

**1 Answer the whole of this question on the insert provided.**

A colony of bats is increasing. The population,  $P$ , is modelled by  $P = a \times 10^{bt}$ , where  $t$  is the time in years after 2000.

- (i) Show that, according to this model, the graph of  $\log_{10} P$  against  $t$  should be a straight line of gradient  $b$ . State, in terms of  $a$ , the intercept on the vertical axis. [3]
- (ii) The table gives the data for the population from 2001 to 2005.

Year	2001	2002	2003	2004	2005
$t$	1	2	3	4	
$P$	7900	8800	10000	11300	12800

Complete the table of values on the insert, and plot  $\log_{10} P$  against  $t$ . Draw a line of best fit for the data. [3]

- (iii) Use your graph to find the equation for  $P$  in terms of  $t$ . [4]
- (iv) Predict the population in 2008 according to this model. [2]

2 (i) Write down the value of  $\log_5 5$ . [1]

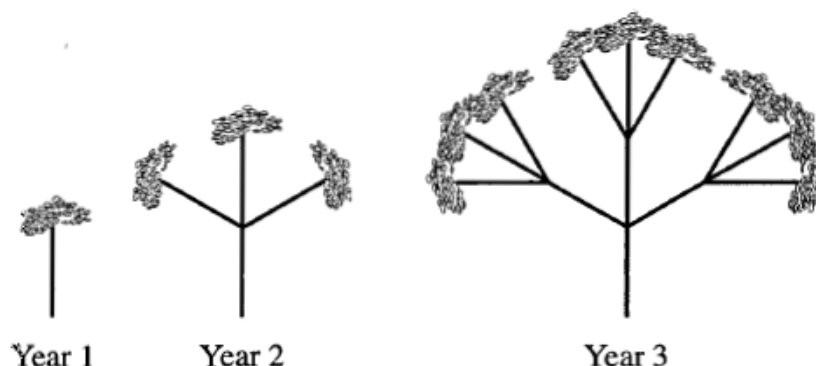
(ii) Find  $\log_3 \left(\frac{1}{9}\right)$ . [2]

(iii) Express  $\log_a x + \log_a (x^5)$  as a multiple of  $\log_a x$ . [2]

3 Sketch the graph of  $y = 2^x$ .

Solve the equation  $2^x = 50$ , giving your answer correct to 2 decimal places. [5]

- 4 There is a flowerhead at the end of each stem of an oleander plant. The next year, each flowerhead is replaced by three stems and flowerheads, as shown in Fig. 11.



**Fig. 11**

- (i) How many flowerheads are there in year 5? [1]
- (ii) How many flowerheads are there in year  $n$ ? [1]
- (iii) As shown in Fig. 11, the total number of stems in year 2 is 4, (that is, 1 old one and 3 new ones). Similarly, the total number of stems in year 3 is 13, (that is, 1 + 3 + 9).

Show that the total number of stems in year  $n$  is given by  $\frac{3^n - 1}{2}$ . [2]

- (iv) Kitty's oleander has a total of 364 stems. Find

(A) its age, [2]

(B) how many flowerheads it has. [1]

- (v) Abdul's oleander has over 900 flowerheads.

Show that its age,  $y$  years, satisfies the inequality  $y > \frac{\log_{10} 900}{\log_{10} 3} + 1$ .

Find the smallest integer value of  $y$  for which this is true. [4]

- 5 (i) Sketch the graph of  $y = 3^x$ . [2]
- (ii) Solve the equation  $3^{5x-1} = 500\,000$ . [3]

6 The table shows population data for a country.

Year	1969	1979	1989	1999	2009
Population in millions ( $p$ )	58.81	80.35	105.27	134.79	169.71

The data may be represented by an exponential model of growth. Using  $t$  as the number of years after 1960, a suitable model is  $p = a \times 10^{kt}$ .

- (i) Derive an equation for  $\log_{10} p$  in terms of  $a$ ,  $k$  and  $t$ . [2]
- (ii) Complete the table and draw the graph of  $\log_{10} p$  against  $t$ , drawing a line of best fit by eye. [3]
- (iii) Use your line of best fit to express  $\log_{10} p$  in terms of  $t$  and hence find  $p$  in terms of  $t$ . [4]
- (iv) According to the model, what was the population in 1960? [1]
- (v) According to the model, when will the population reach 200 million? [3]