

C2 ALGEBRA**Worksheet A**

1 Find the quotient obtained in dividing

a $(x^3 + 2x^2 - x - 2)$ by $(x + 1)$

b $(x^3 + 2x^2 - 9x + 2)$ by $(x - 2)$

c $(20 + x + 3x^2 + x^3)$ by $(x + 4)$

d $(2x^3 - x^2 - 4x + 3)$ by $(x - 1)$

e $(6x^3 - 19x^2 - 73x + 90)$ by $(x - 5)$

f $(-x^3 + 5x^2 + 10x - 8)$ by $(x + 2)$

g $(x^3 - 2x + 21)$ by $(x + 3)$

h $(3x^3 + 16x^2 + 72)$ by $(x + 6)$

2 Find the quotient and remainder obtained in dividing

a $(x^3 + 8x^2 + 17x + 16)$ by $(x + 5)$

b $(x^3 - 15x^2 + 61x - 48)$ by $(x - 7)$

c $(3x^3 + 4x^2 + 7)$ by $(2 + x)$

d $(-x^3 - 5x^2 + 15x - 50)$ by $(x + 8)$

e $(4x^3 + 2x^2 - 16x + 3)$ by $(x - 3)$

f $(1 - 22x^2 - 6x^3)$ by $(x + 2)$

3 Use the factor theorem to determine whether or not

a $(x - 1)$ is a factor of $(x^3 + 2x^2 - 2x - 1)$

b $(x + 2)$ is a factor of $(x^3 - 5x^2 - 9x + 2)$

c $(x - 3)$ is a factor of $(x^3 - x^2 - 14x + 27)$

d $(x + 6)$ is a factor of $(2x^3 + 13x^2 + 2x - 24)$

e $(2x + 1)$ is a factor of $(2x^3 - 5x^2 + 7x - 14)$

f $(3x - 2)$ is a factor of $(2 - 17x + 25x^2 - 6x^3)$

4 $f(x) \equiv x^3 - 2x^2 - 11x + 12.$

a Show that $(x - 1)$ is a factor of $f(x)$.

b Hence, express $f(x)$ as the product of three linear factors.

5 $g(x) \equiv 2x^3 + x^2 - 13x + 6.$

Show that $(x + 3)$ is a factor of $g(x)$ and solve the equation $g(x) = 0$.

6 $f(x) \equiv 6x^3 - 7x^2 - 71x + 12.$

Given that $f(4) = 0$, find all solutions to the equation $f(x) = 0$.

7 $g(x) \equiv x^3 + 7x^2 + 7x - 6.$

Given that $x = -2$ is a solution to the equation $g(x) = 0$,

a express $g(x)$ as the product of a linear factor and a quadratic factor,

b find, to 2 decimal places, the other two solutions to the equation $g(x) = 0$.

8 $f(x) \equiv x^3 + 2x^2 - 11x - 12.$

a Evaluate $f(1)$, $f(2)$, $f(-1)$ and $f(-2)$.

b Hence, state a linear factor of $f(x)$ and fully factorise $f(x)$.

9 By first finding a linear factor, fully factorise

a $x^3 - 2x^2 - 5x + 6$

b $x^3 + x^2 - 5x - 2$

c $20 + 11x - 8x^2 + x^3$

d $3x^3 - 4x^2 - 35x + 12$

e $x^3 + 8$

f $12 + 29x + 8x^2 - 4x^3$

10 Solve each equation, giving your answers in exact form.

a $x^3 - x^2 - 10x - 8 = 0$

b $x^3 + 2x^2 - 9x - 18 = 0$

c $4x^3 - 12x^2 + 9x = 2$

d $x^3 - 5x^2 + 3x + 1 = 0$

e $x^2(x + 4) = 3(3x + 2)$

f $x^3 - 14x + 15 = 0$

11 $f(x) \equiv 2x^3 - x^2 - 15x + c.$

Given that $(x - 2)$ is a factor of $f(x)$,

- a** find the value of the constant c ,
b fully factorise $f(x)$.

12 $g(x) \equiv x^3 + px^2 - 13x + q.$

Given that $(x + 1)$ and $(x - 3)$ are factors of $g(x)$,

- a** show that $p = 3$ and find the value of q ,
b solve the equation $g(x) = 0$.

13 Use the remainder theorem to find the remainder obtained in dividing

- | | |
|--|---|
| a $(x^3 + 4x^2 - x + 6)$ by $(x - 2)$ | b $(x^3 - 2x^2 + 7x + 1)$ by $(x + 1)$ |
| c $(2x^3 + x^2 - 9x + 17)$ by $(x + 5)$ | d $(8x^3 + 4x^2 - 6x - 3)$ by $(2x - 1)$ |
| e $(2x^3 - 3x^2 - 20x - 7)$ by $(2x + 1)$ | f $(3x^3 - 6x^2 + 2x - 7)$ by $(3x - 2)$ |

14 Given that when $(x^3 - 4x^2 + 5x + c)$ is divided by $(x - 2)$ the remainder is 5, find the value of the constant c .

