

Core 2 Integration Questions

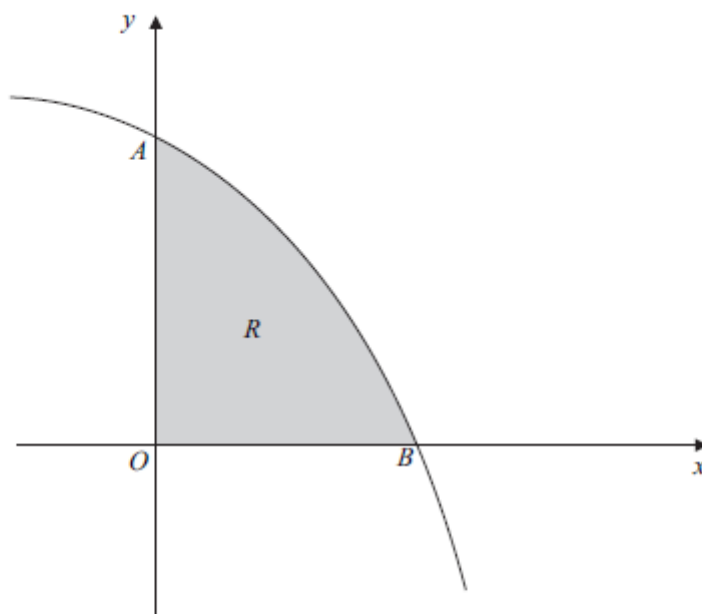
- 2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^4 \frac{1}{x^2 + 1} dx$$

giving your answer to four significant figures. (4 marks)

- (b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. (1 mark)
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- 6 The diagram shows a sketch of the curve with equation $y = 27 - 3^x$.



The curve $y = 27 - 3^x$ intersects the y -axis at the point A and the x -axis at the point B .

- (a) (i) Find the y -coordinate of point A . (2 marks)
- (ii) Verify that the x -coordinate of point B is 3. (1 mark)
- (b) The region, R , bounded by the curve $y = 27 - 3^x$ and the coordinate axes is shaded. Use the trapezium rule with four ordinates (three strips) to find an approximate value for the area of R . (4 marks)
- (c) (i) Use logarithms to solve the equation $3^x = 13$, giving your answer to four decimal places. (3 marks)

- (ii) The line $y = k$ intersects the curve $y = 27 - 3^x$ at the point where $3^x = 13$.
Find the value of k . (1 mark)
- (d) (i) Describe the single geometrical transformation by which the curve with equation $y = -3^x$ can be obtained **from** the curve $y = 27 - 3^x$. (2 marks)
- (ii) Sketch the curve $y = -3^x$. (2 marks)
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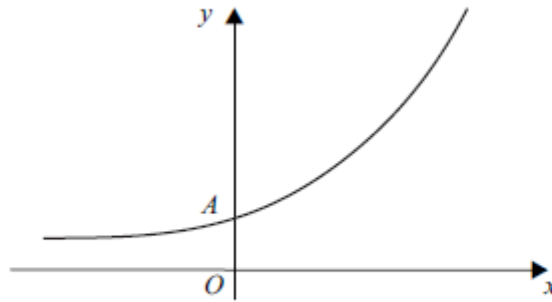
- 2 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{2^x} \, dx$$

giving your answer to three decimal places.

(4 marks)

- 6 The diagram shows a sketch of the curve with equation $y = 3(2^x + 1)$.



The curve $y = 3(2^x + 1)$ intersects the y -axis at the point A .

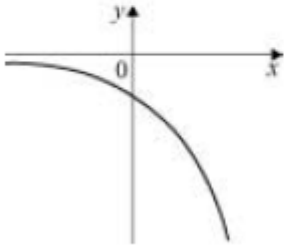
- (a) Find the y -coordinate of the point A . (2 marks)
- (b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^6 3(2^x + 1) \, dx$. (4 marks)
- (c) The line $y = 21$ intersects the curve $y = 3(2^x + 1)$ at the point P .
- (i) Show that the x -coordinate of P satisfies the equation

$$2^x = 6 \quad \text{(1 mark)}$$

- (ii) Use logarithms to find the x -coordinate of P , giving your answer to three significant figures. (3 marks)
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Core 2 integration Answers

