

C2 SEQUENCES AND SERIES

Answers - Worksheet A

- 1** **a** $r = 3$
 $u_8 = 3 \times 3^7 = 6561$
- b** $r = \frac{1}{4}$
 $u_8 = 1024 \times (\frac{1}{4})^7 = \frac{1}{16}$
- c** $r = -2$
 $u_8 = 1 \times (-2)^7 = -128$
- 2** **a** $a = 1, r = 5$
 $u_n = 5^{n-1}$
- b** $a = 3, r = -4$
 $u_n = 3 \times (-4)^{n-1}$
- c** $a = 81, r = \frac{2}{3}$
 $u_n = 81 \times (\frac{2}{3})^{n-1}$
- 3** **a** $a = 2, r = 2, n = 12$
 $S_{12} = \frac{2(2^{12} - 1)}{2 - 1} = 8190$
- b** $a = 640, r = \frac{1}{2}, n = 12$
 $S_{12} = \frac{640[1 - (\frac{1}{2})^{12}]}{1 - \frac{1}{2}} = 1279\frac{11}{16}$
- c** $a = \frac{1}{6}, r = -3, n = 12$
 $S_{12} = \frac{\frac{1}{6}[1 - (-3)^{12}]}{1 - (-3)} = -22\,143\frac{1}{3}$
- 4** **a** $S_8 = \frac{4(3^8 - 1)}{3 - 1} = 13\,120$
- b** $S_{14} = \frac{48[1 - (\frac{1}{2})^{14}]}{1 - \frac{1}{2}} = 95.994$
- c** $S_{12} = \frac{-[1 - (-4)^{12}]}{1 - (-4)} = 3\,355\,443$
- d** $S_{20} = \frac{200[1 - (0.7)^{20}]}{1 - 0.7} = 666.135$
- e** $S_{15} = \frac{120[1 - (\frac{3}{4})^{15}]}{1 - (\frac{3}{4})} = 69.488$
- f** $S_{30} = \frac{-25[(1.2)^{30} - 1]}{1.2 - 1} = -29\,547.039$
- 5** **a** GP: $a = 3$
 $r = 3, n = 9$
 $S_9 = \frac{3(3^9 - 1)}{3 - 1} = 29\,523$
- b** GP: $a = 64$
 $r = 8, n = 6$
 $S_6 = \frac{64(8^6 - 1)}{8 - 1} = 2\,396\,736$
- c** GP: $a = 20$
 $r = 2, n = 10$
 $S_{10} = \frac{20(2^{10} - 1)}{2 - 1} = 20\,460$
- d** GP: $a = 0.8$
 $r = 0.8, n = 8$
 $S_8 = \frac{0.8[1 - (0.8)^8]}{1 - 0.8} = 3.329$ (3dp)
- e** GP: $a = 2$
 $r = \frac{1}{6}, n = 10$
 $S_{10} = \frac{2[1 - (\frac{1}{6})^{10}]}{1 - \frac{1}{6}} = 2.400$ (3dp)
- f** GP: $a = -4$
 $r = -4, n = 9$
 $S_9 = \frac{-4[1 - (-4)^9]}{1 - (-4)} = -209\,716$
- g** GP: $a = \frac{1}{16}$
 $r = \frac{1}{2}, n = 17$
 $S_{17} = \frac{\frac{1}{16}[1 - (\frac{1}{2})^{17}]}{1 - \frac{1}{2}} = 0.125$ (3dp)
- h** GP: $a = -54$
 $r = -3, n = 7$
 $S_7 = \frac{-54[1 - (-3)^7]}{1 - (-3)} = -29\,538$
- 6** **a** $r = 10 \div 2 = 5$
- b** $a \times 5 = 2 \therefore a = 0.4$
- c** $S_8 = \frac{0.4(5^8 - 1)}{5 - 1} = 39\,062.4$
- 7** **a** $a = 2, ar^3 = 54 \therefore r^3 = 54 \div 2 = 27$
 $r = \sqrt[3]{27} = 3$
- b** $u_9 = 2 \times 3^8 = 13\,122$
- 8** **a** $r = 8 \div 24 = \frac{1}{3}$
- b** $a \times (\frac{1}{3})^2 = 24 \therefore a = 216$
- c** $S_{11} = \frac{216[1 - (\frac{1}{3})^{11}]}{1 - \frac{1}{3}} = 323.998$
- 9** **a** $a = 6, ar^2 = 24 \therefore r^2 = 24 \div 6 = 4$
 $r = \pm 2$
- b** $r = 2, S_{15} = \frac{6(2^{15} - 1)}{2 - 1} = 196\,602$
- 10** **a** $a = 768, ar^3 = -96$
 $r^3 = -96 \div 768 = -\frac{1}{8}$
 $r = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$
- b** $u_{10} = 768 \times (-\frac{1}{2})^9 = -1.5$
- 11** **a** $ar = 0.5, ar^4 = 32 \therefore r^3 = 32 \div 0.5 = 64$
 $r = \sqrt[3]{64} = 4, a \times 4 = 0.5 \therefore a = 0.125$
- b** $0.125 \times 4^{n-1} < 10\,000 \therefore 4^{n-1} < 80\,000$
 $(n - 1) \lg 4 < \lg 80\,000$
 $n < \frac{\lg 80000}{\lg 4} + 1$
 $n < 9.14 \therefore 9$ terms

- 12 a** $\frac{a[(\frac{3}{2})^4 - 1]}{\frac{3}{2} - 1} = 130$
 $a = 130 \div \frac{65}{8} = 16$
b $u_8 = 16 \times (\frac{3}{2})^7 = 273\frac{3}{8}$
c $\frac{16[(\frac{3}{2})^n - 1]}{\frac{3}{2} - 1} > 30\,000$
 $(\frac{3}{2})^n > 938.5$
 $n \lg \frac{3}{2} > \lg 938.5$
 $n > \frac{\lg 938.5}{\lg 1.5}$
 $n > 16.9 \therefore$ least $n = 17$
- 13 a** $a + ar = a(1 + r) = 10.8$
 $ar^2 + ar^3 = ar^2(1 + r) = 43.2$
 $\therefore r^2 = 43.2 \div 10.8 = 4$
all terms +ve $\therefore r + \text{ve} \therefore r = 2$
sub. $a = 10.8 \div 3 = 3.6$
b $S_{16} = \frac{3.6(2^{16} - 1)}{2 - 1} = 235\,926$
- 14 a** $a = 12, r = 0.5$
 $S_\infty = \frac{12}{1 - 0.5} = 24$
b $a = 270, r = \frac{1}{3}$
 $S_\infty = \frac{270}{1 - \frac{1}{3}} = 405$
c $a = 25, r = -1.2$
no S_∞ as $r < -1 \therefore$ diverges
- d** $a = 216, r = \frac{2}{3}$
 $S_\infty = \frac{216}{1 - \frac{2}{3}} = 648$
e $a = \frac{8}{25}, r = \frac{5}{4}$
no S_∞ as $r > 1 \therefore$ diverges
f $a = 500, r = -0.6$
 $S_\infty = \frac{500}{1 - (-0.6)} = 312.5$
- 15 a** $a = 0.9, r = 0.9$
 $S_\infty = \frac{0.9}{1 - 0.9} = 9$
b $a = 3, r = \frac{1}{2}$
 $S_\infty = \frac{3}{1 - \frac{1}{2}} = 6$
c $a = 1, r = -\frac{3}{4}$
 $S_\infty = \frac{1}{1 - (-\frac{3}{4})} = \frac{4}{7}$
d $a = 32, r = 0.8$
 $S_\infty = \frac{32}{1 - 0.8} = 160$
- 16 a** $S_\infty = \frac{80}{1 - 0.2} = 100$
b $S_6 = \frac{80[1 - (0.2)^6]}{1 - 0.2} = 99.9936$
 $S_\infty - S_6 = 0.0064$
- 17 a** $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$
b GP: $a = 1, r = \frac{1}{3}$
 $S_\infty = \frac{1}{1 - \frac{1}{3}} = \frac{3}{2}$
- 18 a** $\frac{a}{1 - 0.55} = 40$
 $a = 0.45 \times 40 = 18$
b $18 \times (0.55)^{n-1} < 0.001$
 $(n - 1) \lg 0.55 < \lg 0.0000556$
 $n > \frac{\lg 0.0000556}{\lg 0.55} + 1$
 $n > 17.4 \therefore$ smallest $n = 18$
- 19 a** $u_1 = S_1 = 2^1 - 1 = 1$
 $S_5 = 2^5 - 1 = 31, S_4 = 2^4 - 1 = 15$
 $u_5 = S_5 - S_4 = 31 - 15 = 16$
b $S_{n-1} = 2^{n-1} - 1$
 $u_n = S_n - S_{n-1} = (2^n - 1) - (2^{n-1} - 1)$
 $= 2^n - 2^{n-1} = 2^{n-1}(2 - 1) = 2^{n-1}$
- 20 a** $\frac{k}{k+10} = \frac{k-6}{k}$
 $k^2 = (k+10)(k-6)$
 $4k - 60 = 0$
 $k = 15$
b $u_1 = 25, u_2 = 15 \therefore a = 25, r = 0.6$
 $S_\infty = \frac{25}{1 - 0.6} = 62.5$

