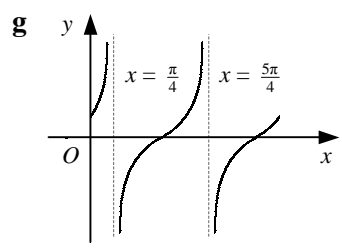
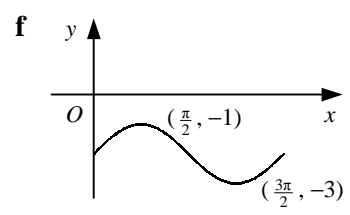
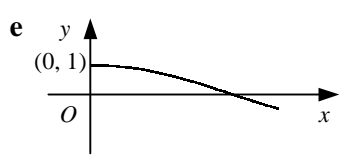
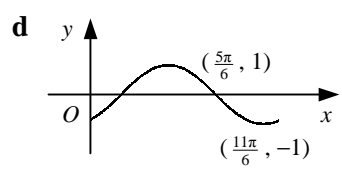
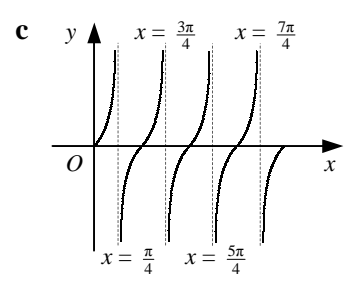
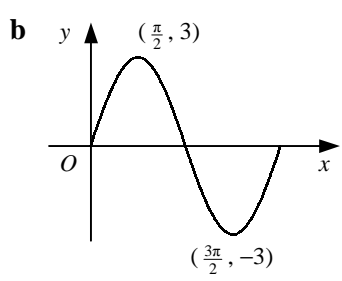
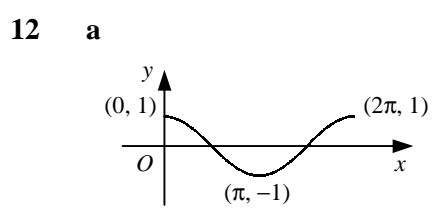
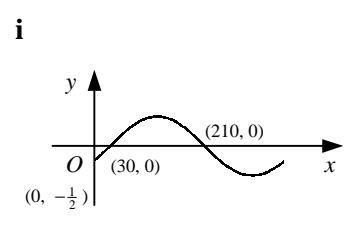
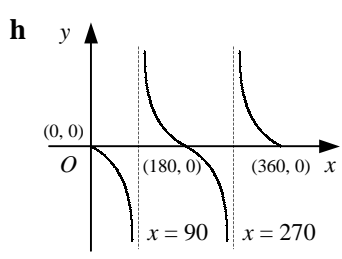
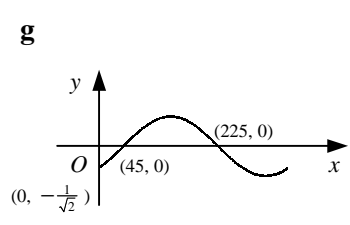
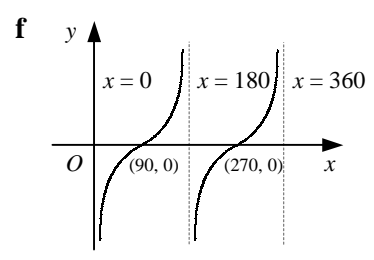
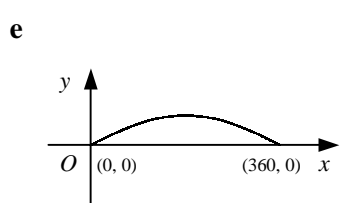
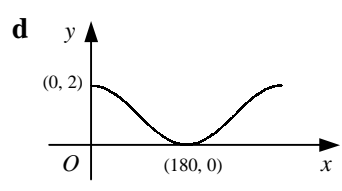
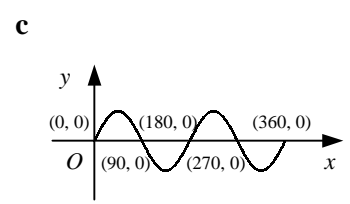
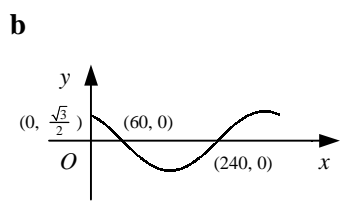
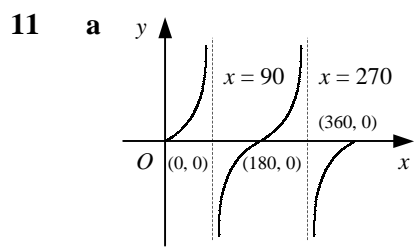
b 180°

