Edexcel Maths C2

Topic Questions from Papers

Integration

Leave blank

6. A river, running between parallel banks, is 20 m wide. The depth, y metres, of the river measured at a point x metres from one bank is given by the formula

$$y = \frac{1}{10}x\sqrt{(20 - x)}, \quad 0 \leqslant x \leqslant 20.$$

(a) Complete the table below, giving values of y to 3 decimal places.

x	0	4	8	12	16	20
y	0		2.771			0

(2)

(b) Use the trapezium rule with all the values in the table to estimate the cross-sectional area of the river.

(4)

Given that the cross-sectional area is constant and that the river is flowing uniformly at $2\ \mathrm{ms}^{-1}$,

(c) estimate, in m^3, the volume of water flowing per minute, giving your answer to 3 significant figures.

(2)

Leave blank

10.

Figure 1

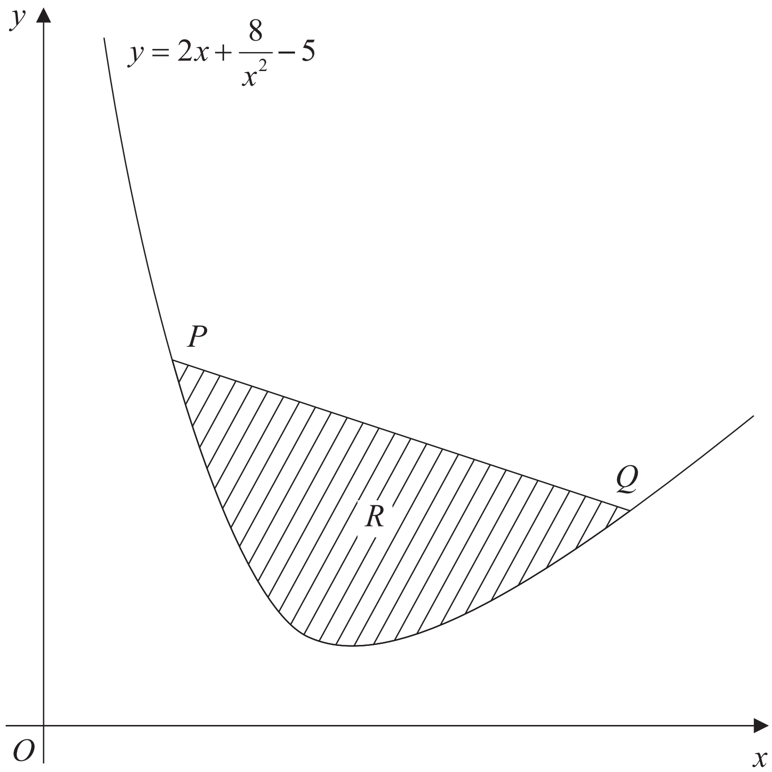


Figure 1 shows part of the curve C with equation $y = 2x + \frac{8}{x^2} - 5$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 4 respectively. The region R, shaded in Figure 1, is bounded by C and the straight line joining P and Q.

(a) Find the exact area of R.

(8)

(b) Use calculus to show that y is increasing for $x > 2$.

(4)

Leave blank

Question 10 continued

Q10

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

6. The speed, v m s^{-1}, of a train at time t seconds is given by

$$v = \sqrt{(1.2^t - 1)}, \quad 0 \leqslant t \leqslant 30.$$

The following table shows the speed of the train at 5 second intervals.

t	0	5	10	15	20	25	30
v	0	1.22	2.28		6.11		

(a) Complete the table, giving the values of v to 2 decimal places.
(3)

The distance, s metres, travelled by the train in 30 seconds is given by

$$s = \int_0^{30} \sqrt{(1.2^t - 1)}\,\mathrm{d}t\,.$$

(b) Use the trapezium rule, with all the values from your table, to estimate the value of s.
(3)

N 2 3 5 5 2 A 0 1 2 2 0

Leave blank

9.

Figure 3

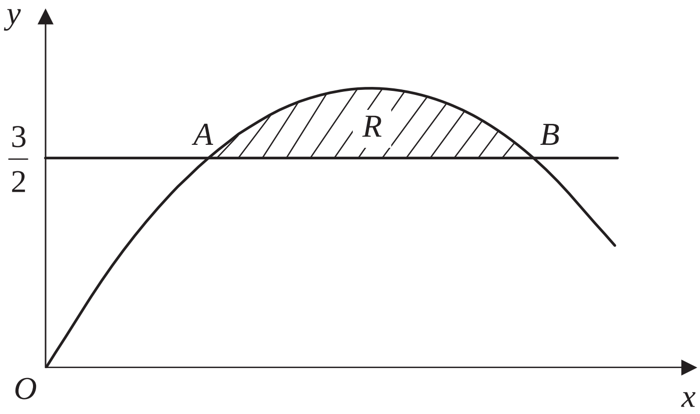


Figure 3 shows the shaded region R which is bounded by the curve $y = -2x^2 + 4x$ and the line $y = \frac{3}{2}$. The points A and B are the points of intersection of the line and the curve.