C2 SEQUENCES AND SERIES

Answers - Worksheet B

- 1 a** $r = 20\frac{1}{4} \div 27 = \frac{3}{4}$
 $a \times (\frac{3}{4})^2 = 27$
 $a = \frac{16}{9} \times 27 = 48$
b $S_{\infty} = \frac{48}{1 - \frac{3}{4}} = 192$
- 2 a** $\frac{k+4}{k-8} = \frac{3k+2}{k+4}$
 $(k+4)^2 = (3k+2)(k-8)$
 $k^2 - 15k - 16 = 0$
 $(k+1)(k-16) = 0$
 $k > 0 \therefore k = 16$
b $u_1 = 8, u_2 = 20 \therefore a = 8, r = \frac{5}{2}$
 $u_6 = 8 \times (\frac{5}{2})^5 = 781\frac{1}{4}$
c $S_{10} = \frac{8[(\frac{5}{2})^{10} - 1]}{\frac{5}{2} - 1} = 50\,857.3$
- 3 a** $ar = 75, ar^4 = 129.6$
 $r^3 = 129.6 \div 75 = 1.728$
 $r = \sqrt[3]{1.728} = 1.2$
 $a = 75 \div 1.2 = 62.5$
b $u_{10} = 62.5 \times (1.2)^9 = 322.5$
c $S_{12} = \frac{62.5[(1.2)^{12} - 1]}{1.2 - 1} = 2473.8$
- 4 a** $S_n = a + ar + ar^2 + \dots + ar^{n-2} + ar^{n-1}$
 $rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n$
subtracting,
 $S_n - rS_n = a - ar^n$
 $(1-r)S_n = a(1-r^n)$
 $S_n = \frac{a(1-r^n)}{1-r}$
b $\frac{2[1 - (\sqrt{2})^n]}{1 - \sqrt{2}} = 126(\sqrt{2} + 1)$
 $1 - (\sqrt{2})^n = 63(\sqrt{2} + 1)(1 - \sqrt{2})$
 $1 - (\sqrt{2})^n = 63(1 - 2)$
 $(\sqrt{2})^n = 64$
 $2^{\frac{1}{2}n} = 2^6$
 $n = 12$
- 5 a** $\frac{18}{1-r} = 15$
 $\therefore 1-r = \frac{18}{15} = 1.2$
 $r = -0.2$
b $u_3 = 18 \times (-0.2)^2 = 0.72$
c $S_8 = \frac{18[1 - (-0.2)^8]}{1 - (-0.2)} = 14.9999616$
 $S_{\infty} - S_8 = 0.000\,0384$
- 6 a** $S_3 = 5(3^3 - 1) = 130$
 $S_2 = 5(3^2 - 1) = 40$
 $u_3 = S_3 - S_2 = 90$
b $S_{n-1} = 5(3^{n-1} - 1)$
 $u_n = S_n - S_{n-1} = 5(3^n - 1) - 5(3^{n-1} - 1)$
 $= 5[3^n - 3^{n-1}] = 5(3^n)[1 - \frac{1}{3}] = \frac{10}{3}(3^n)$
- 7 a** $4 \times (1.25)^7 = 19.1 \text{ mm (3sf)}$
b GP: $a = 4, r = 1.25$
 $S_{20} = \frac{4[(1.25)^{20} - 1]}{1.25 - 1} = 1371.8 \text{ mm}$
 $\therefore \text{length} = 1.37 \text{ m (3sf)}$
- 8 a** $ar = 30, ar^3 = 2.7 \therefore r^2 = 2.7 \div 30 = 0.09$
 $r > 0 \therefore r = \sqrt{0.09} = 0.3$
 $a = 30 \div 0.3 = 100$
b $S_{\infty} = \frac{100}{1 - 0.3} = 142.9 \text{ (1dp)}$

9 a GP: $a = 27, r = 3$

$$S_8 = \frac{27(3^8 - 1)}{3 - 1} = 88\,560$$

b $\sum_{r=1}^{15} 2^r$: GP, $a = 2, r = 2$

$$S_{15} = \frac{2(2^{15} - 1)}{2 - 1} = 65\,534$$

$\sum_{r=1}^{15} 12r$: AP, $a = 12, d = 12$

$$S_{15} = \frac{15}{2} [24 + (14 \times 12)] = 1440$$

$$\sum_{r=1}^{15} (2^r - 12r) = 65\,534 - 1440 = 64\,094$$

11 a $u_8 = 4 \times (\frac{1}{2})^7 = \frac{1}{32}$

b $u_n = 4 \times (\frac{1}{2})^{n-1}$

$$= 2^2 \times 2^{1-n}$$

$$= 2^{3-n}$$

c $S_n = \frac{4[1 - (\frac{1}{2})^n]}{1 - \frac{1}{2}}$

$$= 8(1 - 2^{-n})$$

$$= 8 - (2^3 \times 2^{-n})$$

$$= 8 - 2^{3-n}$$

13 a $a + ar^2 = a(1 + r^2) = 150$

$$ar + ar^3 = ar(1 + r^2) = -75$$

$$\therefore r = -75 \div 150 = -\frac{1}{2}$$

$$a = 150 \div \frac{5}{4} = 120$$

b $S_\infty = \frac{120}{1 - (-\frac{1}{2})} = 80$

10 a $a = 64, ar^2 - ar = 20$

$$\therefore 64r^2 - 64r = 20$$

$$16r^2 - 16r - 5 = 0$$

b $(4r + 1)(4r - 5) = 0$

$$r = -\frac{1}{4} \text{ or } \frac{5}{4}$$

c $r = -\frac{1}{4} \Rightarrow u_4 = 64 \times (-\frac{1}{4})^3 = -1$

$$r = \frac{5}{4} \Rightarrow u_4 = 64 \times (\frac{5}{4})^3 = 125$$

d $r = -\frac{1}{4} \Rightarrow S_\infty = \frac{64}{1 - (-\frac{1}{4})} = 51\frac{1}{5}$

