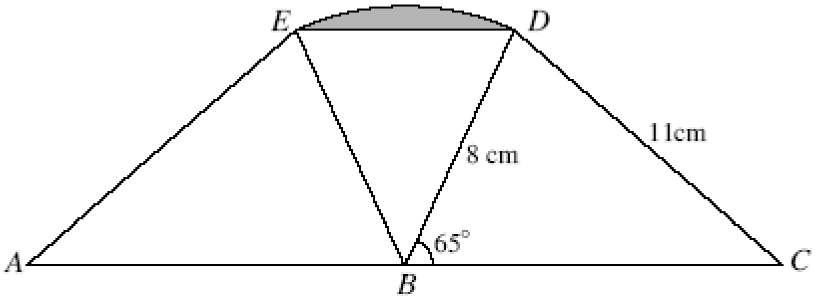
C2 Trigonometry

1. June 2010 qu. 5



The diagram shows two congruent triangles, *BCD* and *BAE*, where *ABC* is a straight line. In triangle *BCD*, *BD* = 8 cm, *CD* = 11 cm and angle *CBD* = 65°. The points *E* and *D* are joined by an arc of a circle with centre *B* and radius 8 cm.

(i) Find angle *BCD*. [2]

(ii) (a) Show that angle *EBD* is 0.873 radians, correct to 3 significant figures. [2]

(b) Hence find the area of the shaded segment bounded by the chord *ED* and the arc *ED*, giving your answer correct to 3 significant figures. [4]

2. June 2010 qu.7

(i) Show that $\dfrac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} \equiv \tan^2 x - 1$. [2]

(ii) Hence solve the equation $\dfrac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = 5 - \tan x$, for $0° \le x \le 360°$. [6]

3. Jan 2010 qu.1

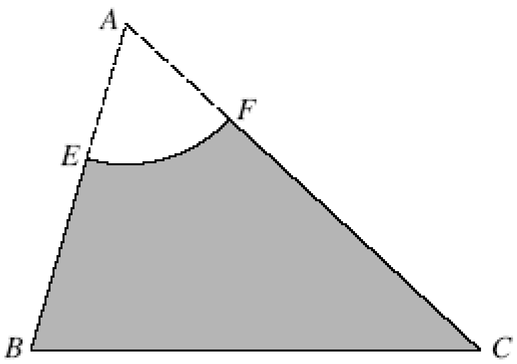
(i) Show that the equation $2\sin^2 x = 5\cos x - 1$

can be expressed in the form $2\cos^2 x + 5\cos x - 3 = 0$. [2]

(ii) Hence solve the equation $2\sin^2 x = 5\cos x - 1$,

giving all values of x between 0° and 360°. [4]

4. Jan 2010 qu.7



The diagram shows triangle ABC, with AB = 10 cm, BC = 13 cm and CA = 14 cm. E and F are points on AB and AC respectively such that $AE = AF$ = 4 cm. The sector AEF of a circle with centre A is removed to leave the shaded region $EBCF$.

(i) Show that angle CAB is 1.10 radians, correct to 3 significant figures. [2]

(ii) Find the perimeter of the shaded region $EBCF$. [3]

(iii) Find the area of the shaded region $EBCF$. [5]

5. June 2009 qu.1

The lengths of the three sides of a triangle are 6.4 cm, 7.0 cm and 11.3 cm.

(i) Find the largest angle in the triangle. [3]

(ii) Find the area of the triangle. [2]

6. June 2009 qu.5

Solve each of the following equations for $0° \le x \le 180°$.

(i) $\sin 2x = 0.5$ [3] (ii) $2\sin^2 x = 2 - \sqrt{3}\cos x$ [5]

7. June 2009 qu.8

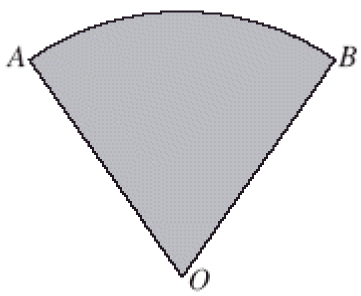


Fig. 1

Fig. 1 shows a sector AOB of a circle, centre O and radius OA. The angle AOB is 1.2 radians and the area of the sector is 60 cm^2.