Find

(a) the x-coordinates of the points A and B,

(4)

(b) the exact area of R.

(6)

Leave blank

Question 9 continued

Q9

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

2. Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) dx.$

(5)

Q2

(Total 5 marks)

Leave blank

5. (a) In the space provided, sketch the graph of $y = 3^x$, $x \in \mathbb{R}$, showing the coordinates of the point at which the graph meets the y-axis.

(2)

(b) Complete the table, giving the values of 3^x to 3 decimal places.

x	0	0.2	0.4	0.6	0.8	1
3^x		1.246	1.552			3

(2)

(c) Use the trapezium rule, with all the values from your table, to find an approximation for the value of $\int_0^1 3^x \, dx$.

(4)

Leave blank

10.

Figure 3

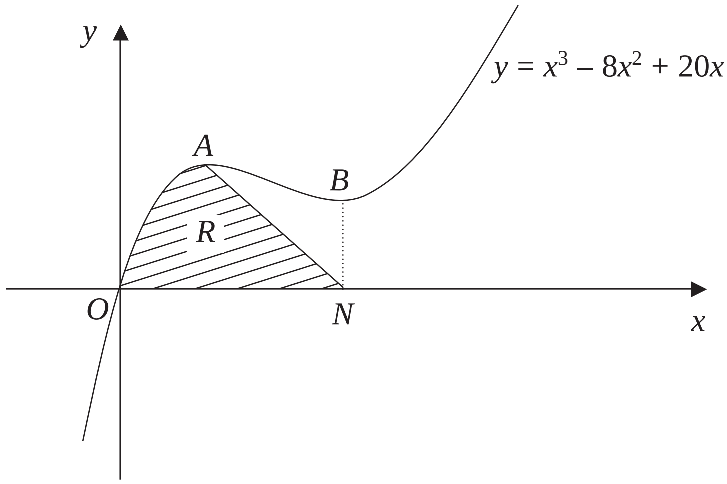


Figure 3 shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$.
The curve has stationary points A and B.

(a) Use calculus to find the x-coordinates of A and B.

(4)

(b) Find the value of $\dfrac{d^2y}{dx^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N.
The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line from A to N.

(c) Find $\int (x^3 - 8x^2 + 20x)\,dx$.

(3)

(d) Hence calculate the exact area of R.

(5)

Leave blank

Question 10 continued

Q10

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

1. $$f(x) = x^3 + 3x^2 + 5.$$

Find

(a) $f''(x)$,

(3)

(b) $\int_1^2 f(x)\,dx$.

(4)

N 2 4 3 2 2 A 0 2 2 4

Leave blank

7.

Figure 1

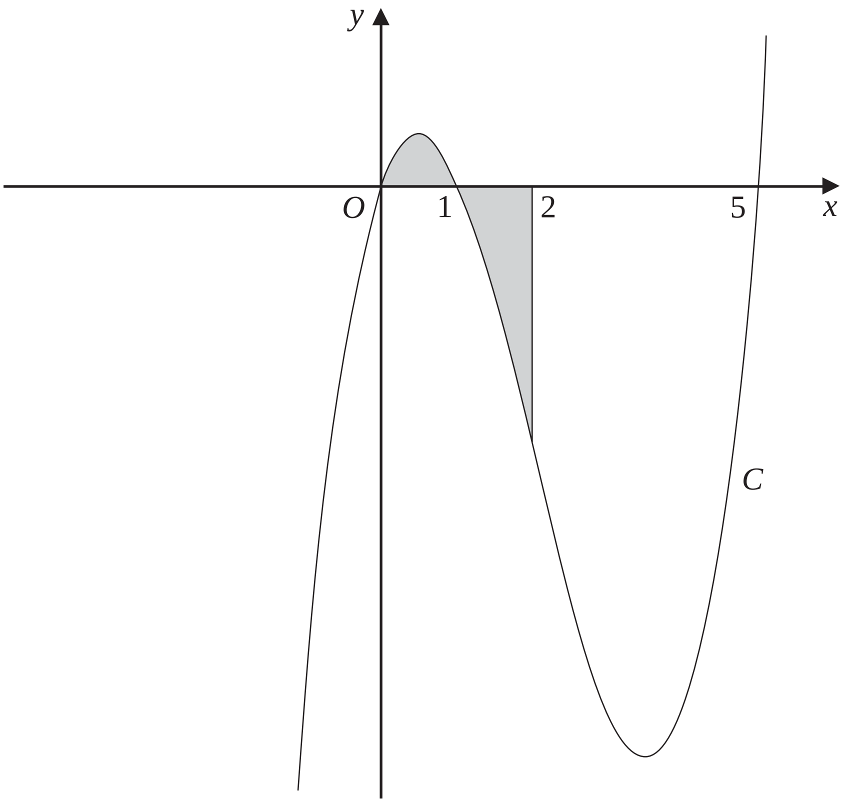


Figure 1 shows a sketch of part of the curve C with equation

$$y = x(x-1)(x-5).$$

Use calculus to find the total area of the finite region, shown shaded in Figure 1, that is between $x = 0$ and $x = 2$ and is bounded by C, the x-axis and the line $x = 2$.

