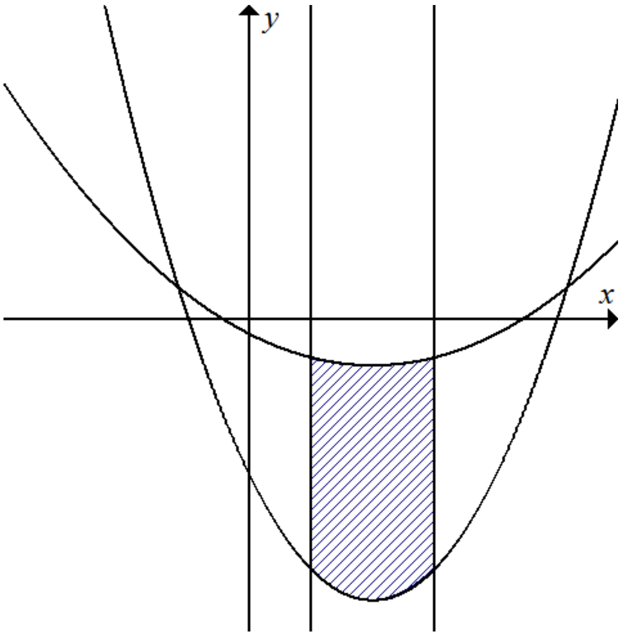
Exercise 8E

1



The area of the shaded region is given by

$$A=\int_1^3\left(\frac{1}{2}x^2-2x-1\right)\mathrm{d}x-\int_1^3\left(2x^2-8x-10\right)\mathrm{d}x$$

$$=\left[\frac{1}{6}x^3-x^2-x\right]_1^3-\left[\frac{2}{3}x^3-4x^2-10x\right]_1^3$$

$$=\left[\left(\frac{1}{6}(3)^3-(3)^2-(3)\right)-\left(\frac{1}{6}(1)^3-(1)^2-(1)\right)\right]-\left[\left(\frac{2}{3}(3)^3-4(3)^2-10(3)\right)-\left(\left(\frac{2}{3}(1)^3-4(1)^2-10(1)\right)\right)\right]$$

$$=\left[\left(-\frac{15}{2}\right)-\left(-\frac{11}{6}\right)\right]-\left[(-48)-\left(-\frac{40}{3}\right)\right]$$

$$=-\frac{17}{3}+\frac{104}{3}$$

$$=29$$

Alternatively, since the limits are the same:

$$A=\int_1^3\left(\frac{1}{2}x^2-2x-1\right)\mathrm{d}x-\int_1^3\left(2x^2-8x-10\right)\mathrm{d}x$$

$$=\int_1^3\left(-\frac{3}{2}x^2+6x+9\right)\mathrm{d}x$$

$$=\left[-\frac{1}{2}x^3+3x^2+9x\right]_1^3$$

$$=\left[\left(-\frac{1}{2}(3)^3+3(3)^2+9(3)\right)-\left(-\frac{1}{2}(1)^3+3(1)^2+9(1)\right)\right]$$

$$=\left[\left(-\frac{81}{2}\right)-\left(-\frac{23}{2}\right)\right]$$

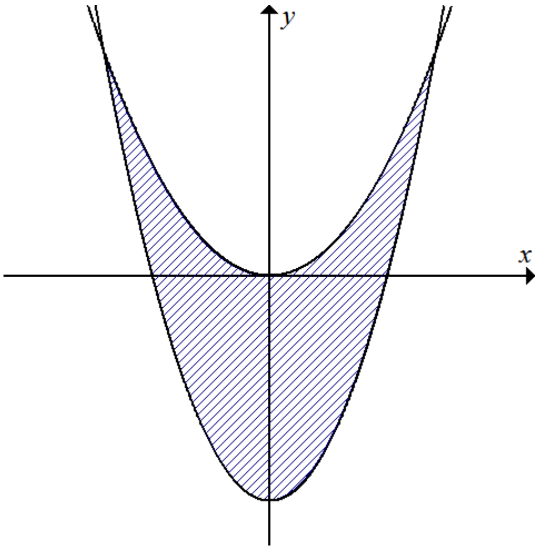
$$=29$$

2 $y = x^2$ and $y = 2x^2 - 25$

To find where the curves intersect

$x^2 = 2x^2 - 25$

$x = \pm 5$



When $x = -5$, $y = 25$ and when $x = 5$, $y = 25$

So the points of intersection are (−5, 25) and (5, 25)