(i) Find the perimeter of the sector. [4]

A pattern on a T-shirt, the start of which is shown in Fig. 2, consists of a sequence of similar sectors. The first sector in the pattern is sector AOB from Fig. 1, and the area of each successive sector is $\frac{3}{5}$ of the area of the previous one.

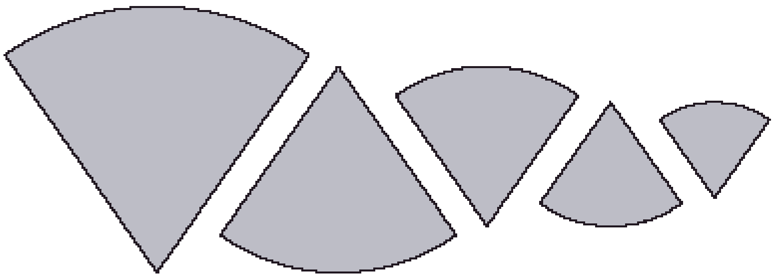


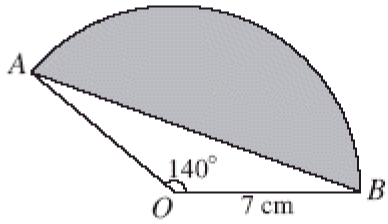
Fig. 2

(ii) (a) Find the area of the fifth sector in the pattern. [2]

(b) Find the total area of the first ten sectors in the pattern. [2]

(c) Explain why the total area will never exceed a certain limit, no matter how many sectors are used, and state the value of this limit. [3]

8. Jan 2009 qu.2

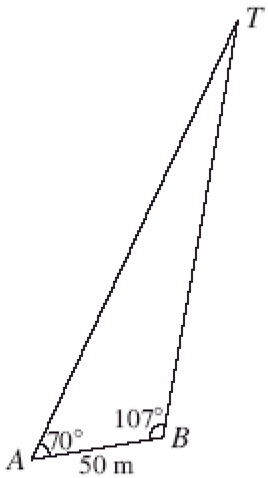


The diagram shows a sector *OAB* of a circle, centre *O* and radius 7 cm. The angle *AOB* is 140°.

(i) Express 140° in radians, giving your answer in an exact form as simply as possible. [2]

(ii) Find the perimeter of the segment shaded in the diagram, giving your answer correct to 3 significant figures. [4]

9. Jan 2009 qu.5



Some walkers see a tower, *T*, in the distance and want to know how far away it is. They take a bearing from a point *A* and then walk for 50m in a straight line before taking another bearing from a point *B*.
They find that angle *TAB* is 70° and angle *TBA* is 107° (see diagram).

(i) Find the distance of the tower from *A*. [2]

(ii) They continue walking in the same direction for another 100m to a point *C*, so that *AC* is 150 m. What is the distance of the tower from *C*? [3]

(iii) Find the shortest distance of the walkers from the tower as they walk from *A* to *C*. [2]

10. Jan 2009 qu.9

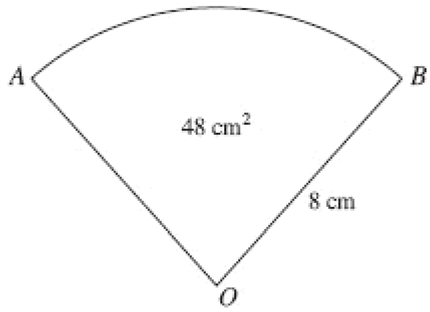
(i) The polynomial $f(x)$ is defined by $f(x) = x^3 - x^2 - 3x + 3$.