(9)

Leave blank

Question 7 continued

Leave blank

1. Evaluate $\int_1^8 \frac{1}{\sqrt{x}}\,dx$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(4)

Q1

(Total 4 marks)

Leave blank

5. The curve C has equation

$$y = x\sqrt{(x^3+1)}, \qquad 0 \leqslant x \leqslant 2.$$

(a) Complete the table below, giving the values of y to 3 decimal places at $x = 1$ and $x = 1.5$.

x	0	0.5	1	1.5	2
y	0	0.530			6

(2)

(b) Use the trapezium rule, with all the y values from your table, to find an approximation for the value of $\int_0^2 x\sqrt{(x^3+1)}\,dx$, giving your answer to 3 significant figures.

(4)

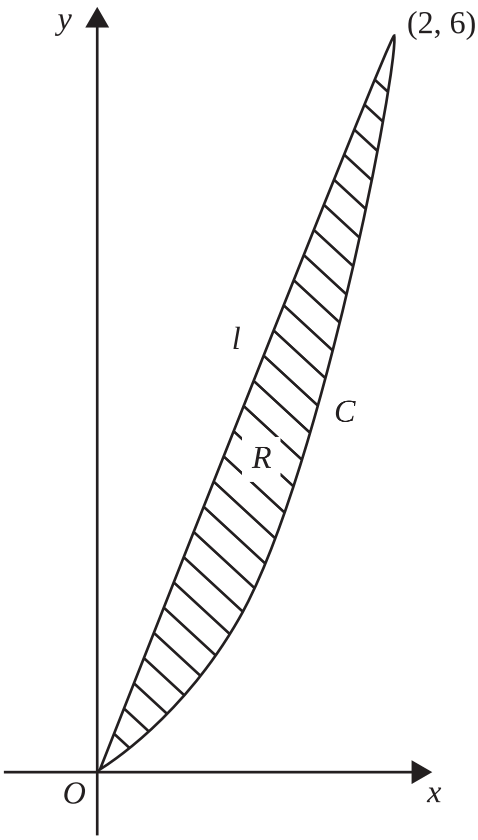


Figure 2

Figure 2 shows the curve C with equation $y = x\sqrt{(x^3+1)}$, $0 \leqslant x \leqslant 2$, and the straight line segment l, which joins the origin and the point (2, 6). The finite region R is bounded by C and l.

(c) Use your answer to part (b) to find an approximation for the area of R, giving your answer to 3 significant figures.

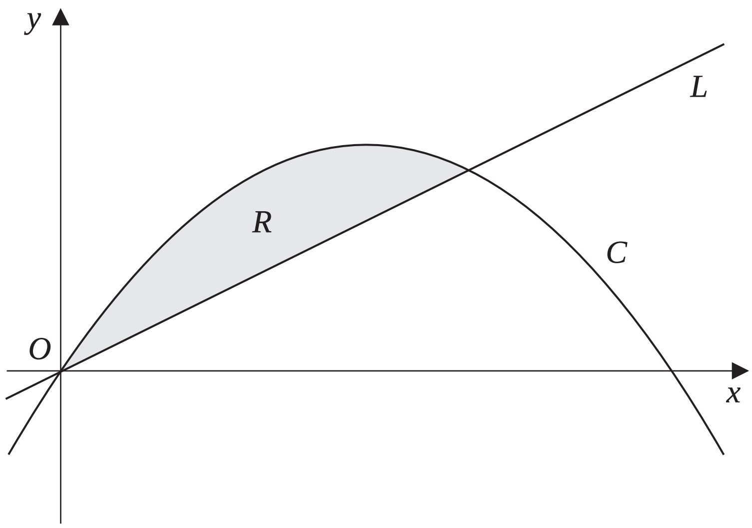
(3)

Leave blank

Question 5 continued

Leave blank

7. **Figure 2**



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$.
(1)

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8).
(3)

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R.
(6)

H 2 6 3 2 0 B 0 1 4 2 4

Leave blank

Question 7 continued

Leave blank

2.

$$y = \sqrt{(5^x + 2)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.5	1	1.5	2
y			2.646	3.630	

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_0^2 \sqrt{(5^x + 2)}\, dx$.

(4)

Leave blank

8.

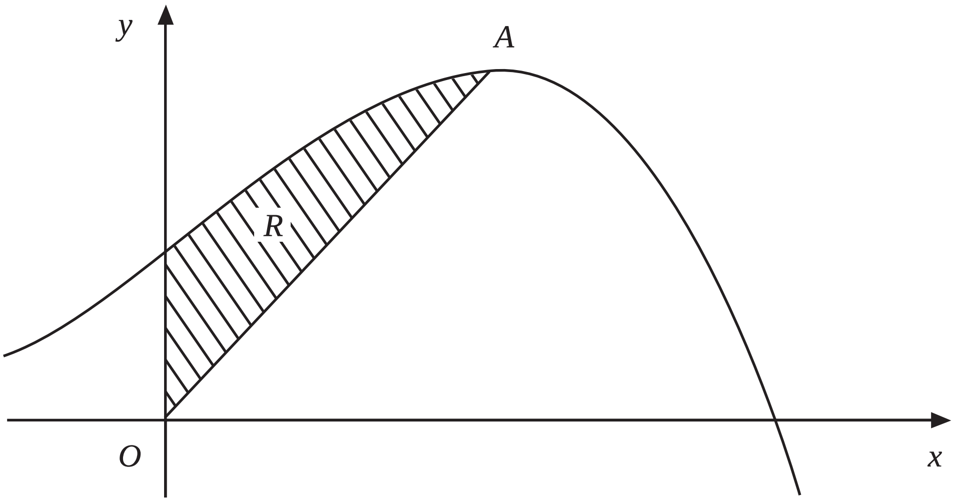


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point *A*.