15 Given that when $(2x^3 - 9x^2 + kx + 5)$ is divided by $(2x - 1)$ the remainder is -2 , find the value of the constant k .

16 Given that when $(2x^3 + ax^2 + 13)$ is divided by $(x + 3)$ the remainder is 22,

- a** find the value of the constant a ,
b find the remainder when $(2x^3 + ax^2 + 13)$ is divided by $(x - 4)$.

17 $f(x) \equiv px^3 + qx^2 + qx + 3.$

Given that $(x + 1)$ is a factor of $f(x)$,

- a** find the value of the constant p .

Given also that when $f(x)$ is divided by $(x - 2)$ the remainder is 15,

- b** find the value of the constant q .

18 $p(x) \equiv x^3 + ax^2 + 9x + b.$

Given that $(x - 3)$ is a factor of $p(x)$,

- a** find a linear relationship between the constants a and b .

Given also that when $p(x)$ is divided by $(x + 2)$ the remainder is -30 ,

- b** find the values of the constants a and b .

19 $f(x) \equiv 4x^3 - 6x^2 + mx + n.$

Given that when $f(x)$ is divided by $(x + 1)$ the remainder is 3 and that when $f(x)$ is divided by $(2x - 1)$ the remainder is 15, find the values of the constants m and n .

20 $g(x) \equiv x^3 + cx + 3.$

Given that when $g(x)$ is divided by $(x - 4)$ the remainder is 39,

- a** find the value of the constant c ,
b find the quotient and remainder when $g(x)$ is divided by $(x + 2)$.

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Worksheet B

1 $f(x) \equiv x^3 - 5x^2 + ax + b.$

Given that $(x + 2)$ and $(x - 3)$ are factors of $f(x)$,

- show that $a = -2$ and find the value of b .
- Hence, express $f(x)$ as the product of three linear factors.

2 $f(x) \equiv 8x^3 - x^2 + 7.$

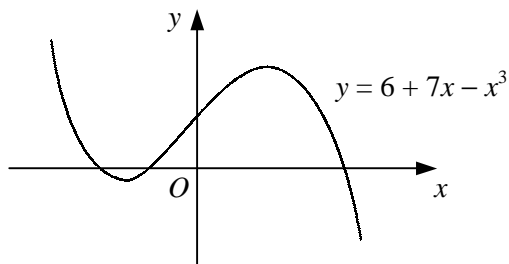
The remainder when $f(x)$ is divided by $(x - k)$ is eight times the remainder when $f(x)$ is divided by $(2x - k)$.

Find the two possible values of the constant k .

3 $f(x) \equiv 3x^3 - x^2 - 12x + 4.$

- Show that $(x - 2)$ is a factor of $f(x)$.
- Solve the equation $f(x) = 0$.

4



The diagram shows the curve with the equation $y = 6 + 7x - x^3$.

Find the coordinates of the points where the curve crosses the x -axis.

5 $f(x) \equiv 3x^3 + px^2 + 8x + q.$

When $f(x)$ is divided by $(x + 1)$ there is a remainder of -4 .

When $f(x)$ is divided by $(x - 2)$ there is a remainder of 80 .

- Find the values of the constants p and q .
- Show that $(x + 2)$ is a factor of $f(x)$.
- Solve the equation $f(x) = 0$.

6 a Solve the equation

$$x^3 - 4x^2 - 7x + 10 = 0.$$

b Hence, solve the equation

$$y^6 - 4y^4 - 7y^2 + 10 = 0.$$

7

$$f(n) \equiv n^3 + 7n^2 + 14n + 3.$$

- Find the remainder when $f(n)$ is divided by $(n + 1)$.
- Express $f(n)$ in the form

$$f(n) \equiv (n + 1)(n + a)(n + b) + c,$$

where a , b and c are integers.

- Hence, show that $f(n)$ is odd for all positive integer values of n .