Show that $x = 1$ is a root of the equation $f(x) = 0$, and hence find the other two roots. [6]

(ii) Hence solve the equation $\tan^3 x - \tan^2 x - 3 \tan x + 3 = 0$

for $0 \le x \le 2\pi$. Give each solution for x in an exact form. [6]

11. June 2008 qu.3

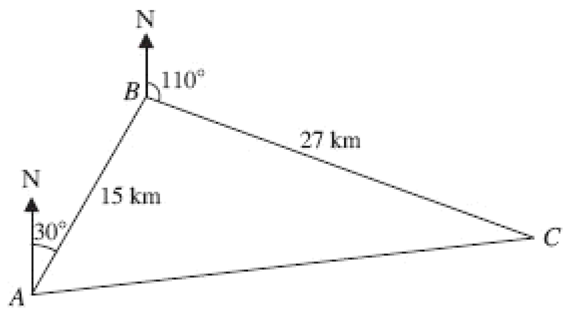


The diagram shows a sector *AOB* of a circle with centre *O* and radius 8 cm. The area of the sector is 48 cm^2.

(i) Find angle *AOB*, giving your answer in radians. [2]

(ii) Find the area of the segment bounded by the arc *AB* and the chord *AB*. [3]

12. June 2008 qu.6



In the diagram, a lifeboat station is at point *A*. A distress call is received and the lifeboat travels 15 km on a bearing of 030º to point *B*. A second call is received and the lifeboat then travels 27 km on a bearing of 110º to arrive at point *C*. The lifeboat then travels back to the station at *A*.

(i) Show that angle *ABC* is 100º. [1]

(ii) Find the distance that the lifeboat has to travel to get from *C* back to *A*. [2]

(iii) Find the bearing on which the lifeboat has to travel to get from *C* to *A*. [4]

13. June 2008 qu.9

(a) (i) Show that the equation $2 \sin x \tan x - 5 = \cos x$

can be expressed in the form $3 \cos^2 x + 5\cos x - 2 = 0$. [3]

(ii) Hence solve the equation $2 \sin x \tan x - 5 = \cos x$,

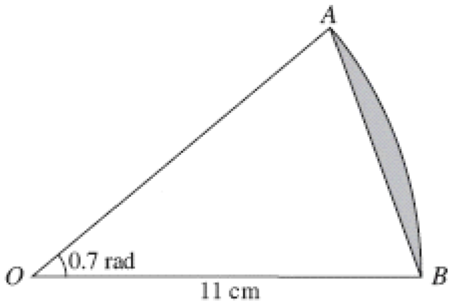
giving all values of x, in radians, for $0 \le x \le 2\pi$. [4]

(b) Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for

$\int_0^1 \cos x \, dx$, where x is in radians. Give your answer correct to 3 significant figures.

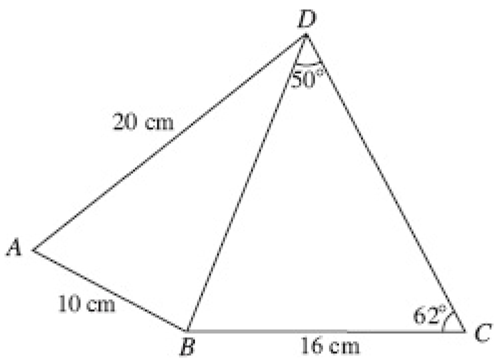
[4]

14. Jan 2008 qu.1



The diagram shows a sector AOB of a circle with centre O and radius 11 cm. The angle AOB is 0.7 radians. Find the area of the segment shaded in the diagram. [4]

15. Jan 2008 qu.4



In the diagram, angle BDC = 50º and angle BCD = 62º. It is given that AB = 10 cm, AD = 20 cm and BC = 16 cm.

(i) Find the length of BD. [2]

(ii) Find angle BAD. [3]

16. Jan 2008 qu.9

(i) Fig. 1 shows the curve $y = 2 \sin x$ for values of x such that $-180º \le x \le 180º$. State the coordinates of the maximum and minimum points on this part of the curve. [2]

(ii)

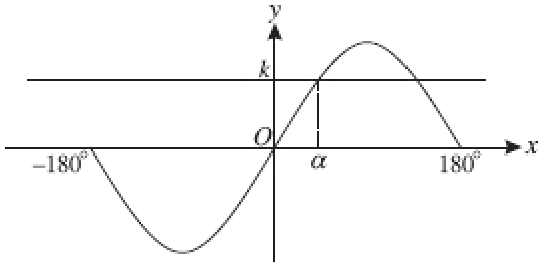


Fig. 2

Fig. 2 shows the curve $y = 2 \sin x$ and the line $y = k$. The smallest positive solution of the equation $2 \sin x = k$ is denoted by α. State, in terms of α, and in the range $-180° \le x \le 180°$,

