1 **(i)**

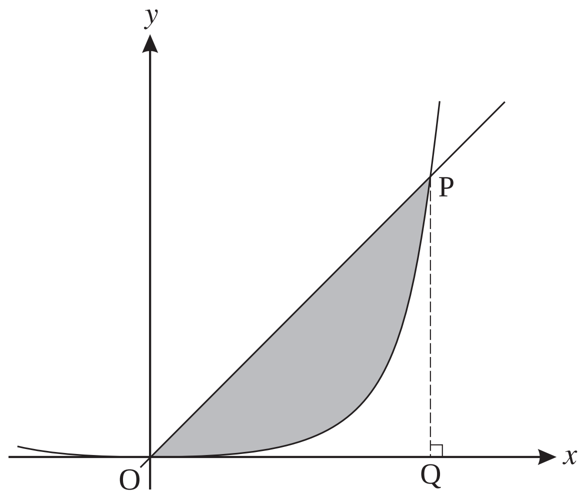


Fig. 12

Fig. 12 shows part of the curve $y = x^4$ and the line $y = 8x$, which intersect at the origin and the point P.

(A) Find the coordinates of P, and show that the area of triangle OPQ is 16 square units. [3]

(B) Find the area of the region bounded by the line and the curve. [3]

(ii) You are given that $\mathrm{f}(x) = x^4$.

(A) Complete this identity for $\mathrm{f}(x + h)$.

$$\mathrm{f}(x+h) = (x+h)^4 = x^4 + 4x^3h + \ \ldots$$ [2]

(B) Simplify $\dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$. [2]

(C) Find $\lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$. [1]

(D) State what this limit represents. [1]

2

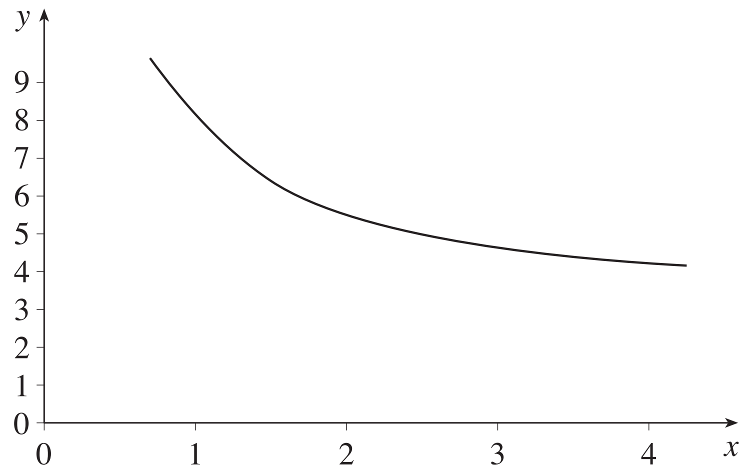


Fig. 4

Fig. 4 shows a curve which passes through the points shown in the following table.

x	1	1.5	2	2.5	3	3.5	4
y	8.2	6.4	5.5	5.0	4.7	4.4	4.2

Use the trapezium rule with 6 strips to estimate the area of the region bounded by the curve, the lines $x = 1$ and $x = 4$, and the x-axis.

State, with a reason, whether the trapezium rule gives an overestimate or an underestimate of the area of this region. [5]

3 (i) A tunnel is 100 m long. Its cross-section, shown in Fig. 9.1, is modelled by the curve

$$y = \frac{1}{4}(10x - x^2),$$

where x and y are horizontal and vertical distances in metres.

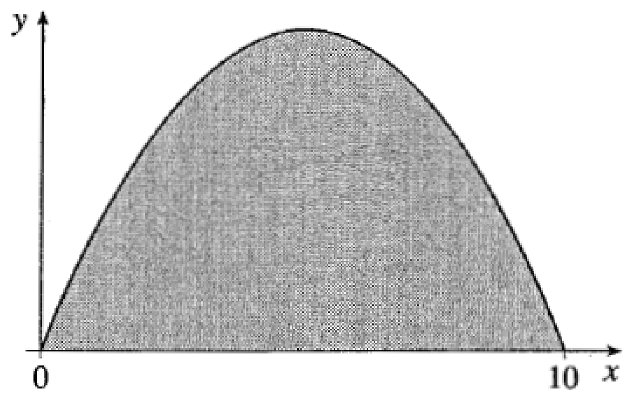


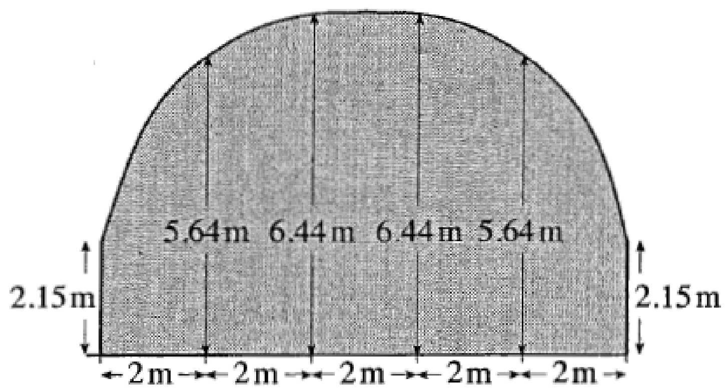
Figure 9.1

Using this model,

(A) find the greatest height of the tunnel, [2]

(B) explain why $100\int_0^{10} y\,dx$ gives the volume, in cubic metres, of earth removed to make the tunnel. Calculate this volume. [5]

(ii) The roof of the tunnel is re-shaped to allow for larger vehicles. Fig. 9.2 shows the new cross-section.



Not to scale

Fig. 9.2

Use the trapezium rule with 5 strips to estimate the new cross-sectional area.

Hence estimate the volume of earth removed when the tunnel is re-shaped. [5]