12 a $u_6 = 4 \times 3^6 = 2916$

b GP: $a = 12, r = 3$

$$S_t = \frac{12(3^t - 1)}{3 - 1} = 6(3^t - 1)$$

$$\therefore 6(3^t - 1) > 10^{25}$$

$$3^t > \frac{10^{25}}{6} + 1$$

$$t \lg 3 > \lg \left(\frac{10^{25}}{6} + 1 \right)$$

$$t > \frac{\lg(\frac{10^{25}}{6} + 1)}{\lg 3}$$

$$t > 50.8 \therefore \text{smallest } t = 51$$

14 a $b - a = (3a + 4) - b$

$$2b = 4a + 4$$

$$b = 2a + 2$$

b $\frac{2a+2}{a} = \frac{6a+1}{2a+2}$

$$(2a + 2)^2 = a(6a + 1)$$

$$2a^2 - 7a - 4 = 0$$

$$(2a + 1)(a - 4) = 0$$

$$a \text{ integer } \therefore a = 4$$

$$\text{sub. } b = 10$$

15 a after 4th bounce,
reaches $3 \times (0.6)^4 = 0.3888$ m

b total distance

$$= h + 2[0.6h + (0.6)^2h + (0.6)^3h + \dots]$$

$$= h + 2 \times S_\infty \text{ of GP, } a = 0.6h, r = 0.6$$

$$= h + \frac{2 \times 0.6h}{1 - 0.6}$$

$$= h + 3h = 4h \text{ metres}$$

C2 SEQUENCES AND SERIES
Answers - Worksheet C

- 1**
- a** $= 1 + 4x + 6x^2 + 4x^3 + x^4$
- b** $= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$
- c** $= 1 + 3(4x) + 3(4x)^2 + (4x)^3$
 $= 1 + 12x + 48x^2 + 64x^3$
- d** $= 1 + 3(-2y) + 3(-2y)^2 + (-2y)^3$
 $= 1 - 6y + 12y^2 - 8y^3$
- e** $= 1 + 4(\frac{1}{2}x) + 6(\frac{1}{2}x)^2 + 4(\frac{1}{2}x)^3 + (\frac{1}{2}x)^4$
 $= 1 + 2x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4$
- f** $= 1 + 3(\frac{1}{3}y) + 3(\frac{1}{3}y)^2 + (\frac{1}{3}y)^3$
 $= 1 + y + \frac{1}{3}y^2 + \frac{1}{27}y^3$
- g** $= 1 + 5(x^2) + 10(x^2)^2 + 10(x^2)^3 + 5(x^2)^4 + (x^2)^5$
 $= 1 + 5x^2 + 10x^4 + 10x^6 + 5x^8 + x^{10}$
- h** $= 1 + 4(-\frac{3}{2}x) + 6(-\frac{3}{2}x)^2 + 4(-\frac{3}{2}x)^3 + (-\frac{3}{2}x)^4$
 $= 1 - 6x + \frac{27}{2}x^2 - \frac{27}{2}x^3 + \frac{81}{16}x^4$
- 2**
- a** $= x^3 + 3x^2y + 3xy^2 + y^3$
- b** $= a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$
- c** $= x^4 + 4x^3(2y) + 6x^2(2y)^2 + 4x(2y)^3 + (2y)^4$
 $= x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$
- d** $= 2^3 + 3(2^2)y + 3(2)y^2 + y^3$
 $= 8 + 12y + 6y^2 + y^3$
- e** $= 3^3 + 3(3^2)(-x) + 3(3)(-x)^2 + (-x)^3$
 $= 27 - 27x + 9x^2 - x^3$
- f** $= 5^4 + 4(5^3)(2x) + 6(5^2)(2x)^2 + 4(5)(2x)^3 + (2x)^4$
 $= 625 + 1000x + 600x^2 + 160x^3 + 16x^4$
- g** $= 3^5 + 5(3^4)(-4y) + 10(3^3)(-4y)^2 + 10(3^2)(-4y)^3 + 5(3)(-4y)^4 + (-4y)^5$
 $= 243 - 1620y + 4320y^2 - 5760y^3 + 3840y^4 - 1024y^5$
- h** $= 3^4 + 4(3^3)(\frac{1}{2}x) + 6(3^2)(\frac{1}{2}x)^2 + 4(3)(\frac{1}{2}x)^3 + (\frac{1}{2}x)^4$
 $= 81 + 54x + \frac{27}{2}x^2 + \frac{3}{2}x^3 + \frac{1}{16}x^4$
- 3**
- a** $= 1 + 10x + \frac{10 \times 9}{2}x^2 + \frac{10 \times 9 \times 8}{3 \times 2}x^3 + \dots$
 $= 1 + 10x + 45x^2 + 120x^3 + \dots$
- b** $= 1 + 6(-x) + \frac{6 \times 5}{2}(-x)^2 + \frac{6 \times 5 \times 4}{3 \times 2}(-x)^3 + \dots$
 $= 1 - 6x + 15x^2 - 20x^3 + \dots$
- c** $= 1 + 8(2x) + \frac{8 \times 7}{2}(2x)^2 + \frac{8 \times 7 \times 6}{3 \times 2}(2x)^3 + \dots$
 $= 1 + 16x + 112x^2 + 448x^3 + \dots$
- d** $= 1 + 7(-\frac{1}{2}x) + \frac{7 \times 6}{2}(-\frac{1}{2}x)^2 + \frac{7 \times 6 \times 5}{3 \times 2}(-\frac{1}{2}x)^3 + \dots$
 $= 1 - \frac{7}{2}x + \frac{21}{4}x^2 - \frac{35}{8}x^3 + \dots$
- e** $= 1 + 6(x^3) + \frac{6 \times 5}{2}(x^3)^2 + \frac{6 \times 5 \times 4}{3 \times 2}(x^3)^3 + \dots$
 $= 1 + 6x^3 + 15x^6 + 20x^9 + \dots$
- f** $= 2^9 + 9(2^8)x + \frac{9 \times 8}{2}(2^7)x^2 + \frac{9 \times 8 \times 7}{3 \times 2}(2^6)x^3 + \dots$
 $= 512 + 2304x + 4608x^2 + 5376x^3 + \dots$
- g** $= 3^7 + 7(3^6)(-x) + \frac{7 \times 6}{2}(3^5)(-x)^2 + \frac{7 \times 6 \times 5}{3 \times 2}(3^4)(-x)^3 + \dots$
 $= 2187 - 5103x + 5103x^2 - 2835x^3 + \dots$
- h** $= 2^{10} + 10(2^9)(5x) + \frac{10 \times 9}{2}(2^8)(5x)^2 + \frac{10 \times 9 \times 8}{3 \times 2}(2^7)(5x)^3 + \dots$
 $= 1024 + 25\,600x + 288\,000x^2 + 1\,920\,000x^3 + \dots$
- 4**
- a** $= \binom{20}{3} = 1140$
- b** $= \binom{14}{4} \times (-1)^4 = 1001$
- c** $= \binom{9}{2} \times 4^2 = 576$
- d** $= \binom{14}{3} \times (-3)^3 = -9828$
- e** $= \binom{12}{4} \times (-\frac{1}{3})^4 = \frac{55}{9}$ or $6\frac{1}{9}$
- f** $= \binom{16}{5} \times (-\frac{1}{2})^5 = -136.5$
- g** $= \binom{15}{2} \times (\frac{2}{5})^2 = \frac{84}{5}$ or 16.8
- h** $= \binom{8}{3} = 56$