(a) Using calculus, show that the *x*-coordinate of *A* is 2.
(3)

The region *R*, shown shaded in Figure 2, is bounded by the curve, the *y*-axis and the line from *O* to *A*, where *O* is the origin.

(b) Using calculus, find the exact area of *R*.
(8)

Leave blank

Question 8 continued

H30722A02128

Turn over

Leave blank

2.

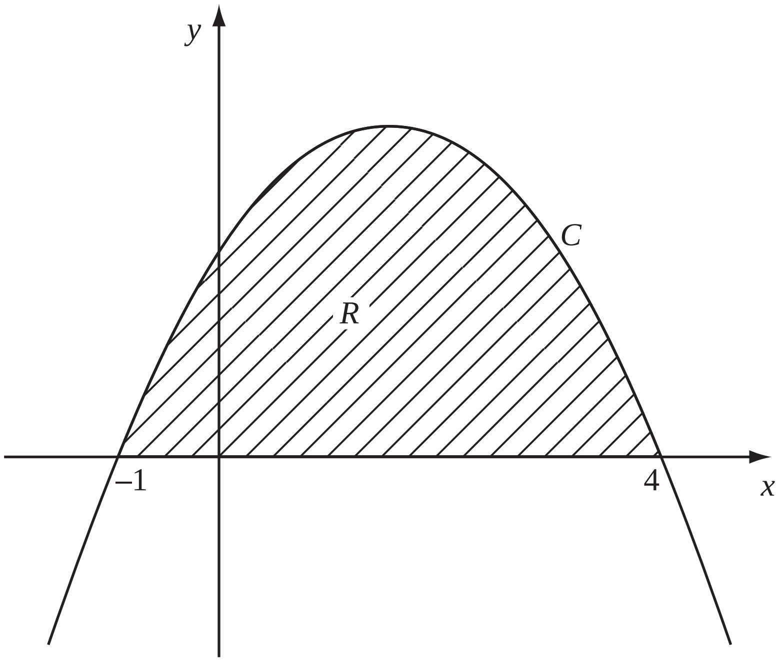


Figure 1

Figure 1 shows part of the curve C with equation $y = (1 + x)(4 - x)$.

The curve intersects the x-axis at $x = -1$ and $x = 4$. The region R, shown shaded in Figure 1, is bounded by C and the x-axis.

Use calculus to find the exact area of R.

(5)

Leave blank

3. $$y = \sqrt{(10x - x^2)}.$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	1	1.4	1.8	2.2	2.6	3
y	3	3.47			4.39	

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_1^3 \sqrt{(10x - x^2)}\, dx$.

(4)

Leave blank

1. Use calculus to find the value of

$$\int_1^4 \left(2x + 3\sqrt{x}\right) \mathrm{d}x\,.$$

(5)

Q1

(Total 5 marks)

Leave blank

4. (a) Complete the table below, giving values of $\sqrt{(2^x + 1)}$ to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
$\sqrt{(2^x + 1)}$	1.414	1.554	1.732	1.957			3

(2)

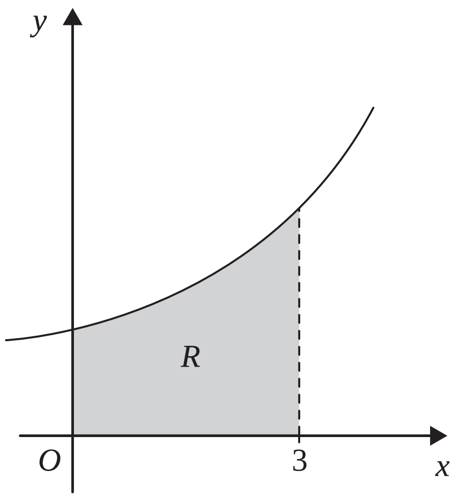


Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \sqrt{(2^x + 1)}$, the x-axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values from your table, to find an approximation for the area of R.

(4)

(c) By reference to the curve in Figure 1 state, giving a reason, whether your approximation in part (b) is an overestimate or an underestimate for the area of R.

(2)

Leave blank

Question 4 continued

Leave blank

7.

