1 The 7th term of an arithmetic progression is 6. The sum of the first 10 terms of the progression is 30.

Find the 5th term of the progression. [5]

2 The first three terms of a geometric progression are 4, 2, 1.

Find the twentieth term, expressing your answer as a power of 2.

Find also the sum to infinity of this progression. [5]

3 A sequence is given by

$$a_1 = 4,$$

$$a_{r+1} = a_r + 3.$$

Write down the first 4 terms of this sequence.

Find the sum of the first 100 terms of the sequence. [5]

4 There is a flowerhead at the end of each stem of an oleander plant. The next year, each flowerhead is replaced by three stems and flowerheads, as shown in Fig. 11.

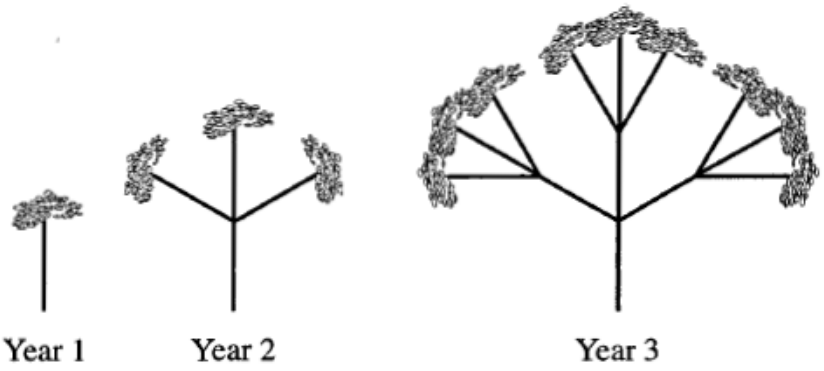

Fig. 11

(i) How many flowerheads are there in year 5? [1]

(ii) How many flowerheads are there in year n? [1]

(iii) As shown in Fig. 11, the total number of stems in year 2 is 4, (that is, 1 old one and 3 new ones). Similarly, the total number of stems in year 3 is 13, (that is, $1 + 3 + 9$).

Show that the total number of stems in year n is given by $\frac{3^n - 1}{2}$. [2]

(iv) Kitty's oleander has a total of 364 stems. Find

(*A*) its age, [2]

(*B*) how many flowerheads it has. [1]

(v) Abdul's oleander has over 900 flowerheads.

Show that its age, y years, satisfies the inequality $y > \frac{\log_{10} 900}{\log_{10} 3} + 1$.

Find the smallest integer value of y for which this is true. [4]