INTEGRATION

1. (a) $\int (x^2+2)\,dx = \dfrac{x^3}{3} + 2x + k$

(b) (i) $\int \dfrac{3}{x^2}\,dx = \int 3x^{-2}\,dx = -3x^{-1} + k$

$= \dfrac{-3}{x} + k$

(ii) $\int_1^{\infty} \dfrac{3}{x^2}\,dx = \left[\dfrac{-3}{x}\right]_1^{\infty} = 0 - (-3) = 3$

2. (a) $\int_4^9 (2x - 3x^{1/2} + 1)\,dx = \left[x^2 - 2x^{3/2} + x\right]_4^9$

$= (81 - 54 + 9) - (16 - 16 + 4)$

$= 32$

(b) $\int_2^{\infty} \dfrac{1}{x^3}\,dx = \int_2^{\infty} x^{-3}\,dx = \left[\dfrac{-x^{-2}}{2}\right]_2^{\infty}$

$= \left[-\dfrac{1}{2x^2}\right]_2^{\infty}$

$= 0 - (-1/8) = 1/8$

3. (i) $\int x(x+1)\,dx = \int x^2 + x\,dx = \dfrac{x^3}{3} + \dfrac{x^2}{2} + k$

(ii) $\int \dfrac{1}{x^2}\,dx = \int x^{-2}\,dx = -x^{-1} + k = \dfrac{-1}{x} + k$

4. $\int \dfrac{1}{x^2} - x\,dx = \int x^{-2} - x\,dx = -x^{-1} - \dfrac{x^2}{2} + k$

$= -\dfrac{1}{x} - \dfrac{x^2}{2} + k$

5. $\int x^3 + 2x + \dfrac{1}{x^2}\,dx = \int x^3 + 2x + x^{-2}\,dx$

$= \dfrac{x^4}{4} + x^2 - x^{-1} + k$

$= \dfrac{x^4}{4} + x^2 - \dfrac{1}{x} + k$

6. (i) $\int_0^3 (3x^2 - 12x + 9)\, dx$

$= \left[x^3 - 6x^2 + 9x\right]_0^3 = (27 - 54 + 27) - 0$

$= \underline{\underline{0}}$

(ii) The area of A = The area of B.

7. (i) $2x^4 + 3 = 5$

$\Rightarrow 2x^4 = 2$

$\Rightarrow x^4 = 1$

$\Rightarrow \underline{\underline{x = \pm 1}}$

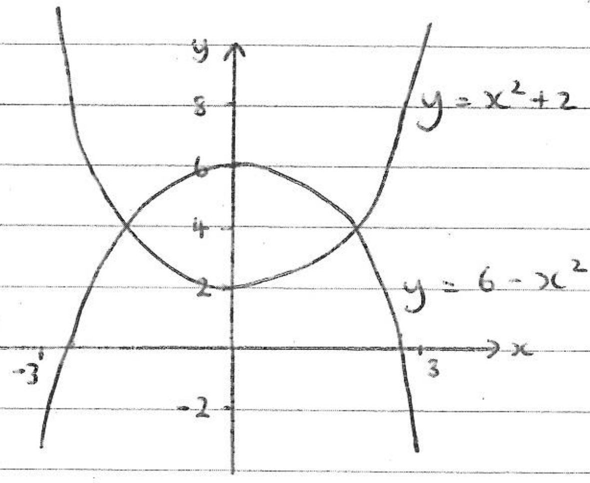
(ii) $\int_{-1}^{1} 2x^4 + 3\, dx = \left[\frac{2x^5}{5} + 3x\right]_{-1}^{1}$

$= \left(\frac{2}{5} + 3\right) - \left(\frac{-2}{5} - 3\right)$

$= 6\,^4/_5$

Area $= 10 - 6\,^4/_5 = \underline{\underline{3\,^1/_5}}$

8. (i)



(ii) $x^2 + 2 = 6 - x^2$

$\Rightarrow 2x^2 = 4$

$\Rightarrow x^2 = 2$

$\Rightarrow \underline{\underline{x = \pm\sqrt{2}}}$

(iii) $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} (6-x^2)-(x^2+2)\ dx$

$= \displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} 4-2x^2\ dx = \left[4x - \frac{2}{3}x^3\right]_{-\sqrt{2}}^{\sqrt{2}}$

$= \left(4\sqrt{2} - \frac{2}{3}\sqrt{2}^3\right) - \left(-4\sqrt{2} + \frac{2}{3}\sqrt{2}^3\right)$

$= 4\sqrt{2} - \frac{4}{3}\sqrt{2} + 4\sqrt{2} - \frac{4}{3}\sqrt{2}$

$= 8\sqrt{2} - \frac{8}{3}\sqrt{2}$

$= \frac{24\sqrt{2} - 8\sqrt{2}}{3} = \underline{\underline{\frac{16\sqrt{2}}{3}}}$

9. (i) $y = x^3 - 8x^2 + 16x$

$= x(x^2 - 8x + 16)$

$= \underline{\underline{x(x-4)^2}}$

x - coordinate at $A = \underline{\underline{4}}$

(ii) $\frac{dy}{dx} = 3x^2 - 16x + 16$

At max. $\frac{dy}{dx} = 0$, $3x^2 - 16x + 16 = 0$

$\Rightarrow (3x-4)(x-4) = 0$

$\Rightarrow x = 4/3$ or 4

At B $\underline{\underline{x = \frac{4}{3}}}$

(iii) $\displaystyle\int_0^4 (x^3 - 8x^2 + 16x)\ dx$

$= \left[\frac{x^4}{4} - \frac{8}{3}x^3 + 8x^2\right]_0^4$

$= \left(64 - \frac{8}{3}.64 + 8.16\right) - 0$

$= \frac{64}{3} = \underline{\underline{21\frac{1}{3}}}$