Factor and Remainder Theorem

1 Given that $x - 2$ is a factor of

$$ax^3 + ax^2 + ax - 42,$$

find the value of the constant a. [3]

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2 Polynomials $\mathrm{P}(x)$ and $\mathrm{Q}(x)$ are defined by

$$\mathrm{P}(x) = x^3 - 4x^2 + ax + 16,$$
$$\mathrm{Q}(x) = x^3 - 4x^2 - 14x + 12.$$

(i) The remainder when $\mathrm{P}(x)$ is divided by $x - 6$ is 4. Show that the value of the constant a is -14. [3]

(ii) Hence solve the equation $\mathrm{Q}(x) = 0$, giving non-integer roots in exact form. [5]

3 The polynomial $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = ax^3 + bx^2 + 4,$$

where a and b are constants. It is given that $x + 2$ is a factor of $\mathrm{f}(x)$. It is also given that, when $\mathrm{f}(x)$ is divided by $x - 3$, the remainder is 130. Find the values of a and b. [7]

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4 The polynomial $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = x^3 + ax^2 - 2ax + c,$$

where a and c are constants.

(i) It is given that $(x - 2)$ is a factor of $\mathrm{f}(x)$. Find the value of c. [2]

(ii) It is further given that, when $\mathrm{f}(x)$ is divided by $(x - 1)$, the remainder is 5. Find the value of a. [3]

5 **(i)** Find the remainder when $x^3 - 8x^2 + 11x$ is divided by $(x - 2)$. [2]

(ii) Find the three roots of the equation

$$x^3 - 8x^2 + 11x + 2 = 0,$$

giving the two non-integer roots in the exact form $p \pm \sqrt{q}$, where p and q are integers. [5]

(iii) Given that $|y| = x$, where x satisfies the cubic equation in part **(ii)**, state the possible values of y. [2]