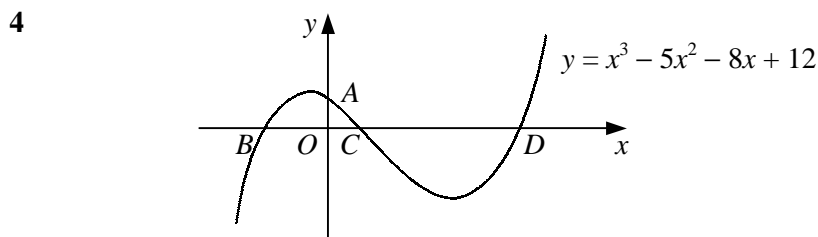
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Worksheet C

- 1 $f(x) \equiv x^3 + x^2 - 22x - 40.$
- Show that $(x + 2)$ is a factor of $f(x).$ (2)
 - Express $f(x)$ as the product of three linear factors. (4)
 - Solve the equation $f(x) = 0.$ (1)

- 2 $f(x) \equiv x^3 - 2x^2 + kx + 1.$
- Given that the remainder when $f(x)$ is divided by $(x - 2)$ and the remainder when $f(x)$ is divided by $(x + 3)$ are equal,
- find the value of the constant $k,$ (4)
 - find the remainder when $f(x)$ is divided by $(x + 2).$ (2)

- 3 The polynomial $p(x)$ is defined by
- $$p(x) \equiv 2x^3 - 9x^2 - 2x + 11.$$
- Find the remainder when $p(x)$ is divided by $(x + 2).$ (2)
 - Find the quotient and remainder when $p(x)$ is divided by $(x - 4).$ (3)



The diagram shows the curve with the equation $y = x^3 - 5x^2 - 8x + 12.$

- State the coordinates of the point A where the curve crosses the y -axis. (1)
- The curve crosses the x -axis at the points B, C and $D.$ Given that C has coordinates $(1, 0),$
- find the coordinates of the points B and $D.$ (6)

- 5 $f(x) \equiv x^3 - 3x^2 + kx + 8.$
- Given that $(x - 1)$ is a factor of $f(x),$
- find the value of $k,$ (2)
 - solve the equation $f(x) = 0.$ (5)

- 6 Solve the equation
- $$2x^3 + x^2 - 13x + 6 = 0. \quad (7)$$

- 7 The polynomial $p(x)$ is defined by
- $$p(x) \equiv bx^3 + ax^2 - 10x + b,$$
- where a and b are constants.
- Given that when $p(x)$ is divided by $(x + 1)$ the remainder is 3,
- find the value of $a.$ (2)
- Given also that when $p(x)$ is divided by $(3x - 1)$ the remainder is $-1,$
- find the value of $b.$ (3)

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- 8** $f(x) \equiv x^3 - 7x^2 + x + 10.$
- a** Find the remainder when $f(x)$ is divided by $(x + 1).$ (2)
- b** Hence, or otherwise, solve the equation $f(x) = 1,$ giving your answers in exact form. (6)
- 9** $f(x) \equiv 3x^3 + kx^2 - 7x + 2k.$
- When $f(x)$ is divided by $(3x - 2)$ the remainder is 6.
- Find the value of the constant $k.$ (3)
- 10** $f(x) \equiv 2x^3 - 7x^2 + 4x - 3.$
- a** Show that $(x - 3)$ is a factor of $f(x).$ (2)
- b** Hence, express $f(x)$ as the product of a linear factor and a quadratic factor. (3)
- c** Show that there is only one real solution to the equation $f(x) = 0.$ (3)
- 11** The polynomial $f(x)$ is defined by
- $$f(x) \equiv x^3 + px + q,$$
- where p and q are constants.
- Given that $(x - 2)$ is a factor of $f(x),$
- a** find an expression for q in terms of $p.$ (2)
- Given also that when $f(x)$ is divided by $(x + 1)$ the remainder is $-15,$
- b** find the values of p and $q.$ (4)
- 12** $f(x) \equiv x^3 + 4x^2 - 9.$
- Given that $x = -3$ is a solution to the equation $f(x) = 0,$ find the other two solutions correct to 2 decimal places. (6)
- 13** $f(x) \equiv (x + k)^3 - 8.$
- Given that when $f(x)$ is divided by $(x + 2)$ the remainder is $-7,$
- a** find the value of the constant $k,$ (3)
- b** show that $(x + 1)$ is a factor of $f(x).$ (2)
- 14** $f(x) \equiv x^3 - 4x^2 - 7x + 8.$
- a** Find the remainder when $f(x)$ is divided by $(x + 2).$ (2)
- Given that
- $$g(x) \equiv f(x) + c,$$
- and that $(x + 2)$ is a factor of $g(x),$
- b** state the value of the constant $c,$ (1)
- c** solve the equation $g(x) = 0.$ (4)
- 15** $f(x) \equiv x^3 - 4x + 1.$
- Given that when $f(x)$ is divided by $(2x - k),$ where k is a constant, the remainder is 4,
- a** show that $k^3 - 16k - 24 = 0.$ (3)
- Given also that when $f(x)$ is divided by $(x + k)$ the remainder is 1,
- b** find the value of $k.$ (3)