c 360°

d 180°

e 180°

f 1080°



C2 TRIGONOMETRY

Answers - Worksheet E

- 1**
- a** $x = 30, 180 - 30$
 $x = 30^\circ, 150^\circ$
- b** $x = 60, 180 + 60$
 $x = 60^\circ, 240^\circ$
- c** $x = 90^\circ, 270^\circ$
- d** $x = 270^\circ$
- e** $x = 30, 360 - 30$
 $x = 30^\circ, 330^\circ$
- f** $x = 45, 180 - 45$
 $x = 45^\circ, 135^\circ$
- g** $x = 180 - 45, 360 - 45$
 $x = 135^\circ, 315^\circ$
- h** $x = 180 - 60, 180 + 60$
 $x = 120^\circ, 240^\circ$
- i** $x = 180 + 60, 360 - 60$
 $x = 240^\circ, 300^\circ$
- j** $x = 30, 180 + 30$
 $x = 30^\circ, 210^\circ$
- k** $x = 180 - 45, 180 + 45$
 $x = 135^\circ, 225^\circ$
- l** $x = 180 - 60, 360 - 60$
 $x = 120^\circ, 300^\circ$
- 2**
- a** $\theta = 66.4, 360 - 66.4$
 $\theta = 66.4^\circ, 293.6^\circ$
- b** $\theta = 15.7, 180 - 15.7$
 $\theta = 15.7^\circ, 164.3^\circ$
- c** $\theta = 58.0, 180 + 58.0$
 $\theta = 58.0^\circ, 238.0^\circ$
- d** $\theta = 54.4, 180 - 54.4$
 $\theta = 54.4^\circ, 125.6^\circ$
- e** $\theta = 5.7, 180 + 5.7$
 $\theta = 5.7^\circ, 185.7^\circ$
- f** $\theta = 79.3, 360 - 79.3$
 $\theta = 79.3^\circ, 280.7^\circ$
- g** $\theta = 180 + 36.9,$
 $360 - 36.9$
 $\theta = 216.9^\circ, 323.1^\circ$
- h** $\theta = 180 - 35.0,$
 $360 - 35.0$
 $\theta = 145.0^\circ, 325.0^\circ$
- i** $\theta = 180 - 67.0,$
 $180 + 67.0$
 $\theta = 113.0^\circ, 247.0^\circ$
- j** $\theta = 180 - 73.6,$
 $360 - 73.6$
 $\theta = 106.4^\circ, 286.4^\circ$
- k** $\theta = 180 - 50.5,$
 $180 + 50.5$
 $\theta = 129.5^\circ, 230.5^\circ$
- l** $\theta = 180 + 11.7,$
 $360 - 11.7$
 $\theta = 191.7^\circ, 348.3^\circ$
- 3**
- a** $x - 60 = 30, 180 - 30$
 $= 30, 150$
 $x = 90, 210$
- b** $x + 30 = 45, 180 + 45$
 $= 45, 225$
 $x = 15, 195$
- c** $x - 45 = 78.5, 360 - 78.5$
 $= 78.5, 281.5$
 $x = 123.5, 326.5$
- d** $x + 30 = 38.0, 180 + 38.0$
 $= 38.0, 218.0$
 $x = 8.0, 188.0$
- e** $x + 45 = 180 - 60, 180 + 60$
 $= 120, 240$
 $x = 75, 195$
- f** $x - 60 = 180 + 62.9, 360 - 62.9$
 $= 242.9, 297.1$
 $x = 302.9, 357.1$
- g** $x + 45 = 360 - 25.8,$
 $360 + 25.8$
 $= 334.2, 385.8$
 $x = 289.2, 340.8$
- h** $x + 30 = 180 - 8.0,$
 $360 + 8.0$
 $= 172.0, 368.0$
 $x = 142.0, 338.0$
- i** $x - 60 = -53.1, 53.1$
 $x = 6.9, 113.1$
- j** $x - 30 = -17.5, 180 + 17.5$
 $= -17.5, 197.5$
 $x = 12.5, 227.5$
- k** $x - 60 = -51.6, 180 - 51.6$
 $= -51.6, 128.4$
 $x = 8.4, 188.4$
- l** $2x = 30, 180 - 30,$
 $360 + 30, 540 - 30$
 $= 30, 150, 390, 510$
 $x = 15, 75, 195, 255$
- m** $2x = 50.208,$
 $360 - 50.208,$
 $360 + 50.208,$
 $720 - 50.208$
 $= 50.208, 309.792,$
 $410.208, 669.792$
 $x = 25.1, 154.9, 205.1, 334.9$
- n** $2x = 180 + 10.370,$
 $360 - 10.370,$
 $540 + 10.370,$
 $720 - 10.370$
 $= 190.370, 349.630,$
 $550.370, 709.630$
 $x = 95.2, 174.8, 275.2, 354.8$
- o** $2x = 180 - 69.950,$
 $360 - 69.950,$
 $540 - 69.950,$
 $720 - 69.950$
 $= 110.050, 290.050,$
 $470.050, 650.050$
 $x = 55.0, 145.0, 235.0, 325.0$
- p** $\frac{1}{2}x = 44.668, 180 - 44.668$
 $= 44.668, 135.332$
 $x = 89.3, 270.7$
- q** $3x = 30.583, 180 + 30.583,$
 $360 + 30.583,$
 $540 + 30.583,$
 $720 + 30.583,$
 $900 + 30.583$
 $= 30.583, 210.583,$
 $390.583, 570.583,$
 $750.583, 930.583$
 $x = 10.2, 70.2, 130.2$
 $190.2, 250.2, 310.2$
- r** $2x = 180 - 65.481,$
 $180 + 65.481,$
 $540 - 65.481,$
 $540 + 65.481$
 $= 114.519, 245.481,$
 $474.519, 605.481$
 $x = 57.3, 122.7, 237.3, 302.7$

4 a $x = 0, \pi, 2\pi$

b $x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$
 $x = \frac{\pi}{3}, \frac{5\pi}{3}$

c $x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$
 $x = \frac{\pi}{4}, \frac{5\pi}{4}$

d $x = \pi$

e $x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$
 $x = \frac{5\pi}{6}, \frac{11\pi}{6}$

f $x = \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$
 $x = \frac{5\pi}{4}, \frac{7\pi}{4}$

g $x + \frac{\pi}{6} = \frac{\pi}{3}, \pi + \frac{\pi}{3}$
 $= \frac{\pi}{3}, \frac{4\pi}{3}$
 $x = \frac{\pi}{6}, \frac{7\pi}{6}$

h $x - \frac{\pi}{4} = \frac{\pi}{6}, \pi - \frac{\pi}{6}$
 $= \frac{\pi}{6}, \frac{5\pi}{6}$
 $x = \frac{5\pi}{12}, \frac{13\pi}{12}$

i $x + \frac{\pi}{3} = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$
 $= \frac{5\pi}{6}, \frac{7\pi}{6}$
 $x = \frac{\pi}{2}, \frac{5\pi}{6}$

j $x + \frac{\pi}{3} = \pi - \frac{\pi}{4}, 2\pi + \frac{\pi}{4}$
 $= \frac{3\pi}{4}, \frac{9\pi}{4}$
 $x = \frac{5\pi}{12}, \frac{23\pi}{12}$

k $2x = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4},$
 $3\pi - \frac{\pi}{4}, 3\pi + \frac{\pi}{4}$
 $= \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}$
 $x = \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$

l $3x = \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi + \frac{\pi}{6},$
 $3\pi + \frac{\pi}{6}, 4\pi + \frac{\pi}{6}, 5\pi + \frac{\pi}{6}$
 $= \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}, \frac{25\pi}{6}, \frac{31\pi}{6}$
 $x = \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18}$

5 a $\theta = -90^\circ, 90^\circ$

b $\tan 2\theta = -1$
 $2\theta = 180 - 45, 360 - 45$
 $-45, -45 - 180$
 $= -225, -45, 135, 315$
 $\theta = -112.5^\circ, -22.5^\circ,$
 $67.5^\circ, 157.5^\circ$

c $\theta + 60 = 16.9, 180 - 16.9$
 $= 16.9, 163.1$
 $\theta = -43.1^\circ, 103.1^\circ$

d $\tan(\theta - 15) = 1.85$
 $\theta - 15 = 61.6, 61.6 - 180$
 $= -118.4, 61.6$
 $\theta = -103.4^\circ, 76.6^\circ$

e $\sin 2\theta = 0.3$
 $2\theta = 17.458, 180 - 17.458,$
 $17.458 - 360,$
 $-17.458 - 180$
 $= -342.542, -197.458,$
 $17.458, 162.542$
 $\theta = -171.3^\circ, -98.7^\circ$
 $8.7^\circ, 81.3^\circ$

f $\cos 3\theta = 0.5$
 $3\theta = 60, 360 - 60, 360 + 60,$
 $-60, 60 - 360, -60 - 360$
 $= -420, -300, -60,$
 $60, 300, 420$
 $\theta = -140^\circ, -100^\circ, -20^\circ$
 $20^\circ, 100^\circ, 140^\circ$

g $\sin(\theta + 110) = -1$
 $\theta + 110 = 270$
 $\theta = 160^\circ$

h $\cos(\theta - 27) = 0.6$
 $\theta - 27 = 53.1, -53.1$
 $\theta = -26.1^\circ, 80.1^\circ$

i $\tan \theta = \frac{7}{3}$
 $\theta = 66.8, 66.8 - 180$
 $\theta = -113.2^\circ, 66.8^\circ$

j $\cos 2\theta = -0.375$
 $2\theta = 180 - 67.976,$
 $180 + 67.976,$
 $67.976 - 180,$
 $-67.976 - 180$
 $= -247.976, -112.024,$
 $112.024, 247.976$
 $\theta = -124.0^\circ, -56.0^\circ,$
 $56.0^\circ, 124.0^\circ$

k $\tan(\theta + 92) = -\frac{1}{3}$
 $\theta + 92 = 180 - 18.4, -18.4$
 $= -18.4, 161.6$
 $\theta = -110.4^\circ, 69.6^\circ$

l $\sin \frac{1}{3}\theta = 0.25$
 $\frac{1}{3}\theta = 14.478$
 $\theta = 43.4^\circ$

