

- 1 Given that $a > 0$, state the values of
- (i) $\log_a 1$, [1]
 - (ii) $\log_a (a^3)^6$, [1]
 - (iii) $\log_a \sqrt{a}$. [1]
- 2 Use logarithms to solve the equation $235 \times 5^x = 987$, giving your answer correct to 3 decimal places. [3]
- 3 Given that $y = a + x^b$, find $\log_{10} x$ in terms of y , a and b . [3]
- 4 Simplify
- (i) $\log_{10} x^5 + 3 \log_{10} x^4$, [2]
 - (ii) $\log_a 1 - \log_a a^b$. [2]

5 The table shows the size of a population of house sparrows from 1980 to 2005.

Year	1980	1985	1990	1995	2000	2005
Population	25 000	22 000	18 750	16 250	13 500	12 000

The 'red alert' category for birds is used when a population has decreased by at least 50% in the previous 25 years.

- (i) Show that the information for this population is consistent with the house sparrow being on red alert in 2005. [1]

The size of the population may be modelled by a function of the form $P = a \times 10^{-kt}$, where P is the population, t is the number of years after 1980, and a and k are constants.

- (ii) Write the equation $P = a \times 10^{-kt}$ in logarithmic form using base 10, giving your answer as simply as possible. [2]
- (iii) Complete the table and draw the graph of $\log_{10} P$ against t , drawing a line of best fit by eye. [3]
- (iv) Use your graph to find the values of a and k and hence the equation for P in terms of t . [4]
- (v) Find the size of the population in 2015 as predicted by this model.
Would the house sparrow still be on red alert? Give a reason for your answer. [3]

- 6 (i) Sketch the graph of $y = 3^x$. [2]
- (ii) Use logarithms to solve $3^{2x+1} = 10$, giving your answer correct to 2 decimal places. [3]

7 Answer part (ii) of this question on the insert provided.

Since 1945 the populations of many countries have been growing. The table shows the estimated population of 15- to 59-year-olds in Africa during the period 1955 to 2005.

Year	1955	1965	1975	1985	1995	2005
Population (millions)	131	161	209	277	372	492

Source: United Nations

Such estimates are used to model future population growth and world needs of resources. One model is $P = a10^{bt}$, where the population is P millions, t is the number of years after 1945 and a and b are constants.

- (i) Show that, using this model, the graph of $\log_{10} P$ against t is a straight line of gradient b . State the intercept of this line on the vertical axis. [3]
- (ii) **On the insert**, complete the table, giving values correct to 2 decimal places, and plot the graph of $\log_{10} P$ against t . Draw, by eye, a line of best fit on your graph. [3]
- (iii) Use your graph to find the equation for P in terms of t . [4]
- (iv) Use your results to estimate the population of 15- to 59-year-olds in Africa in 2050. Comment, with a reason, on the reliability of this estimate. [3]

8 (i) State the value of $\log_a a$. [1]

(ii) Express each of the following in terms of $\log_a x$.

(A) $\log_a x^3 + \log_a \sqrt{x}$ [2]

(B) $\log_a \frac{1}{x}$ [1]