Exercise 8F

1 $\int_1^3 \left(\frac{1}{x^2+1}\right) dx$

x	1	1.5	2	2.5	3
$\frac{1}{x^2+1}$	0.5	0.308	0.2	0.138	0.1

$$\int_a^b y\,dx = \frac{1}{2}h\left(y_0 + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right) + y_n\right)$$

$$\int_1^3 \left(\frac{1}{x^2+1}\right) dx = \frac{1}{2}(0.5)\left(0.5 + 2\left(0.308 + 0.2 + 0.138\right) + 0.1\right)$$

$$= 0.473 \text{ (3 s.f.)}$$

2 $\int_1^{2.5} \sqrt{2x-1}\, dx$

x	1	1.25	1.5	1.75	2	2.25	2.5
$\sqrt{2x-1}$	1	1.225	1.414	1.581	1.732	1.871	2

$$\int_a^b y\,dx = \frac{1}{2}h\left(y_0 + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right) + y_n\right)$$

$$\int_1^{2.5} \sqrt{2x-1}\, dx = \frac{1}{2}(0.25)\left(1 + 2\left(1.225 + 1.414 + 1.581 + 1.732 + 1.871\right) + 2\right)$$

$$= 2.33 \text{ (3 s.f.)}$$

3 $\int_1^3 \sqrt{x^3+1}\, dx$

x	0	0.5	1	1.5	2
$\sqrt{x^3+1}$	1	1.061	1.414	2.092	3

$$\int_a^b y\,dx = \frac{1}{2}h\left(y_0 + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right) + y_n\right)$$

$$\int_1^3 \sqrt{x^3+1}\, dx = \frac{1}{2}(0.5)\left(1 + 2\left(1.061 + 1.414 + 2.092\right) + 3\right)$$

$$= 3.28 \text{ (3 s.f.)}$$

4 $\int_1^3 \frac{1}{\sqrt{x^2+1}}\,dx$

x	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$	3
$\frac{1}{\sqrt{x^2+1}}$	0.707	0.601	0.514	0.447	0.394	0.351	0.316

$$\int_a^b y\,dx = \frac{1}{2}h\left(y_0 + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right) + y_n\right)$$

$$\int_1^3 \frac{1}{\sqrt{x^2+1}}\,dx = \frac{1}{2}\left(\frac{1}{3}\right)\left(0.707 + 2\left(0.601 + 0.514 + 0.447 + 0.394 + 0.351\right) + 0.316\right)$$

$$= 0.940 \text{ (3 s.f.)}$$

5 a $\int_{-1}^{1} \left(\frac{1}{x+2}\right) dx$

x	−1	−0.6	−0.2	0.2	0.6	1
$\frac{1}{x+2}$	1	0.714	0.556	0.455	0.385	0.333

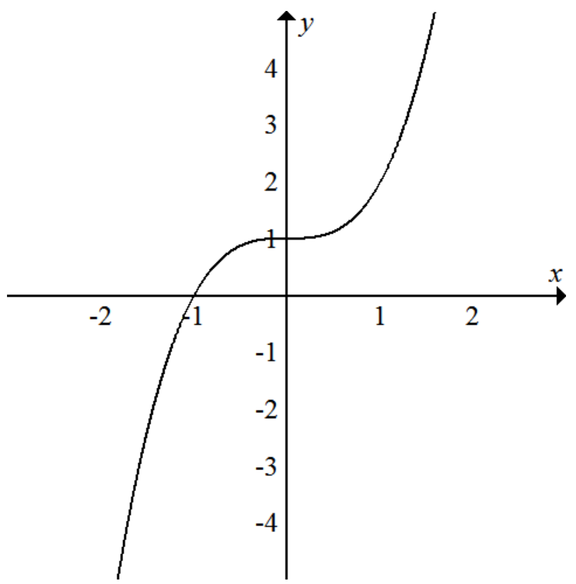
$$\int_a^b y\,dx = \frac{1}{2}h\left(y_0 + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right) + y_n\right)$$

$$\int_{-1}^{1} \left(\frac{1}{x+2}\right) dx = \frac{1}{2}(0.4)\left(1 + 2\left(0.714 + 0.556 + 0.445 + 0.385\right) + 0.333\right)$$

$$= 1.11 \text{ (3 s.f.)}$$

b Overestimate as the curve is convex

6 a



6 b $\int_{-1}^{1}(x^3+1)\,dx$

x	-1	-0.5	0	0.5	1
x^3+1	0	0.875	1	1.125	2

$$\int_a^b y\,dx = \frac{1}{2}h\left(y_0+2\left(y_1+y_2+\ldots+y_{n-1}\right)+y_n\right)$$

