Practice exam paper

1 $n \in \mathbb{Z}$ and $2 \le n \le 7$

$A = n^2 + 2$

When $n = 2$, $A = 2^2 + 2 = 6$

When $n = 3$, $A = 3^2 + 2 = 11$

When $n = 4$, $A = 4^2 + 2 = 18$

When $n = 5$, $A = 5^2 + 2 = 27$

When $n = 6$, $A = 6^2 + 2 = 38$

When $n = 7$, $A = 7^2 + 2 = 51$

So none of 6, 11, 18, 27, 38, 51 can be evenly divided by 4

2 $\log_6 a + \log_6 b = 2$ and $\dfrac{a}{b} = 144$

$\log_6 ab = 2$

$ab = 6^2$

$ab = 36$

$a = \dfrac{36}{b}$

Therefore

$ab = 6^2$

$ab = 36$

$\dfrac{36}{b^2} = 144$

$b^2 = \dfrac{1}{4}$

$b = \pm\dfrac{1}{2}$

Since b must be positive $b = \dfrac{1}{2}$

When $b = \dfrac{1}{2}$, $a = 72$

3 a $f(x) = 2x^3 - 3px^2 + x + 4p$

Since $(x-4)$ is a factor of $f(x)$, $f(4) = 0$

$2(4)^3 - 3p(4)^2 + (4) + 4p = 0$

$128 - 48p + 4 + 4p = 0$

$44p = 132$

$p = 3$

b $f(x) = 2x^3 - 9x^2 + x + 12$

$$f(-2) = 2(-2)^3 - 9(-2)^2 + (-2) + 12$$
$$= -16 - 36 - 2 + 12$$
$$= -42$$

3 c

$$\begin{array}{r} 2x^2 - x - 3 \\ x-4\overline{)2x^3 - 9x^2 + x + 12} \\ \underline{2x^3 - 8x^2} \\ -x^2 + x \\ \underline{x^2 - 4x} \\ -3x + 12 \\ \underline{3x - 12} \\ 0 \end{array}$$

$$2x^3 - 9x^2 + x + 12 = (x-4)(2x^2 - x - 3)$$
$$= (x-4)(2x-3)(x+1)$$

4 $y = (1+x^2)^5$

x	0	0.1	0.2	0.3	0.4
y	1.0000	1.0510	1.2167	1.5386	2.1003

$$\int_a^b y\,dx = \frac{1}{2}h\left(y_0 + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right) + y_n\right)$$

$$\int_0^{0.4} (1+x^2)^5\,dx = \frac{1}{2}(0.1)\left(1 + 2(1.0510 + 1.2167 + 1.5386) + 2.1003\right)$$
$$= 0.536 \text{ (3 d.p.)}$$

5 a i $t_3 + t_6 = \frac{28}{81}$ **(1)** and $t_3 - t_6 = \frac{76}{405}$ **(2)**

Adding **(1)** and **(2)** gives

$$2t_3 = \frac{28}{81} + \frac{76}{405}$$

$$t_3 = \frac{4}{15}$$

Substituting $t_3 = \frac{4}{15}$ into $t_3 + t_6 = \frac{28}{81}$ gives

$$\frac{4}{15} + t_6 = \frac{28}{81}$$

$$t_6 = \frac{32}{405}$$

$t_3 = ar^2 = \frac{4}{15}$ and $t_6 = ar^5 = \frac{32}{405}$

$$\frac{ar^5}{ar^2} = \frac{32}{405} \div \frac{4}{15}$$

$$r^3 = \frac{8}{27}$$

$r = \frac{2}{3}$ as required

5 a ii substituting $r = \frac{2}{3}$ into $ar^2 = \frac{4}{15}$

$a\left(\frac{2}{3}\right)^2 = \frac{4}{15}$

$a = \frac{3}{5}$

b $S_\infty = \frac{a}{1-r}$

$= \frac{\frac{3}{5}}{1-\frac{2}{3}}$

$= \frac{9}{5}$

6 a i $x^2 - 2x + y^2 + 10y - 19 = 0$

$(x-1)^2 - 1 + (y+5)^2 - 25 - 19 = 0$

$(x-1)^2 + (y+5)^2 = 45$

Centre is (1, −5)

