1 The equation of a cubic curve is $y = 2x^3 - 9x^2 + 12x - 2$.

(i) Find $\frac{dy}{dx}$ and show that the tangent to the curve when $x = 3$ passes through the point $(-1, -41)$. [5]

(ii) Use calculus to find the coordinates of the turning points of the curve. You need not distinguish between the maximum and minimum. [4]

(iii) Sketch the curve, given that the only real root of $2x^3 - 9x^2 + 12x - 2 = 0$ is $x = 0.2$ correct to 1 decimal place. [3]

2 A cubic curve has equation $y = x^3 - 3x^2 + 1$.

(i) Use calculus to find the coordinates of the turning points on this curve. Determine the nature of these turning points. [5]

(ii) Show that the tangent to the curve at the point where $x = -1$ has gradient 9.

Find the coordinates of the other point, P, on the curve at which the tangent has gradient 9 and find the equation of the normal to the curve at P.

Show that the area of the triangle bounded by the normal at P and the x- and y-axes is 8 square units. [8]

3 A curve has equation $y = x + \frac{1}{x}$.

Use calculus to show that the curve has a turning point at $x = 1$.

Show also that this point is a minimum. [5]

4 The equation of a curve is $y = 9x^2 - x^4$.

(i) Show that the curve meets the x-axis at the origin and at $x = \pm a$, stating the value of a. **[2]**

(ii) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

Hence show that the origin is a minimum point on the curve. Find the x-coordinates of the maximum points. **[6]**

(iii) Use calculus to find the area of the region bounded by the curve and the x-axis between $x = 0$ and $x = a$, using the value you found for a in part **(i)**. **[4]**

5 Differentiate $4x^2 + \frac{1}{x}$ and hence find the x-coordinate of the stationary point of the curve $y = 4x^2 + \frac{1}{x}$. **[5]**

6

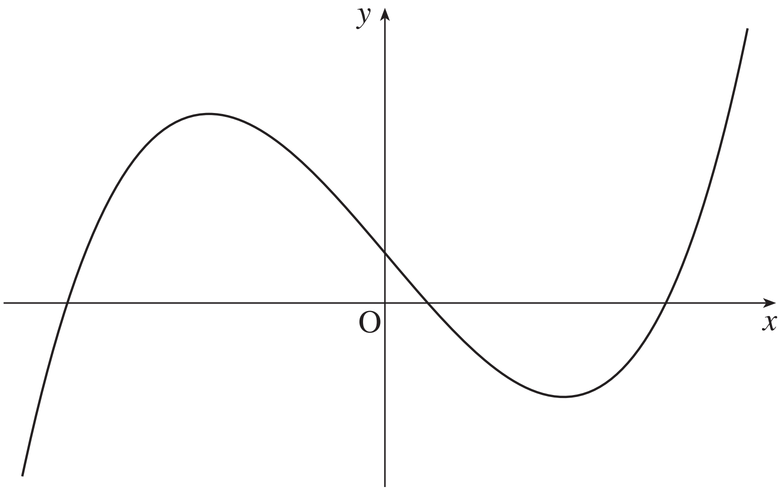

Fig. 11

The equation of the curve shown in Fig. 11 is $y = x^3 - 6x + 2$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) Find, in exact form, the range of values of x for which $x^3 - 6x + 2$ is a decreasing function. [3]

(iii) Find the equation of the tangent to the curve at the point $(-1, 7)$.

Find also the coordinates of the point where this tangent crosses the curve again. [6]