(a) another solution of the equation $2 \sin x = k$, [1]

(b) one solution of the equation $2 \sin x = -k$. [1]

(iii) Find the x-coordinates of the points where the curve $y = 2 \sin x$ intersects the curve $y = 2 - 3\cos^2 x$, for values of x such that $-180° \le x \le 180°$. [6]

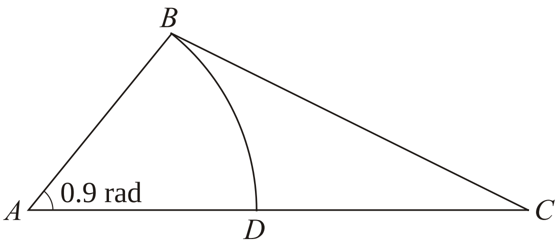
17. June 2007 qu.5

(i) Show that the equation $3\cos^2 \theta = \sin \theta + 1$

can be expressed in the form $3\sin^2 \theta + \sin \theta - 2 = 0$. [2]

(ii) Hence solve the equation $3\cos^2 \theta = \sin \theta + 1$, giving all values of θ between 0° and 360°. [5]

18. June 2007 qu.8



The diagram shows a triangle ABC, where angle BAC is 0.9 radians. BAD is a sector of the circle with centre A and radius AB.

(i) The area of the sector BAD is 16.2 cm^2. Show that the length of AB is 6 cm. [2]

(ii) The area of triangle ABC is twice the area of sector BAD. Find the length of AC. [3]

(iii) Find the perimeter of the region BCD. [6]

19. Jan 2007 qu.2

The diagram shows a sector OAB of a circle, centre O and

radius 8 cm. The angle AOB is 46 .

(i) Express 46° in radians, correct to 3 significant figures. [2]

(ii) Find the length of the arc AB. [1]

(iii) Find the area of the sector OAB. [2]

20. Jan 2007 qu.4

In a triangle ABC, $AB = 5\sqrt{2}$ cm, $BC = 8$ cm and angle $B = 60°$.

(i) Find the exact area of the triangle, giving your answer as simply as possible. [3]

(ii) Find the length of AC, correct to 3 significant figures. [3]

21. Jan 2007 qu.7

(i) (a) Sketch the graph of $y = 2\cos x$ for values of x such that $0° \le x \le 360°$, indicating the coordinates of any points where the curve meets the axes. [2]

(b) Solve the equation2 $\cos x = 0.8$, giving all values of x between 0° and 360°. [3]

(ii) Solve the equation $2\cos x = \sin x$, giving all values of x between −180° and 180°. [3]

22. June 2006 qu.5

Solve each of the following equations, for $0° \le x \le 180°$.

(i) $2\sin^2 x = 1 + \cos x$. [4]

(ii) $\sin 2x = -\cos 2x$. [4]

23. June 2006 qu.7

The diagram shows a triangle ABC,
and a sector ACD of a circle with centre A.
It is given that $AB = 11$ cm, $BC = 8$ cm,
angle $ABC = 0.8$ radians and angle $DAC = 1.7$ radians.
The shaded segment is bounded by the line DC
and the arc DC.

(i) Show that the length of AC is 7.90 cm, correct to 3 significant figures. [3]

(ii) Find the area of the shaded segment. [3]

(iii) Find the perimeter of the shaded segment. [4]

24. Jan 2006 qu.2

Triangle ABC has $AB = 10$ cm, $BC = 7$ cm and angle $B = 80°$. Calculate

(i) the area of the triangle, [2] (ii) the length of CA, [2]

(iii) the size of angle C. [2]

25. Jan 2006 qu.4

The diagram shows a sector OAB of a circle with centre O. The angle AOB is 1.8 radians.
The points C and D lie on OA and OB respectively.
It is given that $OA = OB = 20$ cm and $OC = OD = 15$ cm.
The shaded region is bounded by the arcs AB and CD and by the lines CA and DB.