ii $r = \sqrt{45}$

b Substituting (7, −2) into $(x-1)^2 + (y+5)^2 = 45$ gives

$LHS = (7-1)^2 + (-2+5)^2$

$= 36 + 9$

$= 45 = RHS$

So (7, −2) lies on the circle

c The radius between the points (1, −5) and (7, −2) has gradient

$m = \frac{-2-(-5)}{7-1}$

$= \frac{1}{2}$

So the tangent at (7, −2) has gradient −2

Using $y - y_1 = m(x - x_1)$ gives

$y + 2 = -2(x-7)$

$2x + y - 12 = 0$

7 a $3x + y = 15$ **(1)** and $4x^2 + y^2 = S$ **(2)**

From **(1)** $y = 15 - 3x$

Substituting $y = 15 - 3x$ into $S = 4x^2 + y^2$ gives

$$S = 4x^2 + (15 - 3x)^2$$
$$= 4x^2 + 225 - 90x + 9x^2$$
$$= 13x^2 - 90x + 225$$

b $S = 13x^2 - 90x + 225$

$$\frac{dS}{dx} = 26x - 90$$

At a turning point

$$\frac{dS}{dx} = 26x - 90 = 0$$
$$x = \frac{45}{13}$$

$\frac{d^2S}{dx^2} = 26 > 0$ therefore there is a minimum at $x = \frac{45}{13}$

c $S = 13x^2 - 90x + 225$

at $x = \frac{45}{13}$

$$S = 13\left(\frac{45}{13}\right)^2 - 90\left(\frac{45}{13}\right) + 225$$
$$= \frac{900}{13}$$

8 a $$\left(1 - \frac{x}{4}\right)^9 = 1 + 9\left(-\frac{1}{4}x\right) + \frac{(9)(8)}{2!}\left(-\frac{1}{4}x\right)^2 + \frac{(9)(8)(7)}{3!}\left(-\frac{1}{4}x\right)^3 + \ldots$$
$$= 1 - \frac{9}{4}x + \frac{9}{4}x^2 - \frac{21}{16}x^3 + \ldots$$

b To calculate 0.975^9, let $x = 0.1$

$$0.975^9 = \left(1 - \frac{0.1}{4}\right)^9$$
$$= 1 - \frac{9}{4}(0.1) + \frac{9}{4}(0.1)^2 - \frac{21}{16}(0.1)^3 + \ldots$$
$$= 0.7962 \text{ (4 d.p.)}$$

9 a $y=-x^2+x+13$ and $y=3x+10$

$-x^2+x+13=3x+10$

$x^2+2x-3=0$

$(x-1)(x+3)=0$

$x=1$ or $x=-3$

When $x=1$, $y=13$

When $x=-3$, $y=1$

So P is the point $(-3, 1)$ and Q is the point $(1, 13)$

b

$$A=\int_{-3}^{1}\left(-x^2+x+13\right)\,dx-\int_{-3}^{1}(3x+10)\,dx$$

$$=\int_{-3}^{1}\left(-x^2-2x+3\right)\,dx$$

$$=\left[-\frac{1}{3}x^3-x^2+3x\right]_{-3}^{1}$$

$$=\left(-\frac{1}{3}(1)^3-(1)^2+3(1)\right)-\left(-\frac{1}{3}(-3)^3-(-3)^2+3(-3)\right)$$

$$=\frac{5}{3}-(-9)$$

$$=\frac{32}{3}$$

10 a $2\tan(2x+30)=3,\ 0\le x\le 180$

Let $X=2x+30$

$2\tan X=3,\ 30\le x\le 390$

$\tan X=\frac{3}{2}$

$X=56.3°$ and $X=236.3°$

Since $X=2x+30$

$x=13.2°$ and $x=103.2°$

b $6\cos^2 x+\sin x-4=0,\ 0\le x\le \pi$

$6\left(1-\sin^2 x\right)+\sin x-4=0$

$6-6\sin^2 x+\sin x-4=0$

$6\sin^2 x-\sin x-2=0$

$(3\sin x-2)(2\sin x+1)=0$

$\sin x=\frac{2}{3}$ and $\sin x=-\frac{1}{2}$

When $\sin x=\frac{2}{3}$, $x=0.730$ and $x=2.41$

When $\sin x=-\frac{1}{2}$, x has no solutions in the interval.

