1 A sequence is defined by $u_1 = 2$ and $u_{k+1} = \dfrac{10}{u_k^2}$.

Calculate $\displaystyle\sum_{k=1}^{4} u_k$. **[3]**

2 The nth term of a sequence, u_n, is given by

$$u_n = 12 - \tfrac{1}{2}n.$$

   (i) Write down the values of u_1, u_2 and u_3. State what type of sequence this is. **[2]**

   (ii) Find $\displaystyle\sum_{n=1}^{30} u_n$. **[3]**

3 A sequence is defined by

$$u_1 = 10,$$
$$u_{r+1} = \dfrac{5}{u_r^2}.$$

Calculate the values of u_2, u_3 and u_4.

What happens to the terms of the sequence as r tends to infinity? **[3]**

4 The nth term, t_n, of a sequence is given by

$$t_n = \sin(\theta + 180n)^\circ.$$

Express t_1 and t_2 in terms of $\sin\theta^\circ$. **[2]**

5 Jim and Mary are each planning monthly repayments for money they want to borrow.

   (i) Jim's first payment is £500, and he plans to pay £10 less each month, so that his second payment is £490, his third is £480, and so on.

      (*A*) Calculate his 12th payment. **[2]**

      (*B*) He plans to make 24 payments altogether. Show that he pays £9240 in total. **[2]**

  (ii) Mary's first payment is £460 and she plans to pay 2% less each month than the previous month, so that her second payment is £450.80, her third is £441.784, and so on.

      (*A*) Calculate her 12th payment. **[2]**

      (*B*) Show that Jim's 20th payment is less than Mary's 20th payment but that his 19th is not less than her 19th. **[3]**

      (*C*) Mary plans to make 24 payments altogether. Calculate how much she pays in total. **[2]**

      (*D*) How much would Mary's first payment need to be if she wishes to pay 2% less each month as before, but to pay the same in total as Jim, £9240, over the 24 months? **[2]**

6 You are given that

$$u_1 = 1,$$

$$u_{n+1} = \frac{u_n}{1 + u_n}.$$

Find the values of u_2, u_3 and u_4. Give your answers as fractions. **[2]**

7   **(i)** Evaluate $\displaystyle\sum_{r=2}^{5} \frac{1}{r-1}$. **[2]**

  (ii) Express the series $2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6 + 6 \times 7$ in the form $\displaystyle\sum_{r=2}^{a} \mathrm{f}(r)$ where $\mathrm{f}(r)$ and a are to be determined. **[2]**

8 **(i)** Find $\displaystyle\sum_{k=3}^{8}(k^2 - 1)$. **[2]**

 (ii) State whether the sequence with kth term $k^2 - 1$ is convergent or divergent, giving a reason for your answer. **[1]**

9 **(i)** Find the second and third terms of the sequence defined by the following.

$$t_{n+1} = 2t_n + 5$$
$$t_1 = 3$$

[2]

 (ii) Find $\displaystyle\sum_{k=1}^{3} k(k+1)$. **[2]**

10 For each of the following sequences, state with a reason whether it is convergent, periodic or neither. Each sequence continues in the pattern established by the given terms.

 (i) $3, \ \dfrac{3}{2}, \ \dfrac{3}{4}, \ \dfrac{3}{8}, \ \dots$ **[1]**

 (ii) $3, \ 7, \ 11, \ 15, \ \dots$ **[1]**

 (iii) $3, \ 5, \ -3, \ -5, \ 3, \ 5, \ -3, \ -5, \ \dots$ **[1]**

11 Find $\displaystyle\sum_{r=3}^{6} r(r+2)$. **[2]**

12 Calculate $\displaystyle\sum_{r=3}^{6} \frac{12}{r}$. **[2]**

12 A sequence begins

$$1 \quad 3 \quad 5 \quad 3 \quad 1 \quad 3 \quad 5 \quad 3 \quad 1 \quad 3 \quad \ldots$$

and continues in this pattern.

  (i) Find the 55th term of this sequence, showing your method. **[1]**

  (ii) Find the sum of the first 55 terms of the sequence. **[2]**