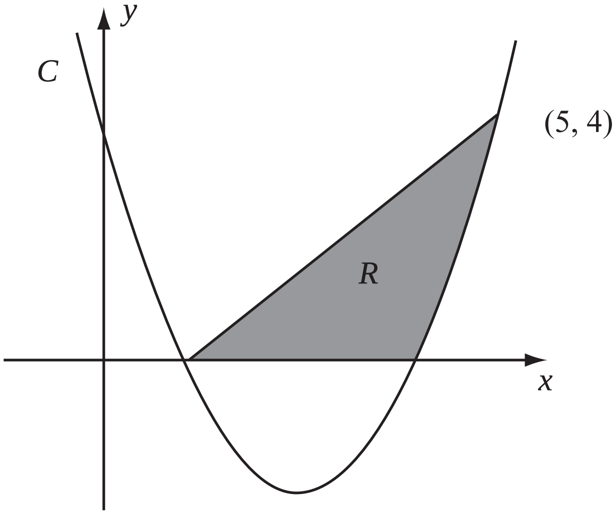


Figure 2

The curve C has equation $y = x^2 - 5x + 4$. It cuts the x-axis at the points and as shown in Figure 2.

(a) Find the coordinates of the point and the point .

(2)

(b) Show that the point (5, 4) lies on C.

(1)

(c) Find $\int (x^2 - 5x + 4)\,dx$.

(2)

The finite region R is bounded by , and the curve C as shown in Figure 2.

(d) Use your answer to part (c) to find the exact value of the area of R.

(5)

N 3 5 1 0 1 A 0 1 4 2 4

Leave blank

Question 7 continued

Leave blank

1. $$y = 3^x + 2x$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	0	0.2	0.4	0.6	0.8	1
y	1	1.65				5

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_0^1 (3^x + 2x)\, dx$.

(4)

8.

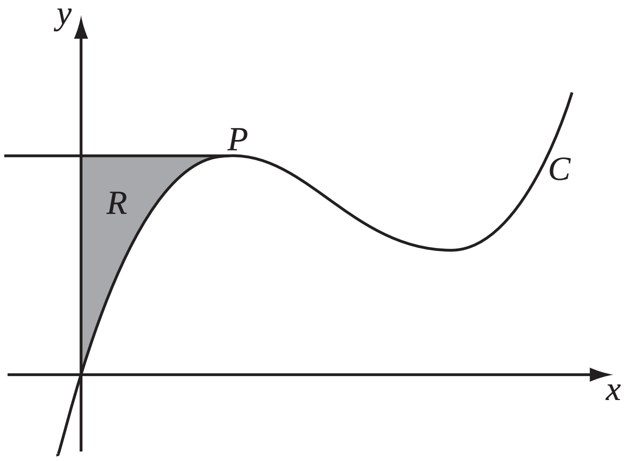


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point .
The region R is bounded by C, the y-axis and P , as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R.

(6)

Leave blank

Question 8 continued

Leave blank

4.

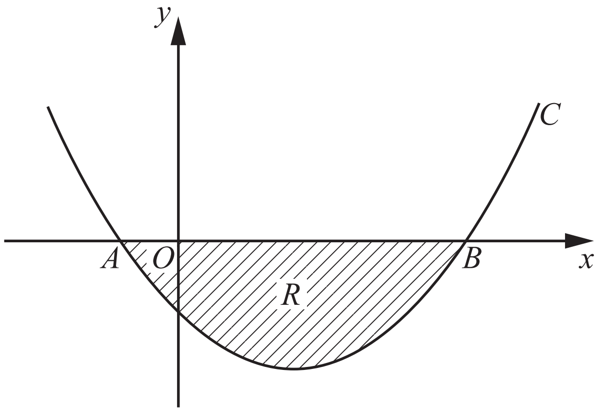


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x+1)(x-5)$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

Leave blank

Question 4 continued

Q4

(Total 7 marks)

H 3 5 4 0 3 A 0 9 2 8

Turn over

Leave blank

6. $$y = \frac{5}{3x^2 - 2}$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	2	2.25	2.5	2.75	3
y	0.5	0.38			0.2

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2 - 2}\,dx$.

(4)

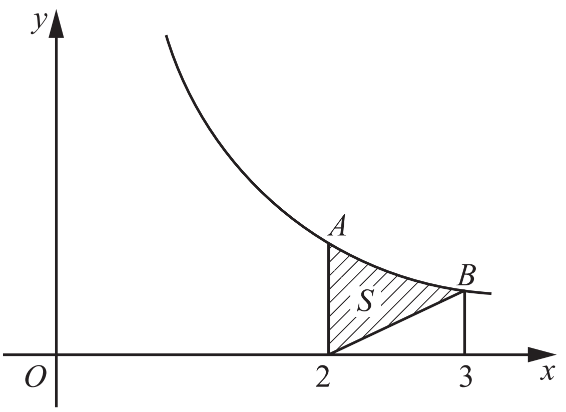


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2 - 2}$, $x > 1$.

At the points A and B on the curve, $x = 2$ and $x = 3$ respectively.

The region S is bounded by the curve, the straight line through B and (2, 0), and the line through A parallel to the y-axis. The region S is shown shaded in Figure 2.

(c) Use your answer to part (b) to find an approximate value for the area of S.

(3)

Leave blank

Question 6 continued

Leave blank

9.