- 5 **a** $= 1 + 3(\sqrt{5}) + 3(\sqrt{5})^2 + (\sqrt{5})^3$
 $= 1 + 3\sqrt{5} + 15 + 5\sqrt{5}$
 $= 16 + 8\sqrt{5}$
- b** $= 1 + 4(-\sqrt{3}) + 6(-\sqrt{3})^2 + 4(-\sqrt{3})^3 + (-\sqrt{3})^4$
 $= 1 - 4\sqrt{3} + 18 - 12\sqrt{3} + 9$
 $= 28 - 16\sqrt{3}$
- c** $= 2^3 + 3(2^2)(\sqrt{2}) + 3(2)(\sqrt{2})^2 + (\sqrt{2})^3$
 $= 8 + 12\sqrt{2} + 12 + 2\sqrt{2}$
 $= 20 + 14\sqrt{2}$
- d** $= 1 + 4(2\sqrt{3}) + 6(2\sqrt{3})^2 + 4(2\sqrt{3})^3 + (2\sqrt{3})^4$
 $= 1 + 8\sqrt{3} + 72 + 96\sqrt{3} + 144$
 $= 217 + 104\sqrt{3}$
- 6 **a** $= 1 + 6x + \frac{6 \times 5}{2}x^2 + \frac{6 \times 5 \times 4}{3 \times 2}x^3 + \dots$
 $= 1 + 6x + 15x^2 + 20x^3 + \dots$
- b i** let $x = 0.02$
 $1.02^6 \approx 1 + 6(0.02) + 15(0.02)^2 + 20(0.02)^3$
 $= 1 + 0.12 + 0.0060 + 0.000160$
 $= 1.1262$ (4dp)
- ii** let $x = -0.01$
 $0.99^6 \approx 1 + 6(-0.01) + 15(-0.01)^2 + 20(-0.01)^3$
 $= 1 - 0.06 + 0.0015 - 0.00020$
 $= 0.9415$ (4dp)
- 7 **a** $= 1 + 8(2y) + \frac{8 \times 7}{2}(2y)^2 + \frac{8 \times 7 \times 6}{3 \times 2}(2y)^3 + \dots$
 $= 1 + 16y + 112y^2 + 448y^3 + \dots$
- b i** let $y = -0.01$
 $0.98^8 \approx 1 + 16(-0.01) + 112(-0.01)^2 + 448(-0.01)^3$
 $= 1 - 0.16 + 0.0112 - 0.000448$
 $= 0.8508$ (4dp)
- ii** let $y = 0.005$
 $1.01^8 \approx 1 + 16(0.005) + 112(0.005)^2 + 448(0.005)^3$
 $= 1 + 0.080 + 0.002800 + 0.000056000$
 $= 1.0829$ (4dp)
- 8 **a** $= 1 + 4x + 6x^2 + 4x^3 + x^4 + (1 - 4x + 6x^2 - 4x^3 + x^4)$
 $= 2 + 12x^2 + 2x^4$
- b** $= 1 + 3(-\frac{1}{3}x) + 3(-\frac{1}{3}x)^2 + (-\frac{1}{3}x)^3 - [1 + 3(\frac{1}{3}x) + 3(\frac{1}{3}x)^2 + (\frac{1}{3}x)^3]$
 $= 1 - x + \frac{1}{3}x^2 - \frac{1}{27}x^3 - (1 + x + \frac{1}{3}x^2 + \frac{1}{27}x^3)$
 $= -2x - \frac{2}{27}x^3$
- 9 **a** $6(ax)^2 = 24x^2$
 $a^2 = 4$
 $a < 0 \therefore a = -2$
- b** $= 4a^3 = -32$

C2 SEQUENCES AND SERIES
Answers - Worksheet D

- 1**
- a** $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$
 $= 1 + 12x + 54x^2 + 108x^3 + 81x^4$
- b** $= 2^5 + 5(2^4)(-x) + 10(2^3)(-x)^2 + 10(2^2)(-x)^3 + 5(2)(-x)^4 + (-x)^5$
 $= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$
- c** $= 3^3 + 3(3^2)(10x^2) + 3(3)(10x^2)^2 + (10x^2)^3$
 $= 27 + 270x^2 + 900x^4 + 1000x^6$
- d** $= a^5 + 5a^4(2b) + 10a^3(2b)^2 + 10a^2(2b)^3 + 5a(2b)^4 + (2b)^5$
 $= a^5 + 10a^4b + 40a^3b^2 + 80a^2b^3 + 80ab^4 + 32b^5$
- e** $= (x^2)^3 + 3(x^2)^2(-y) + 3(x^2)(-y)^2 + (-y)^3$
 $= x^6 - 3x^4y + 3x^2y^2 - y^3$
- f** $= 5^4 + 4(5^3)(\frac{1}{2}x) + 6(5^2)(\frac{1}{2}x)^2 + 4(5)(\frac{1}{2}x)^3 + (\frac{1}{2}x)^4$
 $= 625 + 250x + \frac{75}{2}x^2 + \frac{5}{2}x^3 + \frac{1}{16}x^4$
- g** $= x^4 + 4x^3(\frac{1}{x}) + 6x^2(\frac{1}{x})^2 + 4x(\frac{1}{x})^3 + (\frac{1}{x})^4$
 $= x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$
- h** $= t^3 + 3t^2(-\frac{2}{t^2}) + 3t(-\frac{2}{t^2})^2 + (-\frac{2}{t^2})^3$
 $= t^3 - 6 + \frac{12}{t^3} - \frac{8}{t^6}$
- 2**
- a** $= 1 + 6(3x) + \frac{6 \times 5}{2}(3x)^2 + \frac{6 \times 5 \times 4}{3 \times 2}(3x)^3 + \dots$
 $= 1 + 18x + 135x^2 + 540x^3 + \dots$
- b** $= 1 + 8(-\frac{1}{4}x) + \frac{8 \times 7}{2}(-\frac{1}{4}x)^2 + \frac{8 \times 7 \times 6}{3 \times 2}(-\frac{1}{4}x)^3 + \dots$
 $= 1 - 2x + \frac{7}{4}x^2 - \frac{7}{8}x^3 + \dots$
- c** $= 5^7 + 7(5^6)(-x) + \frac{7 \times 6}{2}(5^5)(-x)^2 + \frac{7 \times 6 \times 5}{3 \times 2}(5^4)(-x)^3 + \dots$
 $= 78\,125 - 109\,375x + 65\,625x^2 - 21\,875x^3 + \dots$
- d** $= 3^{10} + 10(3^9)(2x^2) + \frac{10 \times 9}{2}(3^8)(2x^2)^2 + \frac{10 \times 9 \times 8}{3 \times 2}(3^7)(2x^2)^3 + \dots$
 $= 59\,049 + 393\,660x^2 + 1\,180\,980x^4 + 2\,099\,520x^6 + \dots$
- 3**
- a** $= \binom{15}{3} = 455$
- b** $= \binom{12}{4} \times (-2)^4 = 7920$
- c** $= \binom{7}{2} \times 3^5 = 5103$
- d** $= \binom{10}{5} \times 2^5 \times (-1)^5 = -8064$
- e** $= \binom{8}{5} \times 2^3 = 448$
- f** $= \binom{9}{3} \times (-1)^3 = -84$
- 4**
- a** $= (\sqrt{2})^4 + 4(\sqrt{2})^3(-\sqrt{5}) + 6(\sqrt{2})^2(-\sqrt{5})^2 + 4(\sqrt{2})(-\sqrt{5})^3 + (-\sqrt{5})^4$
 $= 4 - 8\sqrt{10} + 60 - 20\sqrt{10} + 25$
 $= 89 - 28\sqrt{10}$
- b** $= (\sqrt{2})^3 + 3(\sqrt{2})^2(\frac{1}{\sqrt{3}}) + 3(\sqrt{2})(\frac{1}{\sqrt{3}})^2 + (\frac{1}{\sqrt{3}})^3$
 $= 2\sqrt{2} + 2\sqrt{3} + \sqrt{2} + \frac{1}{9}\sqrt{3}$
 $= 3\sqrt{2} + \frac{19}{9}\sqrt{3}$