2(a)	$h=1$ Integral = $\frac{h}{2}\{\dots\}$ $\{\dots\} = f(0) + f(4) + 2[f(1) + f(2) + f(3)]$ $= \left[1 + \frac{1}{17} + 2\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right) \right]$ Integral = 1.329	B1 M1 A1 A1		PI OE summing of areas of the four trapezia. [0.75+0.35+0.15+0.079...] Exact or to 3dp values Condone one numerical slip CSO. Must be 1.329
(b)	Increase the number of ordinates	E1	1	OE
Total			5	

6(a)(i)	y -coordinate of A is $27 - 3^0 = 26$	M1A1	2	
(ii)	When $x = 3$, $y = 27 - 3^3 = 0 \Rightarrow B(3,0)$	B1	1	AG; be convinced
(b)	$h = 1$	B1		PI
	Area $\approx h/2\{\dots\}$ $\{\dots\} = f(0)+f(3)+2[f(1)+f(2)]$ $\{\dots\} = "26" + 0 + 2(24 + 18)$	M1 A1✓		OE summing of areas of the 'trapezia' .. on (a)(i) ($\Sigma_{\text{trap}} = "25"+21+9$)
	(Area \approx) 55	A1✓	4	on $[42 + 0.5 \times "(a)(i)"]$
(c)(i)	$\log_{10} 3^x = \log_{10} 13$	M1		Takes \ln or \log_{10} on both or $x = \log_3 13$
	$x \log_{10} 3 = \log_{10} 13$	m1		Use of $\log 3^x = x \log 3$ or $\log_3 13 = \frac{\lg 13}{\lg 3}$ OE (PI by $\log_3 13 = 2.335$ or better)
	$x = \frac{\lg 13}{\lg 3} = 2.334717 \dots$ $= 2.3347$ to 4dp	A1	3	Must show that logarithms have been used
(ii)	$\{k\} = 14$	B1	1	Condone $y = 14$; Accept final answer 14 with only zeros after decimal point eg 14.000
(d)(i)	Translation;	B1;		'Translation'/'translate(d)' B0 if more than one transformation
	$\begin{bmatrix} 0 \\ -27 \end{bmatrix}$	B1	2	Accept full equivalent in words provided linked to 'translation/move/shift' and negative y -direction (Note: B0 B1 is possible)
(ii)		B1 B1		Correct shape (translation of given curve vertically downwards) Only point of intersection with coord axes is on negative y -axis and curve is asymptotic to the negative x -axis
			2	
Total			15	

2	$h = 1$ $f(x) = \sqrt{2^x}$ Area $\approx h/2\{\dots\}$ $\{\dots\} = f(0)+f(3)+2[f(1)+f(2)]$ $\{\dots\} = 1 + \sqrt{8} + 2(\sqrt{2} + 2)$ (Area \approx) 5.3284... = 5.328 (to 3dp)	B1 M1 A1 A1		PI OE summing of areas of the 'trapezia' .. OE CAO Must be 5.328
Total			4	

6(a)	$y_A = 3(2^0 + 1)$ $= 6$	M1 A1	2	Substituting $x = 0$ PI
(b)	$h = 2$ Integral = $h/2 \{ \dots \}$ $\{ \dots \} = f(0) + 2[f(2) + f(4)] + f(6)$ $\{ \} = 6 + 2[3 \times 5 + 3 \times 17] + 3 \times 65$ $= 6 + 2[15 + 51] + 195$ Integral = 333	B1 M1 A1 A1	4	PI OE summing of areas of the three traps. Condone 1 numerical slip {ft on (a) for $f(0)$ if not recovered} [Sum of 3 traps. = 21 + 66 + 246] CAO
(c)(i)	$21 = 3(2^x + 1) \Rightarrow 2^x = 6$	B1	1	AG (be convinced)
(ii)	$\log_{10} 2^x = \log_{10} 6$ $x \log_{10} 2 = \log_{10} 6$ $x = \frac{\lg 6}{\lg 2} = 2.5849\dots = 2.58 \text{ to 3sf}$	M1 m1 A1	3	Take \ln or \log_{10} of both sides of $a^x = b$ or other relevant base if clear. The equation $a^x = b$ used must be correct. Use of $\log 2^x = x \log 2$ OE Both method marks must have been awarded.
Total			10	