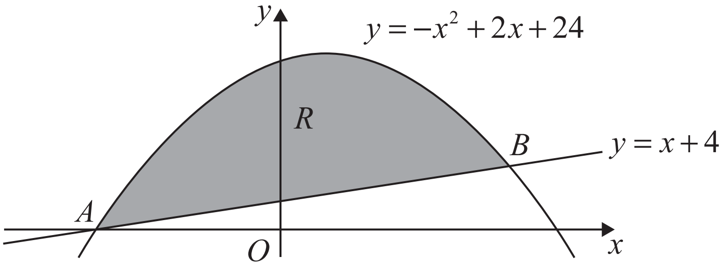


Figure 3

The straight line with equation $y = x + 4$ cuts the curve with equation $y = -x^2 + 2x + 24$ at the points A and B, as shown in Figure 3.

(a) Use algebra to find the coordinates of the points A and B.

(4)

The finite region R is bounded by the straight line and the curve and is shown shaded in Figure 3.

(b) Use calculus to find the exact area of R.

(7)

Leave blank

Question 9 continued

Q9

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

6.

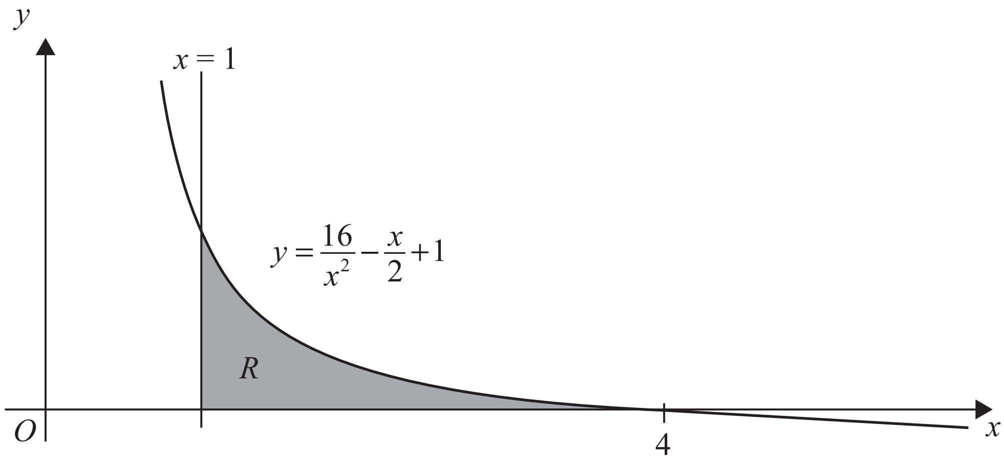


Figure 1

Figure 1 shows the graph of the curve with equation

$$y = \frac{16}{x^2} - \frac{x}{2} + 1, \qquad x > 0$$

The finite region R, bounded by the lines $x = 1$, the x-axis and the curve, is shown shaded in Figure 1. The curve crosses the x-axis at the point $(4, 0)$.

(a) Complete the table with the values of y corresponding to $x = 2$ and 2.5

x	1	1.5	2	2.5	3	3.5	4
y	16.5	7.361			1.278	0.556	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R.

(5)

Leave blank

Question 6 continued

Leave blank

5.

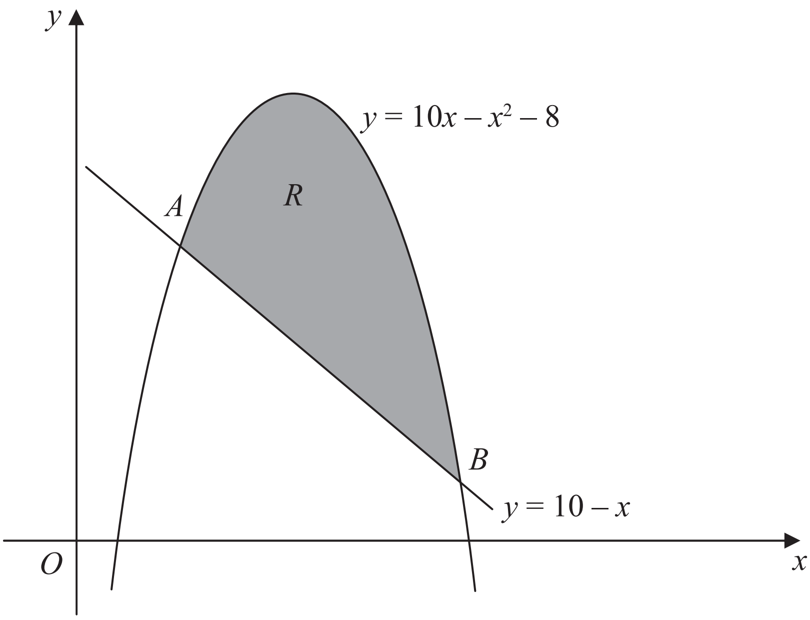


Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$

The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

(7)

Leave blank

Question 5 continued

Leave blank

7. $$y = \sqrt{(3^x + x)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.25	0.5	0.75	1
y	1	1.251			2

(2)

(b) Use the trapezium rule with all the values of y from your table to find an approximation for the value of $\int_0^1 \sqrt{(3^x + x)}\, dx$

You must show clearly how you obtained your answer.