$$\begin{aligned}
 \mathbf{c} &= 1 + 3(\sqrt{5}) + 3(\sqrt{5})^2 + (\sqrt{5})^3 - [1 + 3(-\sqrt{5}) + 3(-\sqrt{5})^2 + (-\sqrt{5})^3] \\
 &= 1 + 3\sqrt{5} + 15 + 5\sqrt{5} - [1 - 3\sqrt{5} + 15 - 5\sqrt{5}] \\
 &= 16 + 8\sqrt{5} - [16 - 8\sqrt{5}] \\
 &= 16\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{5} \quad \mathbf{a} &= 1 + 10\left(\frac{x}{2}\right) + \frac{10 \times 9}{2} \left(\frac{x}{2}\right)^2 + \frac{10 \times 9 \times 8}{3 \times 2} \left(\frac{x}{2}\right)^3 + \dots \\
 &= 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \mathbf{i} \quad \text{let } x &= 0.01 \\
 1.005^{10} &\approx 1 + 0.05 + 0.001125 + 0.000015 \\
 &= 1.05114 \text{ (5dp)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{ii} \quad \text{let } x &= -0.008 \\
 0.996^{10} &\approx 1 - 0.040 + 0.000720 - 0.000007680 \\
 &= 0.96071 \text{ (5dp)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{6} \quad \mathbf{a} &= 3^8 + 8(3^7)x + \frac{8 \times 7}{2} (3^6)x^2 + \frac{8 \times 7 \times 6}{3 \times 2} (3^5)x^3 + \dots \\
 &= 6561 + 17496x + 20412x^2 + 13608x^3 + \dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \mathbf{i} \quad \text{let } x &= 0.001 \\
 3.001^8 &\approx 6561 + 17.496 + 0.020412 + 0.000013608 \\
 &= 6578.516 \text{ (7sf)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{ii} \quad \text{let } x &= -0.005 \\
 2.995^8 &\approx 6561 - 87.480 + 0.510300 - 0.001701000 \\
 &= 6474.029 \text{ (7sf)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{7} \quad \mathbf{a} \quad (1 + 10x)^4 &= 1 + 4(10x) + 6(10x)^2 + 4(10x)^3 + (10x)^4 \\
 &= 1 + 40x + 600x^2 + 4000x^3 + 10000x^4 \\
 \therefore (1 + 10x)^4 + (1 - 10x)^4 &= 1 + 40x + 600x^2 + 4000x^3 + 10000x^4 + (1 - 40x + 600x^2 - 4000x^3 + 10000x^4) \\
 &= 2 + 1200x^2 + 20000x^4
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad (2 + \frac{1}{3}x)^3 &= 2^3 + 3(2^2)(\frac{1}{3}x) + 3(2)(\frac{1}{3}x)^2 + (\frac{1}{3}x)^3 \\
 &= 8 + 4x + \frac{2}{3}x^2 + \frac{1}{27}x^3 \\
 \therefore (2 - \frac{1}{3}x)^3 - (2 + \frac{1}{3}x)^3 &= 8 - 4x + \frac{2}{3}x^2 - \frac{1}{27}x^3 - (8 + 4x + \frac{2}{3}x^2 + \frac{1}{27}x^3) \\
 &= -8x - \frac{2}{27}x^3
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= (1 + 4y)(1 + 3y + 3y^2 + y^3) \\
 &= 1 + 3y + 3y^2 + y^3 + 4y + 12y^2 + 12y^3 + 4y^4 \\
 &= 1 + 7y + 15y^2 + 13y^3 + 4y^4
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} &= (1 - x)\left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}\right) \\
 &= 1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3} - x - 3 - \frac{3}{x} - \frac{1}{x^2} \\
 &= -x - 2 + \frac{2}{x^2} + \frac{1}{x^3}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad \mathbf{a} &= (1+x^2)[1+10(-3x)+\frac{10 \times 9}{2}(-3x)^2+\frac{10 \times 9 \times 8}{3 \times 2}(-3x)^3+\dots] \\
 &= (1+x^2)[1-30x+405x^2-3240x^3+\dots] \\
 &= 1-30x+405x^2-3240x^3+x^2-30x^3+\dots \\
 &= 1-30x+406x^2-3270x^3+\dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} &= (1-2x)[1+8x+\frac{8 \times 7}{2}x^2+\frac{8 \times 7 \times 6}{3 \times 2}x^3+\dots] \\
 &= (1-2x)[1+8x+28x^2+56x^3+\dots] \\
 &= 1+8x+28x^2+56x^3-2x-16x^2-56x^3+\dots \\
 &= 1+6x+12x^2+\dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= (1+x+x^2)[1+6(-x)+\frac{6 \times 5}{2}(-x)^2+\frac{6 \times 5 \times 4}{3 \times 2}(-x)^3+\dots] \\
 &= (1+x+x^2)[1-6x+15x^2-20x^3+\dots] \\
 &= 1-6x+15x^2-20x^3+x-6x^2+15x^3+x^2-6x^3+\dots \\
 &= 1-5x+10x^2-11x^3+\dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} &= (1+3x-x^2)[1+7(2x)+\frac{7 \times 6}{2}(2x)^2+\frac{7 \times 6 \times 5}{3 \times 2}(2x)^3+\dots] \\
 &= (1+3x-x^2)[1+14x+84x^2+280x^3+\dots] \\
 &= 1+14x+84x^2+280x^3+3x+42x^2+252x^3-x^2-14x^3+\dots \\
 &= 1+17x+125x^2+518x^3+\dots
 \end{aligned}$$

$$9 \quad \mathbf{a} = \binom{8}{4} \times y^4 \times \left(\frac{1}{y}\right)^4 = 70$$

$$\mathbf{b} = \binom{12}{6} \times (2y)^6 \times \left(-\frac{1}{2y}\right)^6 = 924$$

$$\mathbf{c} = \binom{6}{2} \times \left(\frac{1}{y}\right)^4 \times (y^2)^2 = 15$$

$$\mathbf{d} = \binom{9}{3} \times (3y)^6 \times \left(-\frac{1}{y^2}\right)^3 = -61\,236$$

$$10 \quad \mathbf{a} \quad \frac{n(n-1)}{2} \times \left(\frac{2}{5}\right)^2 = 1.6$$

$$n(n-1) = \frac{25}{2} \times 1.6 = 20$$

$$n^2 - n - 20 = 0$$

$$(n+4)(n-5) = 0$$

$$n > 0 \quad \therefore n = 5$$

$$\mathbf{b} = 5 \times \left(\frac{2}{5}\right)^4 = \frac{16}{125} \text{ or } 0.128$$

$$11 \quad \mathbf{a} \quad y_1 = (1-2x)[1+10x+\frac{10 \times 9}{2}x^2+\dots]$$

$$= 1+10x+45x^2-2x-20x^2+\dots$$

$$= 1+8x+25x^2+\dots$$

$$\therefore a = 25, b = 8, c = 1$$

$$\mathbf{b} \quad x = 0.2: \quad y_1 = 0.6 \times (1.2)^{10} = 3.71504$$

$$y_2 = (25 \times 0.04) + (8 \times 0.2) + 1 = 3.6$$

$$\% \text{ error} = \frac{3.71504 - 3.6}{3.71504} \times 100\% = 3.1\% \text{ (2sf)}$$

$$12 \quad \mathbf{a} \quad (1+px)^q = 1 + q(px) + \frac{q(q-1)}{2}(px)^2 + \dots$$

$$\therefore pq = -12 \text{ and } \frac{1}{2}p^2q(q-1) = 60$$

$$\text{sub. } p = -\frac{12}{q}$$

$$\Rightarrow \frac{72}{q}(q-1) = 60$$

$$72(q-1) = 60q$$

$$q = 6, p = -2$$

$$\mathbf{b} = \frac{6 \times 5 \times 4}{3 \times 2} \times (-2)^3 = -160$$

$$13 \quad \mathbf{a} = 3^{12} + 12(3^{11})\left(-\frac{x}{3}\right) + \frac{12 \times 11}{2} (3^{10})\left(-\frac{x}{3}\right)^2 + \frac{12 \times 11 \times 10}{3 \times 2} (3^9)\left(-\frac{x}{3}\right)^3 + \dots$$