- 6**
- a** $2x + 30 = 45, 180 + 45$
 $= 45, 225$
 $2x = 15, 195$
 $x = 7.5^\circ, 97.5^\circ$
- b** $2x - 15 = 0, 180$
 $2x = 15, 195$
 $x = 7.5^\circ, 97.5^\circ$
- c** $2x + 70 = 360 - 60, 360 + 60$
 $= 300, 420$
 $2x = 230, 350$
 $x = 115^\circ, 175^\circ$
- d** $2x + 210 = 360 + 15.070,$
 $540 - 15.070$
 $= 375.070, 524.930$
 $2x = 165.070, 314.930$
 $x = 82.5^\circ, 157.5^\circ$
- e** $2x - 38 = 180 - 50.208,$
 $180 + 50.208$
 $= 129.792, 230.208$
 $2x = 167.792, 268.208$
 $x = 83.9^\circ, 134.1^\circ$
- f** $2x - 56 = 180 - 17.745,$
 -17.745
 $= -17.745, 162.256$
 $2x = 38.256, 218.256$
 $x = 19.1^\circ, 109.1^\circ$
- g** $3x - 24 = 42.862,$
 $360 - 42.862,$
 $360 + 42.862$
 $= 42.862, 317.138,$
 402.862
 $3x = 66.862, 341.138,$
 426.862
 $x = 22.3^\circ, 113.7^\circ, 142.3^\circ$
- h** $3x + 60 = 180 - 62.241,$
 $360 - 62.241,$
 $540 - 62.241$
 $= 117.759, 297.759,$
 477.759
 $3x = 57.759, 237.759,$
 417.759
 $x = 19.3^\circ, 79.3^\circ, 139.3^\circ$
- i** $\frac{1}{2}x + 18 = 34.890$
 $\frac{1}{2}x = 16.890$
 $x = 33.8^\circ$
- 7**
- a** $x = 0.48, \pi + 0.4795$
 $x = 0.48^\circ, 3.62^\circ$
- b** $2x = 1.2503, 2\pi - 1.2503,$
 $2\pi + 1.2503, 4\pi - 1.2503$
 $= 1.2503, 5.0328,$
 $7.5335, 11.3160$
 $x = 0.63^\circ, 2.52^\circ, 3.77^\circ, 5.66^\circ$
- c** $x + \frac{\pi}{4} = \pi - 0.7754,$
 $2\pi + 0.7754$
 $= 2.3662, 7.0586$
 $x = 1.58^\circ, 6.27^\circ$
- d** $\cos x = -\frac{1}{3}$
 $x = \pi - 1.2310, \pi + 1.2310$
 $= 1.91^\circ, 4.37^\circ$
- e** $\frac{1}{2}x = 0.0901, \pi - 0.0901$
 $= 0.0901, 3.0515$
 $x = 0.18^\circ, 6.10^\circ$
- f** $2x = \pi - 0.2213, 2\pi - 0.2213$
 $3\pi - 0.2213, 4\pi - 0.2213$
 $= 2.9203, 6.0619,$
 $9.2035, 12.3451$
 $x = 1.46^\circ, 3.03^\circ, 4.60^\circ, 6.17^\circ$
- g** $\sin(x - \frac{\pi}{3}) = 0.75$
 $x - \frac{\pi}{3} = 0.8481, \pi - 0.8481$
 $= 0.8481, 2.2935$
 $x = 1.90^\circ, 3.34^\circ$
- h** $2x + \frac{\pi}{6} = 1.1071, \pi + 1.1071,$
 $2\pi + 1.1071, 3\pi + 1.1071$
 $= 1.1071, 4.2487,$
 $7.3903, 10.5319$
 $2x = 0.5835, 3.7251,$
 $6.8667, 10.0083$
 $x = 0.29^\circ, 1.86^\circ, 3.43^\circ, 5.00^\circ$
- i** $3x = \pi - 0.6266, \pi + 0.6266,$
 $3\pi - 0.6266, 3\pi + 0.6266,$
 $5\pi - 0.6266, 5\pi + 0.6266$
 $= 2.5149, 3.7682, 8.7981,$
 $10.0514, 15.0813, 16.3346$
 $x = 0.84^\circ, 1.26^\circ, 2.93^\circ,$
 $3.35^\circ, 5.03^\circ, 5.44^\circ$
- j** $\tan x = -\frac{5}{3}$
 $x = \pi - 1.0304, 2\pi - 1.0304$
 $x = 2.11^\circ, 5.25^\circ$
- k** $2x - \frac{\pi}{2} = \pi - 1.2239, \pi + 1.2239,$
 $3\pi - 1.2239, 3\pi + 1.2239$
 $= 1.9177, 4.3655,$
 $8.2009, 10.6487$
 $2x = 3.4885, 5.9363,$
 $9.7717, 12.2195$
 $x = 1.74^\circ, 2.97^\circ, 4.89^\circ, 6.11^\circ$
- l** $\sin 2x = -\frac{1}{6}$
 $2x = \pi + 0.1674, 2\pi - 0.1674,$
 $3\pi + 0.1674, 4\pi - 0.1674$
 $= 3.3090, 6.1157,$
 $9.5922, 12.3989$
 $x = 1.65^\circ, 3.06^\circ, 4.80^\circ, 6.20^\circ$