(4)

Leave blank

9.

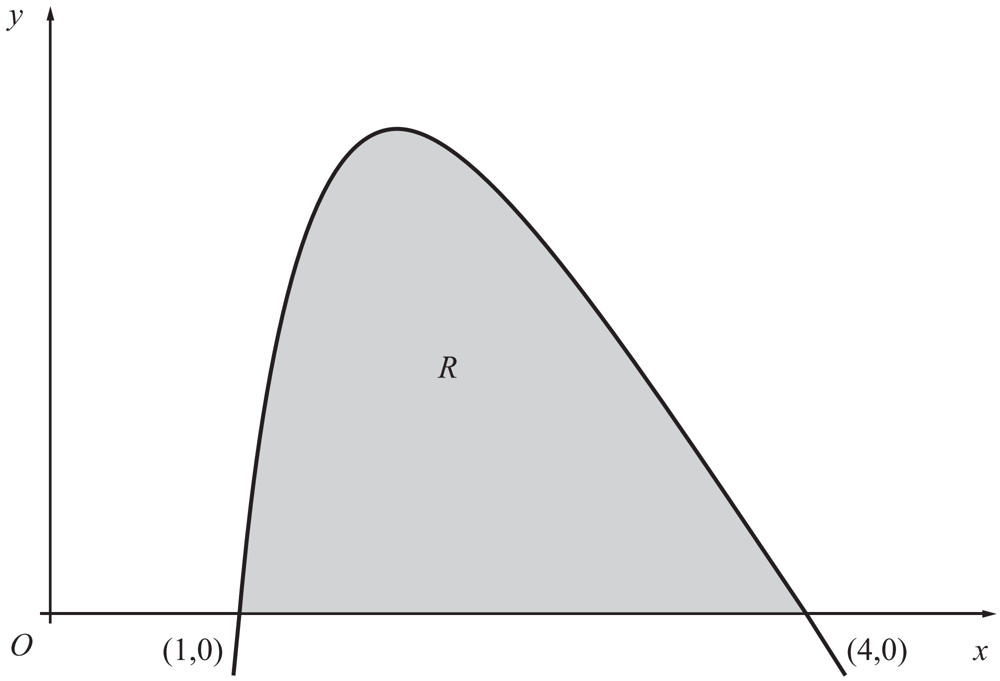


Figure 2

The finite region R, as shown in Figure 2, is bounded by the x-axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \qquad x > 0$$

The curve crosses the x-axis at the points (1, 0) and (4, 0).

(a) Complete the table below, by giving your values of y to 3 decimal places.

x	1	1.5	2	2.5	3	3.5	4
y	0	5.866		5.210		1.856	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R.

(6)

Leave blank

Question 9 continued

Q9

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

2.

$$y = \frac{x}{\sqrt{(1 + x)}}$$

(a) Complete the table below with the value of y corresponding to $x = 1.3$, giving your answer to 4 decimal places.

(1)

x	1	1.1	1.2	1.3	1.4	1.5
y	0.7071	0.7591	0.8090		0.9037	0.9487

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an approximate value for

$$\int_1^{1.5} \frac{x}{\sqrt{(1 + x)}}\, dx$$

giving your answer to 3 decimal places.

You must show clearly each stage of your working.

(4)

Leave blank

7.