$$= 531\,441 - 708\,588x + 433\,026x^2 - 160\,380x^3 + \dots$$

$$\mathbf{b} \text{ let } \frac{x}{3} = 0.002 \quad \therefore x = 0.006$$

$$2.998^{12} \approx 531\,441 - 4251.528 + 15.588\,936 - 0.034\,642\,080$$

$$= 527\,205.03 \text{ (2dp)}$$

$$14 \quad \mathbf{a} = 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$$

$$\mathbf{b} = 3 - 2\sqrt{3} + \sqrt{3} - 2 = 1 - \sqrt{3}$$

$$\mathbf{c} \quad \mathbf{i} = [(\sqrt{3} + 1)(\sqrt{3} - 2)]^5 = (1 - \sqrt{3})^5$$

$$= 1 - 5(\sqrt{3}) + 10(\sqrt{3})^2 - 10(\sqrt{3})^3 + 5(\sqrt{3})^4 - (\sqrt{3})^5$$

$$= 1 - 5\sqrt{3} + 30 - 30\sqrt{3} + 45 - 9\sqrt{3}$$

$$= 76 - 44\sqrt{3}$$

$$\mathbf{ii} = (\sqrt{3} + 1)(76 - 44\sqrt{3})$$

$$= 76\sqrt{3} - 132 + 76 - 44\sqrt{3}$$

$$= -56 + 32\sqrt{3}$$

$$15 \quad \mathbf{a} = 1 + 9\left(\frac{x}{2}\right) + \frac{9 \times 8}{2} \left(\frac{x}{2}\right)^2 + \frac{9 \times 8 \times 7}{3 \times 2} \left(\frac{x}{2}\right)^3 + \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} \left(\frac{x}{2}\right)^4 + \dots$$

$$= 1 + \frac{9}{2}x + 9x^2 + \frac{21}{2}x^3 + \frac{63}{8}x^4 + \dots$$

$$\mathbf{b} = \frac{21}{2} - \left(-\frac{21}{2}\right) = 21$$

$$\mathbf{c} = \left(1 \times \frac{63}{8}\right) + \left(2 \times \frac{21}{2}\right) = 28\frac{7}{8}$$

$$16 \quad 10(x^3)^2 \left(\frac{a}{x^2}\right)^3 = -80$$

$$a^3 = -8$$

$$a = -2$$

$$17 \quad \mathbf{a} \quad \left(1 + \frac{x}{k}\right)^n = 1 + n\left(\frac{x}{k}\right) + \frac{n(n-1)}{2} \left(\frac{x}{k}\right)^2 + \frac{n(n-1)(n-2)}{3 \times 2} \left(\frac{x}{k}\right)^3 + \dots$$

$$\therefore \frac{n(n-1)}{2k^2} = 3 \times \frac{n(n-1)(n-2)}{6k^3}$$

$$kn(n-1) = n(n-1)(n-2)$$

$$n(n-1)[k - (n-2)] = 0$$

$$n > 1 \quad \therefore k - (n-2) = 0$$

$$k = n - 2$$

$$\mathbf{b} \quad k = 7 - 2 = 5$$

$$\left(1 + \frac{x}{5}\right)^7 = 1 + 7\left(\frac{x}{5}\right) + \frac{7 \times 6}{2} \left(\frac{x}{5}\right)^2 + \frac{7 \times 6 \times 5}{3 \times 2} \left(\frac{x}{5}\right)^3 + \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2} \left(\frac{x}{5}\right)^4 + \dots$$

$$= 1 + \frac{7}{5}x + \frac{21}{25}x^2 + \frac{7}{25}x^3 + \frac{7}{125}x^4 + \dots$$

C2 SEQUENCES AND SERIES

Answers - Worksheet E

$$1 \quad = 1 + 4(4x) + 6(4x)^2 + 4(4x)^3 + (4x)^4 \\ = 1 + 16x + 96x^2 + 256x^3 + 256x^4$$

$$2 \quad \mathbf{a} \quad u_5 = 3 \times (-2)^4 = 48$$

$$\mathbf{b} \quad S_{10} = \frac{3[1 - (-2)^{10}]}{1 - (-2)} = -1023$$

\mathbf{c} positive terms form GP:

$$a = 3, r = (-2)^2 = 4$$

$$S_8 = \frac{3(4^8 - 1)}{4 - 1} = 65\,535$$

$$3 \quad \mathbf{a} \quad = 1 + 7(3x) + \frac{7 \times 6}{2} (3x)^2 \\ + \frac{7 \times 6 \times 5}{3 \times 2} (3x)^3 + \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2} (3x)^4 + \dots \\ = 1 + 21x + 189x^2 + 945x^3 + 2835x^4 + \dots$$

\mathbf{b} let $x = 0.01$

$$1.03^7 \approx 1 + 0.21 + 0.0189 \\ + 0.000\,945 + 0.000\,028\,35 \\ = 1.229\,87 \text{ (5dp)}$$

$$4 \quad \text{GP: } a = 8, r = 2, n = 10$$

$$S_{10} = \frac{8(2^{10} - 1)}{2 - 1} = 8184$$

$$5 \quad \mathbf{a} \quad = 2^5 + 5(2^4)x + 10(2^3)x^2 \\ + 10(2^2)x^3 + 5(2)x^4 + x^5 \\ = 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5 \\ \mathbf{b} \quad = 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5 \\ \mathbf{c} \quad (2 + \sqrt{5})^5 = 32 + 80(\sqrt{5}) + 80(\sqrt{5})^2 \\ + 40(\sqrt{5})^3 + 10(\sqrt{5})^4 + (\sqrt{5})^5 \\ = 32 + 80\sqrt{5} + 400 + 200\sqrt{5} + 250 + 25\sqrt{5} \\ = 682 + 305\sqrt{5} \\ \therefore (2 + \sqrt{5})^5 - (2 - \sqrt{5})^5 \\ = (682 + 305\sqrt{5}) - (682 - 305\sqrt{5}) \\ = 610\sqrt{5}, k = 610$$

$$6 \quad \mathbf{a} \quad \text{amount in account after } 3^{\text{rd}} \text{ payment in} \\ = 200 + (1.005 \times 200) + (1.005^2 \times 200) \\ = 603.005$$

interest paid at end of 3rd month

$$= 0.005 \times 603.005 = \text{£}3.02 \text{ (nearest penny)}$$

$$\mathbf{b} \quad \text{amount paid in} = 12 \times 200 = \text{£}2400$$

amount in account after 12 months

$$= 200(1.005 + 1.005^2 + \dots + 1.005^{12}) \\ = 200 \times S_{12} \text{ [GP: } a = 1.005, r = 1.005]$$

$$= 200 \times \frac{1.005(1.005^{12} - 1)}{1.005 - 1} = 2479.45$$

$$\text{total interest} = 2479.45 - 2400 = \text{£}79.45$$

$$7 \quad = 1 + 8(-3x) + \frac{8 \times 7}{2} (-3x)^2 \\ + \frac{8 \times 7 \times 6}{3 \times 2} (-3x)^3 + \dots \\ = 1 - 24x + 252x^2 - 1512x^3 + \dots$$

$$8 \quad \mathbf{a} \quad S_n = a + ar + ar^2 + \dots + ar^{n-1}$$

$$rS_n = ar + ar^2 + \dots + ar^{n-1} + ar^n$$

subtracting, $S_n - rS_n = a - ar^n$

$$S_n(1 - r) = a(1 - r^n)$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$\mathbf{b} \quad r = 6 \div 3 = 2$$

