FACTOR AND REMAINDER THEOREM

1. $f(2) = 8a + 4a + 2a - 42 = 0$

$\Rightarrow 14a = 42$

$\Rightarrow a = 3$

2. (i) $P(6) = 6^3 - 4 \times 6^2 + 6a + 16 = 4$

$\Rightarrow 88 + 6a = 4$

$\Rightarrow 6a = -84$

$a = -14$

(ii) $Q(6)$ must equal 0 as $Q(x) = P(x) - 4$

$x^3 - 4x^2 - 14x + 12 \equiv (x-6)(x^2 + Ax - 2)$

Eq. Coef: x^2 : $-4 = -6 + A$

$\Rightarrow A = 2$

So, $Q(x) \equiv (x-6)(x^2 + 2x - 2)$

Solve $x^2 + 2x - 2$

$$x = \frac{-2 \pm \sqrt{4+8}}{2} = \frac{-2 \pm \sqrt{12}}{2} = \frac{-2 \pm 2\sqrt{3}}{2}$$

$$= -1 \pm \sqrt{3}$$

So solutions are $x = 6$, $-1 + \sqrt{3}$, $-1 - \sqrt{3}$

3. $f(-2) = -8a + 4b + 4 = 0$ ①

$f(3) = 27a + 9b + 4 = 130$ ②

① $\times 9$: $-72a + 36b + 36 = 0$

② $\times 4$: $108a + 36b - 504 = 0$

$180a - 540 = 0$

$a = 3$

$-24 + 4b + 4 = 0$

$\Rightarrow b = 5$

4. (i) $f(2) = 2^3 + 4a - 4a + c = 0$

$\Rightarrow c = -8$

(ii) $f(1) = 1^3 + a - 2a - 8 = 5$

$\Rightarrow -a = 12$

$a = -12$

5. (i) $f(2) = 2^3 - 8\times 2^2 + 11\times 2$

$= 8 - 32 + 22$

$= \underline{\underline{-2}}$

(ii) $g(x) = x^3 - 8x^2 + 11x + 2$

$g(2)$ is a factor of $g(x) = f(x) + 2$

$x^3 - 8x^2 + 11x + 2 \equiv (x-2)(x^2 + Ax - 1)$

Eq. Coef: x^2 : $-8 = -2 + A$

$\Rightarrow A = -6$

$g(x) = (x-2)(x^2 - 6x - 1)$

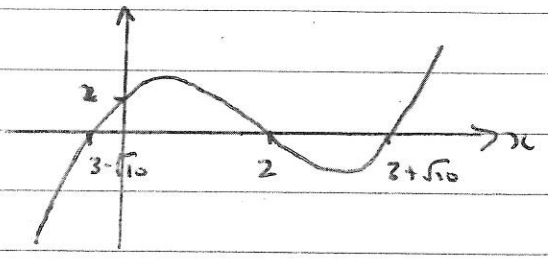
Solve by quad. formula.

$$x = \frac{6 \pm \sqrt{36+4}}{2} = \frac{6 \pm \sqrt{40}}{2} = \frac{6 \pm 2\sqrt{10}}{2}$$

$$= 3 \pm \sqrt{10}$$

S. solutions are $\underline{\underline{2}}$, $\underline{\underline{3+\sqrt{10}}}$, $\underline{\underline{3-\sqrt{10}}}$

(iii)



If $|y| = x$

$y = \underline{\underline{2}}$ or $\underline{\underline{3+\sqrt{10}}}$