8 a $(2y - 1)(y - 1) = 0$

$$y = \frac{1}{2}, 1$$

b $\sin x = \frac{1}{2}$ or 1

$$x = 30, 180 - 30 \text{ or } 90$$

$$x = 30^\circ, 90^\circ, 150^\circ$$

9 a $\sin \theta = \pm \frac{\sqrt{3}}{2}$

$$\theta = 60, 180 - 60 \text{ or } 180 + 60, 360 - 60$$

$$\theta = 60, 120, 240, 300$$

c $\cos \theta(2 \cos \theta + 1) = 0$

$$\cos \theta = 0 \text{ or } -0.5$$

$$\theta = 90, 270 \text{ or } 180 - 60, 180 + 60$$

$$\theta = 90, 120, 240, 270$$

e $\sin \theta(4 - \tan \theta) = 0$

$$\sin \theta = 0 \text{ or } \tan \theta = 4$$

$$\theta = 0, 180, 360 \text{ or } 76.0, 180 + 76.0$$

$$\theta = 0, 76.0, 180, 256.0, 360$$

g $(\tan \theta - 1)(\tan \theta - 2) = 0$

$$\tan \theta = 1 \text{ or } 2$$

$$\theta = 45, 180 + 45 \text{ or } 63.4, 180 + 63.4$$

$$\theta = 45, 63.4, 225, 243.4$$

i $\tan^2 \theta - \tan \theta - 6 = 0$

$$(\tan \theta + 2)(\tan \theta - 3) = 0$$

$$\tan \theta = -2 \text{ or } 3$$

$$\theta = 180 - 63.4, 360 - 63.4 \text{ or } 71.6, 180 + 71.6$$

$$\theta = 71.6, 116.6, 251.6, 296.6$$

k $4 \sin^2 \theta - 8 \sin \theta + 3 = 0$

$$(2 \sin \theta - 1)(2 \sin \theta - 3) = 0$$

$$\sin \theta = 0.5 \text{ or } 1.5 \text{ [no solutions]}$$

$$\theta = 30, 180 - 30$$

$$\theta = 30, 150$$

m $\tan \theta = \frac{-3 \pm \sqrt{9+4}}{2}$

$$\tan \theta = \frac{1}{2}(-3 \pm \sqrt{13})$$

$$\theta = 180 - 73.2, 360 - 73.2 \text{ or } 16.8, 180 + 16.8$$

$$\theta = 16.8, 106.8, 196.8, 286.8$$

b $\tan \theta = \pm 1$

$$\theta = 45, 180 + 45 \text{ or } 180 - 45, 360 - 45$$

$$\theta = 45, 135, 225, 315$$

d $\sin \theta = 0$ or $\cos \theta = 0.25$

$$\theta = 0, 180, 360 \text{ or } 75.5, 360 - 75.5$$

$$\theta = 0, 75.5, 180, 284.5, 360$$

f $\cos \theta = -1$ or 0.5

$$\theta = 180 \text{ or } 60, 360 - 60$$

$$\theta = 60, 180, 300$$

h $(3 \sin \theta - 1)(\sin \theta - 2) = 0$

$$\sin \theta = \frac{1}{3} \text{ or } 2 \text{ [no solutions]}$$

$$\theta = 19.5, 180 - 19.5$$

$$\theta = 19.5, 160.5$$

j $(3 \cos \theta - 2)(2 \cos \theta + 1) = 0$

$$\cos \theta = -0.5 \text{ or } \frac{2}{3}$$

$$\theta = 180 - 60, 180 + 60 \text{ or } 48.2, 360 - 48.2$$

$$\theta = 48.2, 120, 240, 311.8$$

l $\cos \theta = \frac{-2 \pm \sqrt{4+4}}{2}$

$$\cos \theta = -1 + \sqrt{2} \text{ or } -1 - \sqrt{2} \text{ [no solutions]}$$

$$\theta = 65.5, 360 - 65.5$$

$$\theta = 65.5, 294.5$$

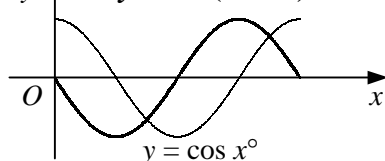
n $3 \sin^2 \theta + \sin \theta - 1 = 0$

$$\sin \theta = \frac{-1 \pm \sqrt{1+12}}{6} = \frac{1}{6}(-1 \pm \sqrt{13})$$

$$\theta = 180 + 50.1, 360 - 50.1 \text{ or } 25.7, 180 - 25.7$$

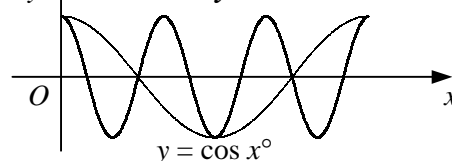
$$\theta = 25.7, 154.3, 230.1, 309.9$$

10 a, b $y = \cos(x + 90^\circ)$



c $x = 135, 315$

11 a $y = \cos 3x^\circ$



b $x = 0, 90, 180, 270, 360$

c $x = 0, 45, 90, 135, 180$

C2 TRIGONOMETRY

Answers - Worksheet F

- 1 a** $4 \sin x = -\cos x$
 $\frac{\sin x}{\cos x} = -\frac{1}{4}$
 $\tan x = -\frac{1}{4}$
- b** $x = 180 - 14.0, 360 - 14.0$
 $x = 166.0^\circ, 346.0^\circ$
- 2 a** LHS = $5 \sin^2 x + 5 \sin x + 4(1 - \sin^2 x)$
 $= \sin^2 x + 5 \sin x + 4$
 $= \text{RHS}$
- b** $(\sin x + 4)(\sin x + 1) = 0$
 $\sin x = -1$ or -4 [no solutions]
 $x = 270^\circ$
- 3 a** $2 \sin x = \cos x$
 $\tan x = 0.5$
 $x = 26.6, 180 + 26.6$
 $x = 26.6^\circ, 206.6^\circ$
- c** $1 - \sin^2 x + 3 \sin x - 3 = 0$
 $\sin^2 x - 3 \sin x + 2 = 0$
 $(\sin x - 1)(\sin x - 2) = 0$
 $\sin x = 1$ or 2 [no solutions]
 $x = 90^\circ$
- d** $3 \cos^2 x - (1 - \cos^2 x) = 2$
 $4 \cos^2 x = 3$
 $\cos x = \pm \frac{\sqrt{3}}{2}$
 $x = 30, 360 - 30$ or $180 - 30, 180 + 30$
 $x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$
- e** $2(1 - \cos^2 x) + 3 \cos x = 3$
 $2 \cos^2 x - 3 \cos x + 1 = 0$
 $(2 \cos x - 1)(\cos x - 1) = 0$
 $\cos x = 0.5$ or 1
 $x = 60, 360 - 60$ or $0, 360$
 $x = 0, 60^\circ, 300^\circ, 360^\circ$
- f** $3(1 - \sin^2 x) = 5(1 - \sin x)$
 $3 \sin^2 x - 5 \sin x + 2 = 0$
 $(3 \sin x - 2)(\sin x - 1) = 0$
 $\sin x = \frac{2}{3}$ or 1
 $x = 41.8, 180 - 41.8$ or 90
 $x = 41.8^\circ, 90^\circ, 138.2^\circ$
- g** $3 \sin^2 x = 8 \cos x$
 $3(1 - \cos^2 x) = 8 \cos x$
 $3 \cos^2 x + 8 \cos x - 3 = 0$
 $(3 \cos x - 1)(\cos x + 3) = 0$
 $\cos x = \frac{1}{3}$ or -3 [no solutions]
 $x = 70.5, 360 - 70.5$
 $x = 70.5^\circ, 289.5^\circ$
- h** $\cos^2 x = 3 \sin x$
 $1 - \sin^2 x = 3 \sin x$
 $\sin^2 x + 3 \sin x - 1 = 0$
 $\sin x = \frac{-3 \pm \sqrt{9+4}}{2}$
 $\sin x = \frac{1}{2}(-3 + \sqrt{13})$ or $\frac{1}{2}(-3 - \sqrt{13})$ [no sols]
 $x = 17.6, 180 - 17.6$
 $x = 17.6^\circ, 162.4^\circ$
- i** $3(1 - \cos^2 x) - 5 \cos x + 2 \cos^2 x = 0$
 $\cos^2 x + 5 \cos x - 3 = 0$
 $\cos x = \frac{-5 \pm \sqrt{25+12}}{2}$
 $\cos x = \frac{1}{2}(-5 + \sqrt{37})$ or $\frac{1}{2}(-5 - \sqrt{37})$ [no sols]
 $x = 57.2, 360 - 57.2$
 $x = 57.2^\circ, 302.8^\circ$
- j** $2 \sin^2 x + 7 \sin x - 2(1 - \sin^2 x) = 0$
 $4 \sin^2 x + 7 \sin x - 2 = 0$
 $(4 \sin x - 1)(\sin x + 2) = 0$
 $\sin x = 0.25$ or -2 [no solutions]
 $x = 14.5, 180 - 14.5$
 $x = 14.5^\circ, 165.5^\circ$
- k** $3 \sin x = 2 \tan x$
 $3 \sin x \cos x = 2 \sin x$
 $\sin x (3 \cos x - 2) = 0$
 $\sin x = 0$ or $\cos x = \frac{2}{3}$
 $x = 0, 180, 360$ or $48.2, 360 - 48.2$
 $x = 0, 48.2^\circ, 180^\circ, 311.8^\circ, 360^\circ$
- l** $(1 - \cos^2 x) - 9 \cos x - \cos^2 x = 5$
 $2 \cos^2 x + 9 \cos x + 4 = 0$
 $(2 \cos x + 1)(\cos x + 4) = 0$
 $\cos x = -0.5$ or -4 [no solutions]
 $x = 180 - 60, 180 + 60$
 $x = 120^\circ, 240^\circ$

