Integration

1 **(a)** Find $\int (x^2 + 2)\,dx$. [3]

(b) **(i)** Find $\int \frac{3}{x^2}\,dx$. [2]

(ii) Evaluate $\int_1^{\infty} \frac{3}{x^2}\,dx$. [2]

2 **(a)** Find $\int_4^9 (2x - 3x^{\frac{1}{2}} + 1)\,dx$. [5]

(b) Find $\int_2^{\infty} \frac{1}{x^3}\,dx$. [3]

3 Find

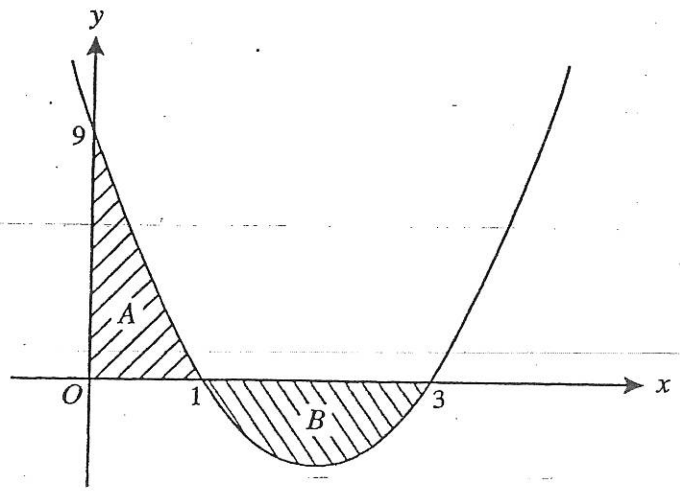
(i) $\int x(x+1)\,dx$, [2]

(ii) $\int \frac{1}{x^2}\,dx$. [2]

4 Find $\int \left(\frac{1}{x^2} - x\right) dx$. [3]

5 Find $\int \left(x^3 + 2x + \frac{1}{x^2}\right) dx$. [4]

6

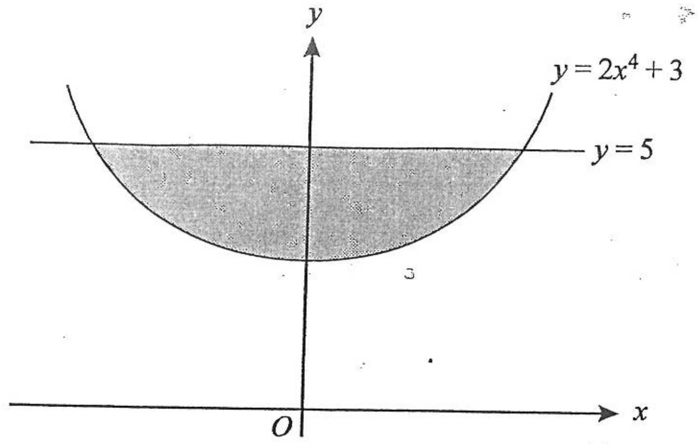


The diagram shows the curve $y = 3x^2 - 12x + 9$.

(i) Show that $\int_0^3 (3x^2 - 12x + 9)\,dx = 0$. [4]

(ii) State what may be deduced from the result in part **(i)** about the areas labelled A and B. [1]

7



The diagram shows the curve $y = 2x^4 + 3$ and the line $y = 5$.

(i) Find the x-coordinates of the points of intersection of the curve and the line. [2]

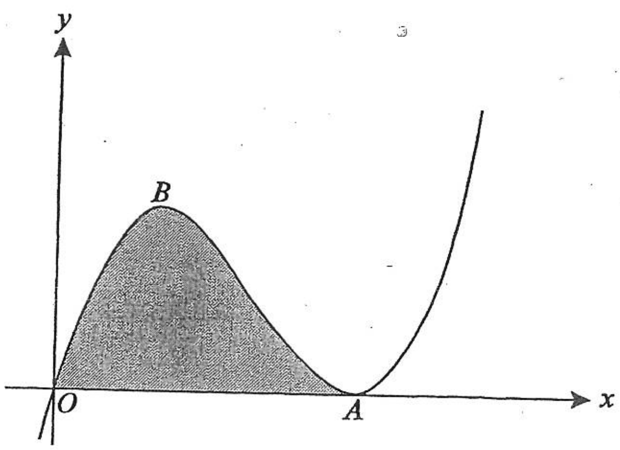
(ii) Calculate the area of the shaded region between the curve and the line. [7]

8 (i) Sketch, on the same diagram, the graph of $y = x^2 + 2$ and the graph of $y = 6 - x^2$, for values of x such that $-3 \leqslant x \leqslant 3$. [2]

(ii) Find the exact values of the x-coordinates of the points of intersection of the curves $y = x^2 + 2$ and $y = 6 - x^2$. [2]

(iii) Show that the area of the region enclosed by the curve $y = x^2 + 2$ and the curve $y = 6 - x^2$ is $\frac{16}{3}\sqrt{2}$. [6]

9



The diagram shows the curve $y = x^3 - 8x^2 + 16x$. The curve passes through the origin, touches the x-axis at A and has a maximum turning point at B.

(i) Show that the equation of the curve may be written in the form $y = x(x - p)^2$, and hence write down the x-coordinate of A. [3]

(ii) Find $\frac{dy}{dx}$, and hence find the x-coordinate of B. [4]

(iii) Calculate the area of the shaded region between the curve and the x-axis. [4]