$$a \times 2^3 = 3 \quad \therefore a = \frac{3}{8}$$

$$S_{16} = \frac{\frac{3}{8}(2^{16} - 1)}{2 - 1} = 24\,575\frac{5}{8}$$

- 9** a $= 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \dots$
 $= 1 + anx + \frac{1}{2}a^2n(n-1)x^2 + \dots$
 b $\frac{1}{2}a^2n(n-1) = 3an$
 $a^2n(n-1) = 6an$
 $an[a(n-1) - 6] = 0$
 $n \neq 0 \therefore a(n-1) - 6 = 0$
 $an - a = 6$
 $n = \frac{6+a}{a}$
 c $n = 10 \therefore \text{coeff. of } x^3 = \frac{10 \times 9 \times 8}{3 \times 2} \times \left(\frac{2}{3}\right)^3 = 35\frac{5}{9}$
- 10** $= 2^6 + 6(2^5)(5x) + \frac{6 \times 5}{2}(2^4)(5x)^2 + \dots$
 $= 64 + 960x + 6000x^2 + \dots$
- 11** a $\frac{162}{1-r} = 486$
 $1-r = \frac{162}{486} = \frac{1}{3} \therefore r = \frac{2}{3}$
 b $u_6 = 162 \times \left(\frac{2}{3}\right)^5 = \frac{64}{3}$ or $21\frac{1}{3}$
 c $S_{10} = \frac{162[1 - (\frac{2}{3})^{10}]}{1 - \frac{2}{3}} = 477.572$
- 12** a $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$
 $= 1 + 12x + 54x^2 + 108x^3 + 81x^4$
 b term in $x^2 = (1)(54x^2) + (4x)(12x) + (-x^2)(1)$
 coefficient of $x^2 = 54 + 48 - 1 = 101$
- 13** a time $= 120 \times (0.9)^3 = 87.48$ seconds
 b GP: $a = 120, r = 0.9, n = 12$
 $S_{12} = \frac{120[1 - (0.9)^{12}]}{1 - 0.9}$
 $= 861.08$ seconds
 $= 14$ mins 21 secs (nearest sec.)
- 14** $= [1 + 8(\frac{x}{2}) + \frac{8 \times 7}{2}(\frac{x}{2})^2 + \dots][1 + 6(-x) + \frac{6 \times 5}{2}(-x)^2 + \dots]$
 $= [1 + 4x + 7x^2 + \dots][1 - 6x + 15x^2 + \dots]$
 $= 1 - 6x + 15x^2 + 4x - 24x^2 + 7x^2 + \dots$
 $= 1 - 2x - 2x^2 + \dots$
 $\therefore A = -2, B = -2$
- 15** a 6, 12, 24, 48
 b GP: $a = 6, r = 2, n = 10$
 $S_{10} = \frac{6(2^{10} - 1)}{2 - 1} = 6138$
- 16** a $= 1 + 4x + 6x^2 + 4x^3 + x^4$
 b $= 1 - 4x + 6x^2 - 4x^3 + x^4$
 c $(1 + 4x + 6x^2 + 4x^3 + x^4)$
 $+ (1 - 4x + 6x^2 - 4x^3 + x^4) = 82$
 $2 + 12x^2 + 2x^4 = 82$
 $x^4 + 6x^2 - 40 = 0$
 $(x^2 + 10)(x^2 - 4) = 0$
 $x^2 = -10$ [no real solutions] or $x^2 = 4$
 $x = \pm 2$
- 17** a $a \times (1.5)^2 = 18$
 $a = 18 \div 2.25 = 8$
 b $S_6 = \frac{8[(1.5)^6 - 1]}{1.5 - 1} = 166.25$
 c $8 \times (1.5)^{k-1} > 8000$
 $(k-1) \lg 1.5 > \lg 1000$
 $k > \frac{\lg 1000}{\lg 1.5} + 1$
 $k > 18.04 \therefore$ smallest $k = 19$
- 18** $(1 + \frac{ax}{2})^{10} + (1 + bx)^{10}$
 $= 1 + 10(\frac{ax}{2}) + \frac{10 \times 9}{2}(\frac{ax}{2})^2 + \dots$
 $+ 1 + 10(bx) + \frac{10 \times 9}{2}(bx)^2 + \dots$
 $= 2 + (5a + 10b)x + (\frac{45}{4}a^2 + 45b^2)x^2 + \dots$
 $\therefore 5a + 10b = 0 \Rightarrow a = -2b$
 and $\frac{45}{4}a^2 + 45b^2 = 90 \Rightarrow a^2 + 4b^2 = 8$
 sub. $(-2b)^2 + 4b^2 = 8$
 $b^2 = 1$
 $a < b \therefore b = 1, a = -2$

C2 SEQUENCES AND SERIES

Answers - Worksheet F

- 1 a** $a = 108, ar^3 = 32$
 $\therefore r^3 = 32 \div 108 = \frac{8}{27}$
 $r = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$
 $u_3 = 108 \times \left(\frac{2}{3}\right)^2 = 48$
- b** $S_\infty = \frac{108}{1 - \frac{2}{3}} = 324$
- 3 a** new subscribers in 4th week
 $= 200 \times (1.15)^3 = 304.175$
 $= 304$ (nearest unit)
- b** new subscribers: GP, $a = 200, r = 1.15$
 $S_{10} = \frac{200[(1.15)^{10} - 1]}{1.15 - 1} = 4060.74$
 total no. of subscribers $= 3600 + S_{10}$
 $= 7661$ (nearest unit)
- 5 a** $= 1 + 2n\left(\frac{x}{k}\right) + \frac{2n(2n-1)}{2}\left(\frac{x}{k}\right)^2$
 $+ \frac{2n(2n-1)(2n-2)}{3 \times 2}\left(\frac{x}{k}\right)^3 + \dots$
 $= 1 + \frac{2n}{k}x + \frac{n(2n-1)}{k^2}x^2 + \frac{2n(n-1)(2n-1)}{3k^3}x^3 + \dots$
- b** $\frac{2n(n-1)(2n-1)}{3k^3} = \frac{1}{2} \times \frac{n(2n-1)}{k^2}$
 $4n(n-1)(2n-1) = 3kn(2n-1)$
 $n(2n-1)[4(n-1) - 3k] = 0$
 $n > 1 \therefore 4(n-1) - 3k = 0$
 $3k = 4(n-1)$
- c** $\frac{2n}{k} = 2 \therefore n = k$
 $\therefore 3k = 4k - 4$
 $k = 4, n = 4$
- 7** $\sum_{r=1}^9 3^r$: GP, $a = 3, r = 3$
 $S_9 = \frac{3(3^9 - 1)}{3 - 1} = 29523$
 $\therefore \sum_{r=1}^9 (3^r - 1) = 29523 - 9$
 $= 29514$
- 8 a** $= 1 + 9(2x) + \frac{9 \times 8}{2}(2x)^2 + \frac{9 \times 8 \times 7}{3 \times 2}(2x)^3 + \dots$
 $= 1 + 18x + 144x^2 + 672x^3 + \dots$
- b** $(1 - 2x)^9 = 1 - 18x + 144x^2 - 672x^3 + \dots$
 $\therefore (1 + 2x)^9 + (1 - 2x)^9$
 $= (1 + 18x + 144x^2 + 672x^3 + \dots)$
 $+ (1 - 18x + 144x^2 - 672x^3 + \dots)$
 $= 2 + 288x^2$ (ignoring terms in x^4 and higher)
- c** let $x = 0.001$
 $\therefore 1.002^9 + 0.998^9 \approx 2 + 0.000288$
 $= 2.000288$ (7sf)
- 2** $= 1 + 5(-2x) + 10(-2x)^2$
 $+ 10(-2x)^3 + 5(-2x)^4 + (-2x)^5$
 $= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$
- 4 a** $= 1 + 7(4x) + \frac{7 \times 6}{2}(4x)^2 + \dots$
 $= 1 + 28x + 336x^2 + \dots$
- b** $(1 + 2x)^2(1 + 4x)^7$
 $= (1 + 4x + 4x^2)(1 + 28x + 336x^2 + \dots)$
 term in x^2
 $= (1)(336x^2) + (4x)(28x) + (4x^2)(1)$
 coefficient of $x^2 = 336 + 112 + 4 = 452$
- 6 a** $r = 3\sqrt{2} \div \sqrt{6} = \sqrt{3}$
 $a = \sqrt{6} \div \sqrt{3} = \sqrt{2}$
- b** $S_8 = \frac{\sqrt{2}[(\sqrt{3})^8 - 1]}{\sqrt{3} - 1}$
 $= \frac{80\sqrt{2}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$
 $= \frac{80\sqrt{2}(\sqrt{3} + 1)}{3 - 1}$
 $= 40\sqrt{2}(\sqrt{3} + 1)$