- 4 a** $\cos \theta = \pm 0.5$
 $\theta = \frac{\pi}{3}, -\frac{\pi}{3}$ or $\pi - \frac{\pi}{3}, -\pi + \frac{\pi}{3}$
 $\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$
- b** $(2 \sin \theta + 1)^2 = 0$
 $\sin \theta = -0.5$
 $\theta = -\frac{\pi}{6}, -\pi + \frac{\pi}{6}$
 $\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}$
- c** $(\cos \theta + 3)(\cos \theta - 1) = 0$
 $\cos \theta = 1$ or -3 [no solutions]
 $\theta = 0$
- d** $3 \sin^2 \theta - (1 - \sin^2 \theta) = 0$
 $4 \sin^2 \theta = 1$
 $\sin \theta = \pm 0.5$
 $\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$ or $-\frac{\pi}{6}, -\pi + \frac{\pi}{6}$
 $\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$
- e** $4 \sin^2 \theta - 5 \sin \theta + 2(1 - \sin^2 \theta) = 0$
 $2 \sin^2 \theta - 5 \sin \theta + 2 = 0$
 $(2 \sin \theta - 1)(\sin \theta - 2) = 0$
 $\sin \theta = 0.5$ or 2 [no solutions]
 $\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$
 $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$
- f** $(1 - \cos^2 \theta) - 3 \cos \theta - \cos^2 \theta = 2$
 $2 \cos^2 \theta + 3 \cos \theta + 1 = 0$
 $(2 \cos \theta + 1)(\cos \theta + 1) = 0$
 $\cos \theta = -0.5$ or -1
 $\theta = \pi - \frac{\pi}{3}, -\pi + \frac{\pi}{3}$ or $-\pi, \pi$
 $\theta = -\pi, -\frac{2\pi}{3}, \frac{2\pi}{3}, \pi$
- 5 a** LHS = $\sin^2 x + 2 \sin x \cos x + \cos^2 x$
 $= (\sin^2 x + \cos^2 x) + 2 \sin x \cos x$
 $= 1 + 2 \sin x \cos x$
 $= \text{RHS}$
- b** LHS = $\frac{1 - \cos^2 x}{\cos x}$
 $= \frac{\sin^2 x}{\cos x}$
 $= \sin x \times \frac{\sin x}{\cos x}$
 $= \sin x \tan x$
 $= \text{RHS}$
- c** LHS = $\frac{1 - \sin^2 x}{1 - \sin x}$
 $= \frac{(1 + \sin x)(1 - \sin x)}{1 - \sin x}$
 $= 1 + \sin x$
 $= \text{RHS}$
- d** LHS = $\frac{(1 + \sin x)(1 - \sin x)}{\cos x(1 - \sin x)}$
 $= \frac{1 - \sin^2 x}{\cos x(1 - \sin x)}$
 $= \frac{\cos^2 x}{\cos x(1 - \sin x)}$
 $= \frac{\cos x}{1 - \sin x}$
 $= \text{RHS}$
- 6 a** LHS = $\cos^2 x - 2 \cos x \tan x + \tan^2 x$
 $+ \sin^2 x + 2 \sin x + 1$
 $= \cos^2 x - 2 \sin x + \tan^2 x$
 $+ \sin^2 x + 2 \sin x + 1$
 $= (\cos^2 x + \sin^2 x) + \tan^2 x + 1$
 $= 2 + \tan^2 x = \text{RHS}$
- b** $2 + \tan^2 x = 3$
 $\tan^2 x = 1$
 $\tan x = \pm 1$
 $x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$ or $\pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$
 $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
- 7 a** $f(x) = (1 - \sin^2 x) + 2 \sin x$
 $= 2 - (\sin^2 x - 2 \sin x + 1)$
 $= 2 - (\sin x - 1)^2$
- b** max. value of $f(x) = 2$
occurs when $\sin x = 1 \therefore x = \frac{\pi}{2}$

C2 TRIGONOMETRY

Answers - Worksheet G

1 a $\tan x = \frac{1}{\sqrt{3}}$

$$x = \frac{\pi}{6}, \pi + \frac{\pi}{6}$$

$$x = \frac{\pi}{6}, \frac{7\pi}{6}$$

b $\cos(x + \frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$

$$x + \frac{\pi}{3} = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$$

$$= \frac{5\pi}{6}, \frac{7\pi}{6}$$

$$x = \frac{\pi}{2}, \frac{5\pi}{6}$$

3 a $45^\circ = \frac{\pi}{4}$

$$P = (2 \times 8) + (8 \times \frac{\pi}{4}) = 22.3 \text{ cm}$$

b area of sector $= \frac{1}{2} \times 8^2 \times \frac{\pi}{4} = 8\pi$

$$\begin{aligned} \text{area of triangle} &= \frac{1}{2} \times 8^2 \times \sin \frac{\pi}{4} \\ &= 32 \times \frac{1}{\sqrt{2}} = 16\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{area of segment} &= 8\pi - 16\sqrt{2} \\ &= 8(\pi - 2\sqrt{2}) \text{ cm}^2 \end{aligned}$$

5 $3 \sin^2 x + 4 \sin x - 4 = 0$

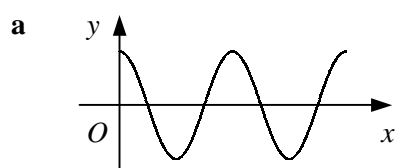
$$(3 \sin x - 2)(\sin x + 2) = 0$$

$$\sin x = \frac{2}{3} \text{ or } -2 \text{ [no solutions]}$$

$$x = 0.73, \pi - 0.7297$$

$$x = 0.73^\circ, 2.41^\circ$$

7



b $2x = 180 - 60, 180 + 60,$
 $540 - 60, 540 + 60$
 $= 120, 240, 480, 600$
 $x = 60, 120, 240, 300$