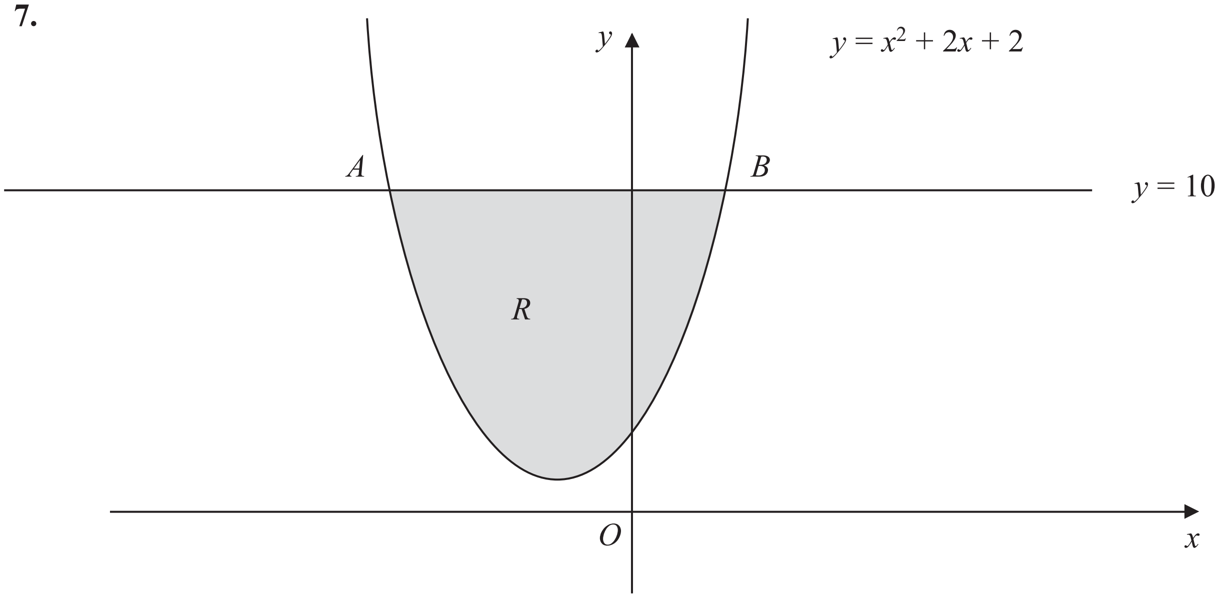


Figure 1

The line with equation $y = 10$ cuts the curve with equation $y = x^2 + 2x + 2$ at the points A and B as shown in Figure 1. The figure is not drawn to scale.

(a) Find by calculation the x-coordinate of A and the x-coordinate of B.

(2)

The shaded region R is bounded by the line with equation $y = 10$ and the curve as shown in Figure 1.

(b) Use calculus to find the exact area of R.

(7)

Leave blank

Question 7 continued

P 4 2 8 2 6 A 0 2 1 3 2

Turn over

Leave blank

4.
$$y = \frac{5}{(x^2 + 1)}$$

(a) Complete the table below, giving the missing value of y to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
y	5	4	2.5		1	0.690	0.5

(1)

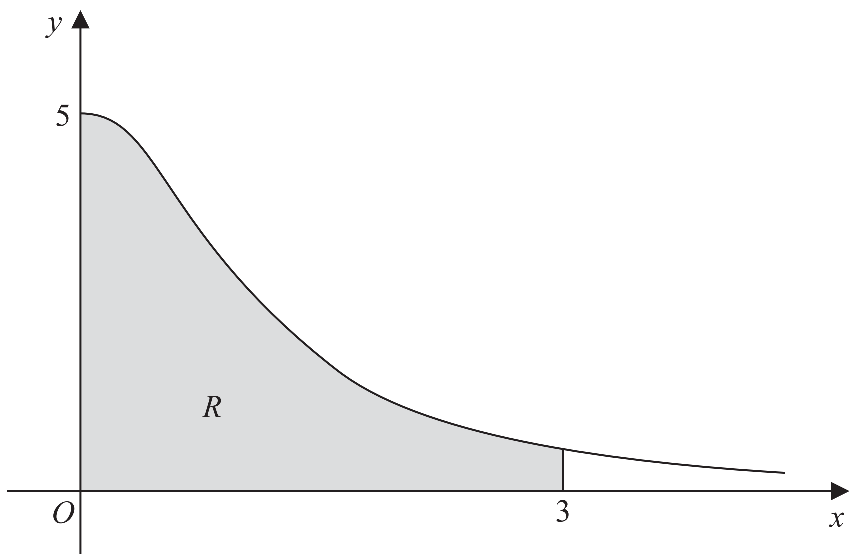


Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \frac{5}{(x^2 + 1)}$, the x-axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for the area of R.

(4)

(c) Use your answer to part (b) to find an approximate value for

$$\int_0^3 \left(4 + \frac{5}{(x^2 + 1)}\right) dx$$

giving your answer to 2 decimal places.

(2)

Leave blank

Question 4 continued

Q4

(Total 7 marks)

P 4 1 8 5 9 A 0 1 1 3 2

Turn over

Leave blank

6.

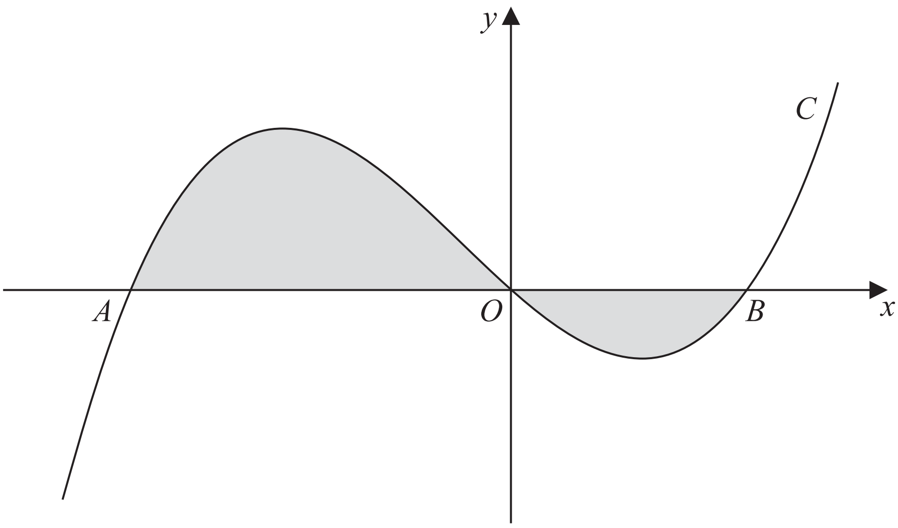


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

Leave blank

Question 6 continued

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2} h\{(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$