- 9 $(k-x)^9 = k^9 + 9(k^8)(-x) + \frac{9 \times 8}{2}(k^7)(-x)^2 + \dots$
 $= k^9 - 9k^8x + 36k^7x^2 + \dots$
 $\therefore -b = -9k^8$ and $b = 36k^7$
 $9k^8 = 36k^7$
 $9k^7(k-4) = 0$
 $k \neq 0 \therefore k = 4$
 $a = k^9 = 262\,144$
 $b = 9k^8 = 589\,824$
- 10 $= 3^4 + 4(3)^3(2x) + 6(3)^2(2x)^2$
 $+ 4(3)(2x)^3 + (2x)^4$
 $= 81 + 216x + 216x^2 + 96x^3 + 16x^4$
- 11 a $\frac{t}{1-r} = 3t$
 $1-r = \frac{t}{3t} = \frac{1}{3} \therefore r = \frac{2}{3}$
b $\frac{t[1-(\frac{2}{3})^4]}{1-\frac{2}{3}} = 130$
 $t = (\frac{1}{3} \times 80) \div \frac{65}{81} = 54$
- 12 a $= 1 + 4(-2x) + 6(-2x)^2 + 4(-2x)^3 + (-2x)^4$
 $= 1 - 8x + 24x^2 - 32x^3 + 16x^4$
b let $x = y^2 - 2y$
 $(1 + 4y - 2y^2)^4$
 $= 1 - 8(y^2 - 2y) + 24(y^2 - 2y)^2 + \dots$
term in $y^2 = -8y^2 + 24(-2y)^2$
coefficient of $y^2 = -8 + 96 = 88$
- 13 a $= 12000 \times (0.75)^4$
 $= 3796.875$
 $= \text{£}3800$ (3sf)
b GP: $a = 12000, r = 0.75$
 $S_8 = \frac{12000[1-(0.75)^8]}{1-0.75}$
 $= \text{£}43\,200$ (3sf)
- 14 a $p(-2) = 1^4 - (-1)^4 = 1 - 1 = 0$
 $\therefore (x+2)$ is a factor of $p(x)$
b $p(x) = [x^4 + 4(x^3)(3) + 6(x^2)(3^2) + 4(x)(3^3) + 3^4]$
 $- [x^4 + 4x^3 + 6x^2 + 4x + 1]$
 $= 8x^3 + 48x^2 + 104x + 80$
 $= 8(x^3 + 6x^2 + 13x + 10)$

$$\begin{array}{r} x^2 + 4x + 5 \\ x+2 \overline{) x^3 + 6x^2 + 13x + 10} \\ \underline{x^3 + 2x^2} \\ 4x^2 + 13x \\ \underline{4x^2 + 8x} \\ 5x + 10 \\ \underline{5x + 10} \\ 0 \end{array}$$

 $p(x) = 8(x+2)(x^2 + 4x + 5)$
c $8(x+2)(x^2 + 4x + 5) = 0$
 $x = -2$ or $(x^2 + 4x + 5) = 0$
 $b^2 - 4ac = 16 - 20 = -4$
 $b^2 - 4ac < 0 \therefore$ no real sols to $(x^2 + 4x + 5) = 0$
 \therefore only one real solution to $p(x) = 0$

$$\begin{aligned}
 15 \quad \mathbf{a} \quad & (1-x)(1+2x)^n \\
 & = (1-x)\left[1 + n(2x) + \frac{n(n-1)}{2}(2x)^2 + \dots\right] \\
 & = (1-x)[1 + 2nx + 2n(n-1)x^2 + \dots] \\
 & \therefore 2n(n-1) - 2n = 198 \\
 & \quad n^2 - 2n - 99 = 0 \\
 & \quad (n+9)(n-11) = 0 \\
 & n \geq 0 \quad \therefore n = 11
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & (1-x)(1+2x)^{11} \\
 & = (1-x)\left[\dots + \frac{11 \times 10}{2}(2x)^2 + \frac{11 \times 10 \times 9}{3 \times 2}(2x)^3 + \dots\right] \\
 & = (1-x)[\dots + 220x^2 + 1320x^3 + \dots] \\
 & \therefore \text{coefficient of } x^3 = 1320 - 220 = 1100
 \end{aligned}$$

$$\begin{aligned}
 17 \quad \mathbf{a} \quad & S_4 = 3^4 - 1 = 80 \\
 & S_3 = 3^3 - 1 = 26 \\
 & u_4 = S_4 - S_3 = 80 - 26 = 54
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & S_{n-1} = 3^{n-1} - 1 \\
 & u_n = S_n - S_{n-1} \\
 & \quad = (3^n - 1) - (3^{n-1} - 1) \\
 & \quad = 3^n - 3^{n-1} \\
 & \quad = 3^n\left(1 - \frac{1}{3}\right) = \frac{2}{3}(3^n) \quad \left[k = \frac{2}{3}\right]
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad & u_{n-1} = \frac{2}{3}(3^{n-1}) \\
 & u_n \div u_{n-1} = \frac{2}{3}(3^n) \div \frac{2}{3}(3^{n-1}) = 3 \\
 & u_n \div u_{n-1} \text{ is constant } \therefore \text{geometric}
 \end{aligned}$$

$$\begin{aligned}
 16 \quad & = \left(\frac{3}{x}\right)^4 + 4\left(\frac{3}{x}\right)^3(-x) + 6\left(\frac{3}{x}\right)^2(-x)^2 \\
 & \quad + 4\left(\frac{3}{x}\right)(-x)^3 + (-x)^4 \\
 & = x^4 - 12x^2 + 54 - \frac{108}{x^2} + \frac{81}{x^4}
 \end{aligned}$$

$$\begin{aligned}
 18 \quad \mathbf{a} \quad & 3(x-3) = y-3 \\
 & y = 3x-6
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & \left(\frac{x}{3}\right)^3 = \frac{y}{3} \\
 & x^3 = 9y = 9(3x-6) \\
 & x^3 - 27x + 54 = 0
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad & \text{trying } x = 1, 2 \text{ etc. } \Rightarrow x = 3 \text{ is a solution} \\
 & \therefore (x-3) \text{ is a factor}
 \end{aligned}$$

$$\begin{array}{r}
 x^2 + 3x - 18 \\
 x-3 \overline{) x^3 + 0x^2 - 27x + 54} \\
 \underline{x^3 - 3x^2} \\
 3x^2 - 27x \\
 \underline{3x^2 - 9x} \\
 -18x + 54 \\
 \underline{-18x + 54} \\
 0
 \end{array}$$

$$\begin{aligned}
 (x-3)(x^2 + 3x - 18) & = 0 \\
 (x-3)(x+6)(x-3) & = 0 \\
 x & = -6 \text{ or } 3
 \end{aligned}$$