(i) Find the perimeter of the shaded region. [3]

(ii) Find the area of the shaded region. [3]

26. Jan 2006 qu.9

(i) Sketch, on a single diagram showing values of x from –180° to +180°, the graphs of $y = \tan x$ and $y = 4 \cos x$. [3]

The equation $\tan x = 4 \cos x$

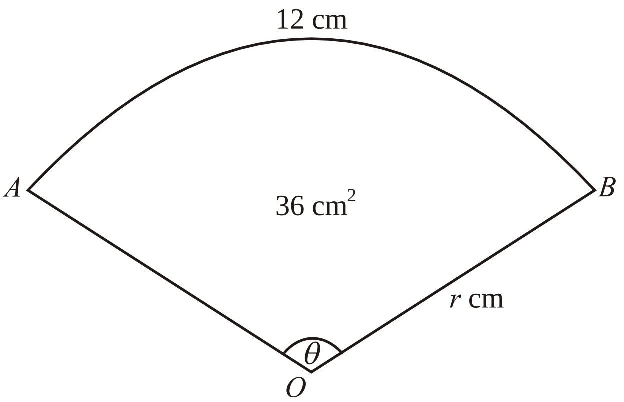
has two roots in the interval $-180° \le x \le 180°$. These are denoted by α and β, where $\alpha < \beta$.

(ii) Show α and β on your sketch, and express β in terms of α. [3]

(iii) Show that the equation $\tan x = 4 \cos x$ may be written as $4 \sin^2 x + \sin x - 4 = 0$.

Hence find the value of $\beta - \alpha$, correct to the nearest degree. [6]

27. June 2005 qu.2



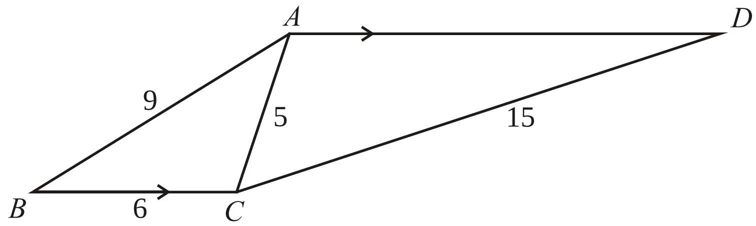
A sector OAB of a circle of radius r cm has angle θ radians. The length of the arc of the sector is 12 cm and the area of the sector is 36 cm^2 (see diagram).

(i) Write down two equations involving r and θ. [2]

(ii) Hence show that $r = 6$, and state the value of θ. [2]

(iii) Find the area of the segment bounded by the arc AB and the chord AB. [3]

28. June 2005 qu.4



In the diagram, $ABCD$ is a quadrilateral in which AD is parallel to BC. It is given that $AB = 9$, $BC = 6$, $CA = 5$ and $CD = 15$.

(i) Show that $\cos BCA = -\frac{1}{3}$, and hence find the value of $\sin BCA$. [4]

(ii) Find the angle ADC correct to the nearest 0.1°. [4]

29. June 2005 qu.9

(a) (i) Write down the exact values of $\cos \frac{1}{6}\pi$ and $\tan \frac{1}{3}\pi$ (where the angles are in radians). Hence verify that $x = \frac{1}{6}\pi$ a solution of the equation $2\cos x = \tan 2x$. [3]

(ii) Sketch, on a single diagram, the graphs of $y = 2\cos x$ and $y = \tan 2x$, for x (radians) such that $0 \le x \le \pi$. Hence state, in terms of π, the other values of x between 0 and π satisfying the equation $2\cos x = \tan 2x$. [4]

(b) (i) Use the trapezium rule, with 3 strips, to find an approximate value for the area of the region bounded by the curve $y = \tan x$, the x-axis, and the lines $x = 0.1$ and $x = 0.4$. (Values of x are in radians.) [4]

(ii) State with a reason whether this approximation is an underestimate or an overestimate. [1]