2 a $\cos^2 A = (\sqrt{3} - 1)^2 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$

$$\sin^2 A = 1 - \cos^2 A = 2\sqrt{3} - 3$$

b $\tan^2 A = \frac{\sin^2 A}{\cos^2 A}$
 $= \frac{2\sqrt{3}-3}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}} = \frac{(2\sqrt{3}-3)(4+2\sqrt{3})}{16-12}$
 $= \frac{8\sqrt{3}+12-12-6\sqrt{3}}{4} = \frac{2\sqrt{3}}{4}$
 $= \frac{\sqrt{3}}{2}$

4 $2 \sin^2 \theta + \sin \theta - (1 - \sin^2 \theta) = 2$

$$3 \sin^2 \theta + \sin \theta - 3 = 0$$

$$\sin \theta = \frac{-1 \pm \sqrt{1+36}}{6}$$

$$\sin \theta = \frac{1}{6}(-1 + \sqrt{37})$$

$$\text{or } \frac{1}{6}(-1 - \sqrt{37}) \text{ [no solutions]}$$

$$\theta = 57.9, 180 - 57.9$$

$$\theta = 57.9^\circ, 122.1^\circ$$

6 $2 \sin x = 3 \cos x$

$$\tan x = 1.5$$

$$x = 0.98, \pi + 0.9828 = 0.98, 4.12$$

$$\therefore (0.98, 1.66), (4.12, -1.66)$$

8 $12 \cos^2 \theta = 7 \sin \theta$

$$12(1 - \sin^2 \theta) = 7 \sin \theta$$

$$12 \sin^2 \theta + 7 \sin \theta - 12 = 0$$

$$(4 \sin \theta - 3)(3 \sin \theta + 4) = 0$$

$$\sin \theta = 0.75 \text{ or } -\frac{4}{3} \text{ [no solutions]}$$

$$\theta = 48.6, 180 - 48.6$$

$$\theta = 48.6, 131.4$$

$$\begin{aligned}
 9 \quad \mathbf{a} \quad \tan 15^\circ &= \frac{\sqrt{3}-1}{1+(\sqrt{3}\times 1)} = \frac{\sqrt{3}-1}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}} \\
 &= \frac{(\sqrt{3}-1)(1-\sqrt{3})}{1-3} \\
 &= -\frac{1}{2}(\sqrt{3}-3-1+\sqrt{3}) \\
 &= 2 - \sqrt{3}
 \end{aligned}$$

$$\mathbf{b} \quad \tan 345^\circ = -\tan 15^\circ = \sqrt{3} - 2$$

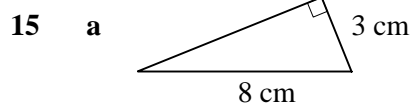
$$\begin{aligned}
 11 \quad \mathbf{a} \quad \angle ABC &= 180 - (41 + 26) = 113 \\
 \frac{BC}{\sin 41} &= \frac{18}{\sin 113} \\
 BC &= \frac{18 \times \sin 41}{\sin 113} = 12.8 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} &= \frac{1}{2} \times 18 \times 12.829 \times \sin 26 \\
 &= 50.6 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 13 \quad \sin^2 x + 5 \sin x &= 2(1 - \sin^2 x) \\
 3 \sin^2 x + 5 \sin x - 2 &= 0 \\
 (3 \sin x - 1)(\sin x + 2) &= 0 \\
 \sin x &= \frac{1}{3} \quad \text{or} \quad -2 \text{ [no solutions]}
 \end{aligned}$$

$$x = 19.5, 180 - 19.5$$

$$x = 19.5^\circ, 160.5^\circ$$



$$\cos(\angle PQR) = \frac{3}{8}$$

$$\therefore \angle PQR = 1.186^\circ$$

$$\begin{aligned}
 \mathbf{b} \quad RS^2 &= 8^2 - 3^2 = 55 \\
 RS &= \sqrt{55} = 7.42 \text{ cm (3sf)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad \text{obtuse } \angle SPU &= 2 \times \angle PQR = 2.3728 \\
 \text{reflex } \angle RQT &= 2\pi - \angle SPU = 3.9104 \\
 \text{length of rubber band} \\
 &= (2 \times 7.4162) + (2 \times 2.3728) + (5 \times 3.9104) \\
 &= 39.1 \text{ cm (3sf)}
 \end{aligned}$$

$$10 \quad (1 - \cos^2 x) + 5 \cos x - 3 \cos^2 x = 2$$

$$4 \cos^2 x - 5 \cos x + 1 = 0$$

$$(4 \cos x - 1)(\cos x - 1) = 0$$

$$\cos x = 0.25 \quad \text{or} \quad 1$$

$$x = 75.5, 360 - 75.5 \quad \text{or} \quad 0, 360$$

$$x = 0, 75.5^\circ \text{ (1dp)}, 284.5^\circ \text{ (1dp)}, 360^\circ$$

$$12 \quad 6 \cos^2 \theta + 5 \cos \theta - 4 = 0$$

$$(3 \cos \theta + 4)(2 \cos \theta - 1) = 0$$

$$\cos \theta = 0.5 \quad \text{or} \quad -\frac{4}{3} \text{ [no solutions]}$$

$$\theta = 60, 360 - 60$$

$$\theta = 60^\circ, 300^\circ$$

$$\begin{aligned}
 14 \quad \mathbf{a} \quad \text{LHS} &= (1 - \cos^2 \theta)^2 - 2(1 - \cos^2 \theta) \\
 &= 1 - 2 \cos^2 \theta + \cos^4 \theta - 2 + 2 \cos^2 \theta \\
 &= \cos^4 \theta - 1 \\
 &= \text{RHS}
 \end{aligned}$$

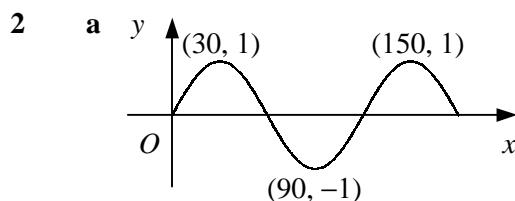
$$\begin{aligned}
 \mathbf{b} \quad \text{LHS} &= \frac{\sin^2 \theta + (1 + \cos \theta)^2}{\sin \theta (1 + \cos \theta)} \\
 &= \frac{\sin^2 \theta + 1 + 2 \cos \theta + \cos^2 \theta}{\sin \theta (1 + \cos \theta)} \\
 &= \frac{2 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)} \\
 &= \frac{2(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)} \\
 &= \frac{2}{\sin \theta} \\
 &= \text{RHS}
 \end{aligned}$$

C2 TRIGONOMETRY

Answers - Worksheet H

1 a $\theta + \frac{\pi}{4} = \pi - 0.4115, 2\pi + 0.4115$
 $= 2.7301, 6.6947$
 $\theta = 1.94^\circ, 5.91^\circ$

b $\cos 2\theta = \frac{1}{3}$
 $2\theta = 1.2310, 2\pi - 1.2310$
 $2\pi + 1.2310, 4\pi - 1.2310$
 $= 1.2310, 5.0522, 7.5141, 11.3354$
 $\theta = 0.62^\circ, 2.53^\circ, 3.76^\circ, 5.67^\circ$



b $(\tan \theta + 1)(\tan \theta - 3) = 0$
 $\tan \theta = -1$ or 3
 $\theta = 180 - 45, 360 - 45$ or $71.6, 180 + 71.6$
 $\theta = 71.6^\circ$ (1dp), $135^\circ, 251.6^\circ$ (1dp), 315°

