Edexcel Maths C2

Topic Questions from Papers

Series and Sequences

Leave blank

9. (a) A geometric series has first term a and common ratio r. Prove that the sum of the first n terms of the series is

$$\frac{a(1-r^n)}{1-r}.$$

(4)

Mr. King will be paid a salary of £35 000 in the year 2005. Mr. King's contract promises a 4% increase in salary every year, the first increase being given in 2006, so that his annual salaries form a geometric sequence.

(b) Find, to the nearest £100, Mr. King's salary in the year 2008.

(2)

Mr. King will receive a salary each year from 2005 until he retires at the end of 2024.

(c) Find, to the nearest £1000, the total amount of salary he will receive in the period from 2005 until he retires at the end of 2024.

(4)

Leave blank

Question 9 continued

Leave blank

4. The first term of a geometric series is 120. The sum to infinity of the series is 480.

(a) Show that the common ratio, r, is $\frac{3}{4}$.

(3)

(b) Find, to 2 decimal places, the difference between the 5th and 6th term.

(2)

(c) Calculate the sum of the first 7 terms.

(2)

The sum of the first n terms of the series is greater than 300.

(d) Calculate the smallest possible value of n.

(4)

Leave blank

9. A geometric series has first term a and common ratio r.
The second term of the series is 4 and the sum to infinity of the series is 25.

(a) Show that $25r^2 - 25r + 4 = 0$.

(4)

(b) Find the two possible values of r.

(2)

(c) Find the corresponding two possible values of a.

(2)

(d) Show that the sum, S_n, of the first n terms of the series is given by

$$S_n = 25(1 - r^n).$$

(1)

Given that r takes the larger of its two possible values,

(e) find the smallest value of n for which S_n exceeds 24.

(2)

Leave blank

Question 9 continued

Q9

(Total 11 marks)

Leave blank

2. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1-2x)^5$. Give each term in its simplest form.

(4)

(b) If x is small, so that x^2 and higher powers can be ignored, show that

$$(1+x)(1-2x)^5 \approx 1-9x\,.$$

(2)

Leave blank

10. A geometric series is $a + ar + ar^2 + \ldots$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

(4)

(b) Find

$$\sum_{k=1}^{10} 100(2^k).$$

(3)

(c) Find the sum to infinity of the geometric series

$$\frac{5}{6} + \frac{5}{18} + \frac{5}{54} + \ldots.$$

(3)

(d) State the condition for an infinite geometric series with common ratio r to be convergent.

(1)

Leave blank

Question 10 continued

Q10

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

8. A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r, $r > 1$.

The model therefore predicts that in 2007 (Year 2) a profit of £50 000r will be made.

(a) Write down an expression for the predicted profit in Year n.

(1)

The model predicts that in Year n, the profit made will exceed £200 000.

(b) Show that $n > \frac{\log 4}{\log r} + 1$.

(3)

Using the model with $r = 1.09$,

(c) find the year in which the profit made will first exceed £200 000,

(2)

(d) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000.

(3)

Leave blank

Question 8 continued

Q8

(Total 9 marks)

H26108A01924

Turn over

Leave blank

2. The fourth term of a geometric series is 10 and the seventh term of the series is 80.

For this series, find

(a) the common ratio,

(2)

(b) the first term,

(2)

(c) the sum of the first 20 terms, giving your answer to the nearest whole number.

(2)

Leave blank

6. A geometric series has first term 5 and common ratio $\frac{4}{5}$.

Calculate

(a) the 20th term of the series, to 3 decimal places, **(2)**

(b) the sum to infinity of the series. **(2)**

Given that the sum to k terms of the series is greater than 24.95,

(c) show that $k > \frac{\log 0.002}{\log 0.8}$, **(4)**

(d) find the smallest possible value of k. **(1)**

Leave blank

Question 6 continued

Leave blank

9. The first three terms of a geometric series are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

(a) Show that $k^2 - 7k - 60 = 0$.

(4)

(b) Hence show that $k = 12$.