$$\int_{-1}^{1}(x^3+1)\,dx = \frac{1}{2}(0.5)\left(0+2\left(0.875+1+1.125\right)+2\right)$$

$$=2$$

c $A=\int_{-1}^{1}(x^3+1)\,dx$

$$A=\left[\frac{1}{4}x^4+x\right]_{-1}^{1}$$

$$=\left(\frac{1}{4}(1)^4+(1)\right)-\left(\frac{1}{4}(-1)^4+(-1)\right)$$

$$=\frac{5}{4}+\frac{3}{4}$$

$$=2$$

d Same; the trapezium rule gives an underestimate of the area between $x=-1$ and $x=0$, and an overestimate between $x=0$ and $x=1$, and these cancel out.

7 $\int_0^2\sqrt{3^x-1}\,dx$

x	0	0.5	1	1.5	2
$\sqrt{3^x-1}$	0	0.856	1.414	2.048	2.828

$$\int_a^b y\,dx = \frac{1}{2}h\left(y_0+2\left(y_1+y_2+\ldots+y_{n-1}\right)+y_n\right)$$

$$\int_0^2\sqrt{3^x-1}\,dx = \frac{1}{2}(0.5)\left(0+2\left(0.856+1.414+2.048\right)+2.828\right)$$

$$=2.87\text{ (3 s.f.)}$$

8 a $\int_1^3 \left(\frac{x}{x+1}\right) dx$

x	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$	3
$\frac{x}{x+1}$	0.5	0.571	0.625	0.667	0.700	0.727	0.75

$$\int_a^b y\, dx = \frac{1}{2}h\left(y_0 + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right) + y_n\right)$$

$$\int_1^3 \left(\frac{x}{x+1}\right) dx = \frac{1}{2}\left(\frac{1}{3}\right)\left(0.5 + 2\left(0.571 + 0.625 + 0.667 + 0.7 + 0.727\right) + 0.75\right)$$

$$= 1.31 \text{ (3 s.f.)}$$

b Underestimate as the curve is convex.

9 a $\int_0^2 \sqrt{x}\, dx$

i Four strips

x	0	0.5	1	1.5	2
$\sqrt{x}$	0	0.707	1	1.225	1.414

$$\int_a^b y\, dx = \frac{1}{2}h\left(y_0 + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right) + y_n\right)$$

$$\int_1^3 \sqrt{x}\, dx = \frac{1}{2}(0.5)\left(0 + 2\left(0.707 + 1 + 1.225\right) + 1.414\right)$$

$$= 1.82 \text{ (3 s.f.)}$$

ii Six strips

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
$\sqrt{x}$	0	0.577	0.816	1	1.155	1.291	1.414

$$\int_a^b y\, dx = \frac{1}{2}h\left(y_0 + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right) + y_n\right)$$

$$\int_0^2 \sqrt{x}\, dx = \frac{1}{2}\left(\frac{1}{3}\right)\left(0 + 2\left(0.577 + 0.816 + 1 + 1.155 + 1.291\right) + 1.414\right)$$

$$= 1.85 \text{ (3 s.f.)}$$

9 b $A=\int_0^2 \sqrt{x}\,\mathrm{d}x$

$$=\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^2$$

$$=\frac{2}{3}(2)^{\frac{3}{2}}$$

$$=\frac{4\sqrt{2}}{3}$$

i $$\frac{\frac{4\sqrt{2}}{3}-1.82}{\frac{4\sqrt{2}}{3}}\times100=3.5\%$$

ii $$\frac{\frac{4\sqrt{2}}{3}-1.85}{\frac{4\sqrt{2}}{3}}\times100=1.89\%$$

10 a $\int_0^2 2^x\,\mathrm{d}x$

x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
2^x	1	1.189	1.414	1.682	2	2.378	2.828	3.364	4

$$\int_a^b y\,\mathrm{d}x=\frac{1}{2}h\left(y_0+2\left(y_1+y_2+\ldots+y_{n-1}\right)+y_n\right)$$

$$\int_0^2 \sqrt{x}\,\mathrm{d}x=\frac{1}{2}(0.25)\left(1+2\left(1.189+1.414+1.682+2+2.378+2.828+3.364\right)+4\right)$$

$$=4.34\text{ (3 s.f.)}$$

b Overestimate because the curve is convex.