3 a $260^\circ = \frac{260}{180}\pi = 4.538$ radians

b $P = (2 \times 6.4) + (6.4 \times 4.538)$
 $= 41.8$ cm (3sf)

c $A = \frac{1}{2} \times (6.4)^2 \times 4.538$
 $= 92.9$ cm² (3sf)

4 $3 \cos^2 \theta + 6 \cos \theta = 2(1 - \cos^2 \theta) + 6$
 $5 \cos^2 \theta + 6 \cos \theta - 8 = 0$
 $(5 \cos \theta - 4)(\cos \theta + 2) = 0$
 $\cos \theta = 0.8$ or -2 [no solutions]
 $\theta = 36.9, 360 - 36.9$
 $\theta = 36.9^\circ, 323.1^\circ$

5 a $\text{area} = \frac{1}{2} \times 4 \times 5 \times \sin 60^\circ$
 $= 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$ cm²

b $AB^2 = 4^2 + 5^2 - (2 \times 4 \times 5 \times \cos 60^\circ)$
 $= 16 + 25 - (40 \times \frac{1}{2}) = 21$
 $\therefore AB = \sqrt{21}$ cm

c $\frac{\sin(\angle ABC)}{4} = \frac{\sin 60^\circ}{\sqrt{21}}$
 $\therefore \sin(\angle ABC) = \frac{4 \times \frac{\sqrt{3}}{2}}{\sqrt{3}\sqrt{7}} = \frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$
 $= \frac{2}{7}\sqrt{7}$

6 $2x + 15 = 63.435, 180 + 63.435,$
 $360 + 63.435, 540 + 63.435$
 $= 63.435, 243.435, 423.435, 603.435$
 $2x = 48.435, 228.435, 408.435, 588.435$
 $x = 24.2, 114.2, 204.2, 294.2$

7 $\sin^2 \theta - \cos^2 \theta = \cos \theta$
 $(1 - \cos^2 \theta) - \cos^2 \theta = \cos \theta$
 $2 \cos^2 \theta + \cos \theta - 1 = 0$
 $(2 \cos \theta - 1)(\cos \theta + 1) = 0$
 $\cos \theta = 0.5$ or -1
 $\theta = 60, 360 - 60$ or 180
 $\theta = 60^\circ, 180^\circ, 300^\circ$

8 a $(x - 5)^2 - 25 + (y - 1)^2 - 1 - 3 = 0$
 $(x - 5)^2 + (y - 1)^2 = 29$
 \therefore centre $(5, 1)$ radius $\sqrt{29}$

b sub. $x^2 + 36 - 10x - 12 - 3 = 0$
 $x^2 - 10x + 21 = 0$
 $(x - 3)(x - 7) = 0$
 $x = 3, 7$
 $\therefore (3, 6)$ and $(7, 6)$

c mid-point of chord = $(5, 6)$
angle of sector = $2 \times \tan^{-1} \frac{2}{5} = 0.761^\circ$
area = $\frac{1}{2} r^2 (\theta - \sin \theta)$
 $= \frac{29}{2} (0.761 - \sin 0.761) = 1.03$ (3sf)

C2 TRIGONOMETRY*Answers - Worksheet H page 2*

9 $5 \sin^2 \theta + 5 \sin \theta + 2(1 - \sin^2 \theta) = 0$
 $3 \sin^2 \theta + 5 \sin \theta + 2 = 0$
 $(3 \sin \theta + 2)(\sin \theta + 1) = 0$
 $\sin \theta = -\frac{2}{3}$ or -1
 $\theta = 180 + 41.8, 360 - 41.8$ or 270
 $\theta = 221.8^\circ$ (1dp), 270° , 318.2° (1dp)

10 **a** $(158^\circ, 0), (338^\circ, 0)$
b $(0, \tan 22^\circ) = (0, 0.404)$ [y-coord to 3sf]
c $x = 68^\circ$ and $x = 248^\circ$

11 **a** $\tan x = 0.4$
 $x = 21.8, 180 + 21.8$
 $x = 21.8^\circ, 201.8^\circ$
b $2 \sin^2 y - \sin y - 1 = 0$
 $(2 \sin y + 1)(\sin y - 1) = 0$
 $\sin y = -0.5$ or 1
 $y = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$ or $\frac{\pi}{2}$
 $y = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

12 $3 \cos^2 \theta - 5 \cos \theta + 2(1 - \cos^2 \theta) = 0$
 $\cos^2 \theta - 5 \cos \theta + 2 = 0$
 $\cos \theta = \frac{5 \pm \sqrt{25 - 8}}{2}$
 $\cos \theta = \frac{1}{2}(5 - \sqrt{17})$ or $\frac{1}{2}(5 + \sqrt{17})$ [no sols]
 $\theta = -64.0^\circ, 64.0^\circ$

13 **a** $60^\circ = \frac{\pi}{3}$
 $\text{area} = \frac{1}{2} \times a^2 \times \frac{\pi}{3} = \frac{1}{6} \pi a^2$
b $OC = OA \cos 60^\circ = \frac{1}{2} a$
c $\text{area of triangle } OAC = \frac{1}{2} \times a \times \frac{1}{2} a \times \sin 60^\circ$
 $= \frac{1}{4} a^2 \times \frac{\sqrt{3}}{2} = \frac{1}{8} a^2 \sqrt{3}$
 $\text{shaded area} = \frac{1}{6} \pi a^2 - \frac{1}{8} a^2 \sqrt{3}$
 $= \frac{1}{24} a^2 (4\pi - 3\sqrt{3})$

C2 TRIGONOMETRY

Answers - Worksheet 1

1 a $x + 40 = \pm 72.5$

$$x = -112.5^\circ, 32.5^\circ$$

b $\tan 2x = -2$

$$2x = 180 - 63.435, 360 - 63.435, \\ -63.435, -180 - 63.435$$

$$= -243.435, -63.435, 116.565, 296.565$$

$$x = -121.7^\circ, -31.7^\circ, 58.3^\circ, 148.3^\circ$$

2 $\tan x = \frac{4 \pm \sqrt{16-8}}{4} = 1 \pm \frac{1}{2}\sqrt{2}$

$$x = 59.6, 180 + 59.6 \text{ or } 16.3, 180 + 16.3$$

$$x = 16.3, 59.6, 196.3, 239.6$$

3 a $15\theta = 32.1$

$$\theta = 32.1 \div 15 = 2.14$$

b $A = \frac{1}{2} \times 15^2 \times 2.14$

$$= 240.75 \text{ cm}^2$$

4 $2x - \frac{\pi}{3} = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$$= \frac{\pi}{6}, \frac{5\pi}{6}$$

$$2x = \frac{\pi}{2}, \frac{7\pi}{6}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{12}$$

