1 Express $\log_a x^3 + \log_a \sqrt{x}$ in the form $k \log_a x$. **[2]**

2 The points (2, 6) and (3, 18) lie on the curve $y = ax^n$.

Use logarithms to find the values of a and n, giving your answers correct to 2 decimal places. **[5]**

3

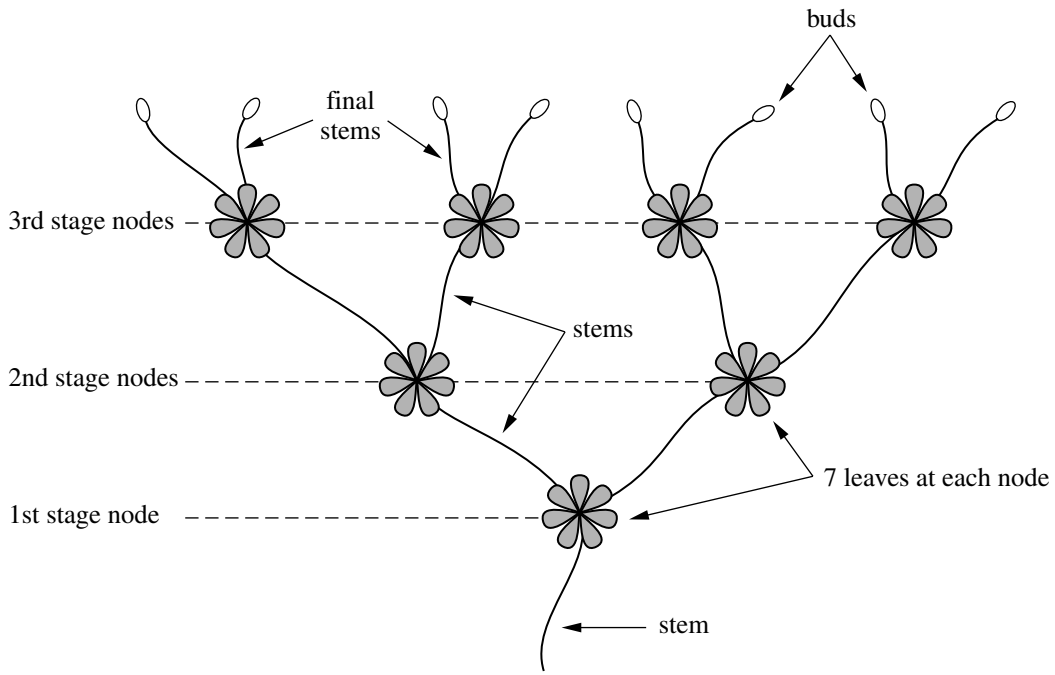

Fig. 12

A branching plant has stems, nodes, leaves and buds.

- There are 7 leaves at each node.
- From each node, 2 new stems grow.
- At the end of each final stem, there is a bud.

Fig. 12 shows one such plant with 3 stages of nodes. It has 15 stems, 7 nodes, 49 leaves and 8 buds.

(i) One of these plants has 10 stages of nodes.

(*A*) How many buds does it have? **[2]**

(*B*) How many stems does it have? **[2]**

(ii) (*A*) Show that the number of leaves on one of these plants with n stages of nodes is

$$7(2^n - 1).$$ **[2]**

(*B*) One of these plants has n stages of nodes and more than 200 000 leaves. Show that n satisfies the inequality $n > \dfrac{\log_{10} 200\,007 - \log_{10} 7}{\log_{10} 2}$. Hence find the least possible value of n. **[4]**

4 Simplify

(i) $10 - 3\log_a a$, **[1]**

(ii) $\dfrac{\log_{10} a^5 + \log_{10}\sqrt{a}}{\log_{10} a}$. **[2]**

5 **Answer part (i) of this question on the insert provided.**

Ash trees grow quickly for the first years of their life, then more slowly. This table shows the height of a tree at various ages.

Age (t years)	4	7	10	15	20	40
Height (h m)	4	9	12	17	19	26

The height, h m, of an ash tree when it is t years old may be modelled by an equation of the form

$$h = a\log_{10} t + b.$$

(i) **On the insert**, complete the table and plot h against $\log_{10} t$, drawing by eye a line of best fit. **[3]**

(ii) Use your graph to find an equation for h in terms of $\log_{10} t$ for this model. **[3]**

(iii) Find the height of the tree at age 100 years, as predicted by this model. **[1]**

(iv) Find the age of the tree when it reaches a height of 29 m, according to this model. **[3]**

(v) Comment on the suitability of the model when the tree is very young. **[2]**

6 Use logarithms to solve the equation $5^x = 235$, giving your answer correct to 2 decimal places. [3]

7 **(i)** Write down the values of $\log_a 1$ and $\log_a a$, where $a > 1$. [2]

(ii) Show that $\log_a x^{10} \quad 2\log_a\left(\frac{x^3}{4}\right) = 4\log_a(2x)$. [3]

8 **(i)** Sketch the graph of $y = 3^x$. [2]

(ii) Use logarithms to solve the equation $3^x = 20$. Give your answer correct to 2 decimal places. [3]

9 Write down the values of $\log_a a$ and $\log_a (a^3)$. [2]

10 Use logarithms to solve the equation $5^{3x} = 100$. Give your answer correct to 3 decimal places. [4]