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum to infinity of this series.

(2)

Leave blank

Question 9 continued

Leave blank

5. The third term of a geometric sequence is 324 and the sixth term is 96

(a) Show that the common ratio of the sequence is $\frac{2}{3}$
(2)

(b) Find the first term of the sequence.
(2)

(c) Find the sum of the first 15 terms of the sequence.
(3)

(d) Find the sum to infinity of the sequence.
(2)

H 3 4 2 6 3 A 0 1 2 2 4

Leave blank

Question 5 continued

Q5

(Total 9 marks)

H 3 4 2 6 3 A 0 1 3 2 4

Turn over

6. A car was purchased for 18 000 on 1st January.
 n 1st January each following year, the value of the car is 80 of its value on 1st January in the previous year.

(a) Show that the value of the car exactly 3 years after it was purchased is 9216.
(1)

The value of the car falls below 1000 for the first time years after it was purchased.

(b) Find the value of .
(3)

An insurance company has a scheme to cover the maintenance of the car.
The cost is 200 for the first year, and for every following year the cost increases by 12 so that for the 3rd year the cost of the scheme is 250.88

(c) Find the cost of the scheme for the 5th year, giving your answer to the nearest penny.
(2)

(d) Find the total cost of the insurance scheme for the first 15 years.
(3)

Leave blank

Question 6 continued

Leave blank

9. The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

(a) Show that the predicted adult population at the end of Year 2 is 25 750.
(1)

(b) Write down the common ratio of the geometric sequence.
(1)

The model predicts that Year will be the first year in which the adult population of the town exceeds 40 000.

(c) Show that

$$(\quad -1)\log 1.03 > \log 1.6$$
(3)

(d) Find the value of .
(2)

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(e) find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000.
(3)

Leave blank

Question 9 continued

H35384A02328

Turn over

Leave blank

3. The second and fifth terms of a geometric series are 750 and –6 respectively.

Find

(a) the common ratio of the series,
(3)

(b) the first term of the series,
(2)

(c) the sum to infinity of the series.
(2)

Leave blank

6. The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(a) the common ratio,
(2)

(b) the first term,
(2)

(c) the sum to infinity,
(2)

(d) the smallest value of n for which the sum of the first n terms of the series exceeds 1000.
(4)

Leave blank

Question 6 continued

Leave blank

1. A geometric series has first term $a = 360$ and common ratio $r = \frac{7}{8}$

Giving your answers to 3 significant figures where appropriate, find

(a) the 20th term of the series,

(2)

(b) the sum of the first 20 terms of the series,

(2)

(c) the sum to infinity of the series.

(2)

Leave blank

9. A geometric series is $a + ar + ar^2 + ...$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4)

The third and fifth terms of a geometric series are 5.4 and 1.944 respectively and all the terms in the series are positive.

For this series find,

(b) the common ratio,

(2)

(c) the first term,

(2)

(d) the sum to infinity.

(3)

P 4 0 6 8 5 A 0 2 6 2 8

Leave blank

Question 9 continued

Q9

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

3. A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05

(a) Show that the predicted profit in the year 2016 is £138 915

(1)

(b) Find the first year in which the yearly predicted profit exceeds £200 000

(5)

(c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.

(3)

Leave blank

5. The first three terms of a geometric series are $4p$, $(3p + 15)$ and $(5p + 20)$ respectively, where p is a **positive** constant.

(a) Show that $11p^2 - 10p - 225 = 0$

(4)

(b) Hence show that $p = 5$

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum of the first ten terms of the series, giving your answer to the nearest integer.

(3)

Leave blank

Question 5 continued

Turn over

Leave blank

1. The first three terms of a geometric series are

$$18, 12 \text{ and } p$$

respectively, where p is a constant.

Find

(a) the value of the common ratio of the series,

(1)

(b) the value of p,

(1)

(c) the sum of the first 15 terms of the series, giving your answer to 3 decimal places.

(2)

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times\ldots\times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$u_n = ar^{n-1}$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Numerical integration

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$