5 a $\sin^2 A = (1 - \sqrt{2})^2$

$$= 1 - 2\sqrt{2} + 2 = 3 - 2\sqrt{2}$$

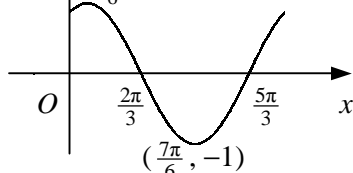
$$\cos^2 A = 1 - \sin^2 A = 2\sqrt{2} - 2$$

$$\therefore \cos^2 A + 2 \sin A$$

$$= 2\sqrt{2} - 2 + 2(1 - \sqrt{2})$$

$$\therefore \cos^2 A + 2 \sin A = 0$$

b y $(\frac{\pi}{6}, 1)$



6 $2 \sin^2 x + \sin x + 1 = 1 - \sin^2 x$

$$3 \sin^2 x + \sin x = 0$$

$$\sin x (3 \sin x + 1) = 0$$

$$\sin x = 0 \text{ or } -\frac{1}{3}$$

$$x = 0, 180, 360 \text{ or } 180 + 19.5, 360 - 19.5$$

$$x = 0, 180^\circ, 199.5^\circ (1\text{dp}), 340.5^\circ (1\text{dp}), 360^\circ$$

7 a $\frac{\sin(\angle PRQ)}{10} = \frac{\sin 0.7}{14}$

$$\sin(\angle PRQ) = \frac{10 \times \sin 0.7}{14} = 0.4602$$

$$\angle PRQ = 0.48^\circ$$

b $\angle PQR = \pi - (0.7 + 0.4782) = 1.963$

$$\text{area of } \Delta = \frac{1}{2} \times 10 \times 14 \times \sin 1.963 \\ = 64.67$$

$$\text{area of sector} = \frac{1}{2} \times 10^2 \times 0.7 \\ = 35$$

$$\text{shaded area} = 64.67 - 35 \\ = 29.7 \text{ cm}^2 (3\text{sf})$$

8 a i $\cos^2 A = 1 - \sin^2 A = 1 - \frac{5}{9} = \frac{4}{9}$

$$\cos A = \pm \sqrt{\frac{4}{9}} = \pm \frac{2}{3}$$

$$0 < A < 90 \therefore \cos A = \frac{2}{3}$$

ii $\tan A = \frac{\sin A}{\cos A} = \frac{\sqrt{5}}{3} \div \frac{2}{3} = \frac{1}{2}\sqrt{5}$

b $\cos x (5 \sin x + 1) = 0$

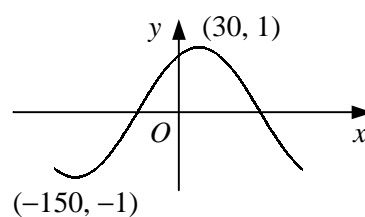
$$\cos x = 0 \text{ or } \sin x = -0.2$$

$$x = 90, 270 \text{ or } 180 + 11.5, 360 - 11.5$$

$$x = 90^\circ, 191.5^\circ (1\text{dp}), 270^\circ, 348.5^\circ (1\text{dp})$$

$$\begin{aligned}
 9 \quad 2\theta + 30 &= 180 - 60, 180 + 60 \\
 &= 120, 240 \\
 2\theta &= 90, 210 \\
 \theta &= 45, 105
 \end{aligned}$$

10 a



$$\begin{aligned}
 \text{b} \quad \cos(x - 30) &= 0.2 \\
 x - 30 &= \pm 78.5 \\
 x &= -48.5, 108.5
 \end{aligned}$$

$$\begin{aligned}
 11 \quad 4\cos^2 x - \cos x - 2(1 - \cos^2 x) &= 0 \\
 6\cos^2 x - \cos x - 2 &= 0 \\
 (3\cos x - 2)(2\cos x + 1) &= 0 \\
 \cos x &= \frac{2}{3} \text{ or } -0.5 \\
 x &= 48.2, 360 - 48.2 \text{ or } 180 - 60, 180 + 60 \\
 x &= 48.2^\circ \text{ (1dp), } 120^\circ, 240^\circ, 311.8^\circ \text{ (1dp)}
 \end{aligned}$$

$$\begin{aligned}
 12 \quad \text{a} \quad \text{area of sector} &= \frac{1}{2} \times r^2 \times \theta \\
 \text{area of triangle} &= \frac{1}{2} \times r^2 \times \sin \theta \\
 A_1 &= \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta \\
 &= \frac{1}{2} r^2 (\theta - \sin \theta) \text{ cm}^2 \\
 \text{b} \quad \theta &= \frac{5\pi}{6} \quad \therefore A_1 = \frac{1}{2} r^2 \left(\frac{5\pi}{6} - \frac{1}{2} \right) \\
 &= \frac{1}{12} r^2 (5\pi - 3) \\
 A_2 &= \pi r^2 - A_1 = \pi r^2 - \left(\frac{5}{12} \pi r^2 - \frac{1}{4} r^2 \right) \\
 &= \frac{7}{12} \pi r^2 + \frac{1}{4} r^2 \\
 &= \frac{1}{12} r^2 (7\pi + 3) \\
 \therefore A_1 : A_2 &= \frac{1}{12} r^2 (5\pi - 3) : \frac{1}{12} r^2 (7\pi + 3) \\
 &= (5\pi - 3) : (7\pi + 3)
 \end{aligned}$$

$$\begin{aligned}
 13 \quad 3\sin x - 2\cos^2 x &= 0 \\
 3\sin x - 2(1 - \sin^2 x) &= 0 \\
 2\sin^2 x + 3\sin x - 2 &= 0 \\
 (2\sin x - 1)(\sin x + 2) &= 0 \\
 \sin x &= 0.5 \text{ or } -2 \text{ [no solutions]} \\
 x &= \frac{\pi}{6}, \pi - \frac{\pi}{6} \\
 x &= \frac{\pi}{6}, \frac{5\pi}{6}
 \end{aligned}$$

$$\begin{aligned}
 14 \quad \text{a} \quad 7^2 &= 5^2 + 8^2 - [2 \times 5 \times 8 \times \cos(\angle ABC)] \\
 \cos(\angle ABC) &= \frac{25 + 64 - 49}{80} \\
 &= \frac{1}{2} \\
 \text{b} \quad \sin(\angle ABC) &= \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2} \\
 \text{area} &= \frac{1}{2} \times 5 \times 8 \times \frac{\sqrt{3}}{2} \\
 &= 10\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 15 \quad \text{a} \quad \text{LHS} &= 2 + 2\tan^2 \theta + \cos^2 \theta + \sin^2 \theta \\
 &= 2 + 2\tan^2 \theta + 1 \\
 &= 3 + 2\tan^2 \theta \\
 &= \text{RHS} \\
 \text{b} \quad 3 + 2\tan^2 \theta &= 7 \\
 \tan^2 \theta &= 2 \\
 \tan \theta &= \pm\sqrt{2} \\
 \theta &= 54.7, 180 + 54.7 \\
 &\text{or } 180 - 54.7, 360 - 54.7 \\
 \theta &= 54.7^\circ, 125.3^\circ, 234.7^\circ, 305.3^\circ \text{ (1dp)}
 \end{aligned}$$