The area of the shaded region is given by

$$A = \int_{-5}^{5} x^2 dx - \int_{-5}^{5} (2x^2 - 25) dx$$

$$= \left[\frac{1}{3}x^3\right]_{-5}^{5} - \left[\frac{2}{3}x^3 - 25x\right]_{-5}^{5}$$

$$= \left[\left(\frac{1}{3}(5)^3\right) - \left(\frac{1}{3}(-5)^3\right)\right] - \left[\left(\frac{2}{3}(5)^3 - 25(5)\right) - \left(\frac{2}{3}(-5)^3 - 25(-5)\right)\right]$$

$$= \frac{250}{3} + \frac{250}{3}$$

$$= \frac{500}{3}$$

Alternatively, since the limits are the same:

$$A = \int_{-5}^{5} x^2 dx - \int_{-5}^{5} (2x^2 - 25) dx$$

$$= \int_{-5}^{5} -x^2 + 25 dx$$

$$= \left[-\frac{1}{3}x^3 + 25x\right]_{-5}^{5}$$

$$= \left[\left(-\frac{1}{3}(5)^3 + 25(5)\right) - \left(-\frac{1}{3}(-5)^3 + 25(5)\right)\right]$$

$$= \left(\frac{250}{3}\right) - \left(-\frac{250}{3}\right)$$

$$= \frac{500}{3}$$

3 $y = 10 - 2x^2$ and $y = 2x^2 - 6$

$10 - 2x^2 = 2x^2 - 6$

$4x^2 = 16$

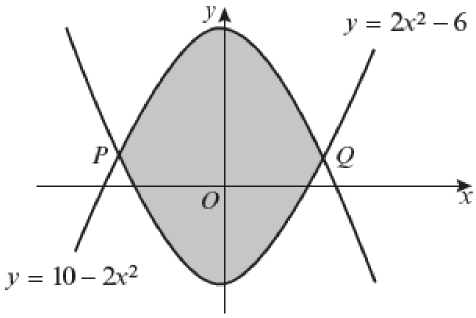
$x^2 = 4$

$x = \pm 2$

When $x = 2$, $y = 2$

When $x = -2$, $y = 2$

So the curves intersect at $(-2, 2)$ and $(2, 2)$



The shaded area is given by

$$A = \int_{-2}^{2} (10 - 2x^2)\,dx - \int_{-2}^{2} (2x^2 - 6)\,dx$$

$$= \left[10x - \frac{2}{3}x^3\right]_{-2}^{2} - \left[\frac{2}{3}x^3 - 6x\right]_{-2}^{2}$$

$$= \left[\left(10(2) - \frac{2}{3}(2)^3\right) - \left(10(-2) - \frac{2}{3}(-2)^3\right)\right] - \left[\left(\frac{2}{3}(2)^3 - 6(2)\right) - \left(\frac{2}{3}(-2)^3 - 6(-2)\right)\right]$$

$$= \left[\left(20 - \frac{16}{3}\right) - \left(-20 + \frac{16}{3}\right)\right] - \left[\left(\frac{16}{3} - 12\right) - \left(-\frac{16}{3} + 12\right)\right]$$

$$= \left[\frac{44}{3} - \left(-\frac{44}{3}\right)\right] - \left[-\frac{20}{3} - \frac{20}{3}\right]$$

$$= \frac{88}{3} + \frac{40}{3}$$

$$= \frac{128}{3}$$

Alternatively, since the limits are the same:

$$A = \int_{-2}^{2} (10 - 2x^2)\,dx - \int_{-2}^{2} (2x^2 - 6)\,dx = \int_{-2}^{2} (16 - 4x^2)\,dx$$

$$= \left[16x - \frac{4}{3}x^3\right]_{-2}^{2}$$

$$= \left[\left(16(2) - \frac{4}{3}(2)^3\right) - \left(16(-2) - \frac{4}{3}(-2)^3\right)\right]$$

$$= \left[\frac{64}{3} - \left(-\frac{64}{3}\right)\right]$$

$$= \frac{128}{3}$$

4 $y=x^3+x^2$ and $y=2x^2+2x$

$x^3+x^2=2x^2+2x$

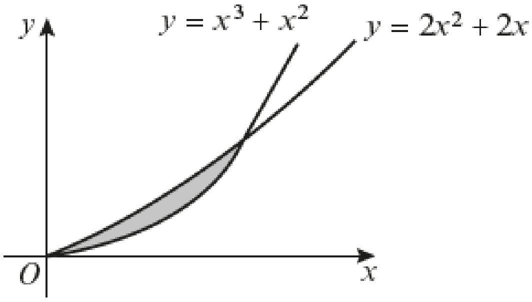
$x^3-x^2-2x=0$

$x\left(x^2-x-2\right)=0$

$x(x+1)(x-2)=0$

$x=-1,\ x=0$ or $x=2$

When $x=0$, $y=0$
When $x=2$, $y=12$
So the curves intersect at (0, 0) and (2, 12)



The shaded area is given by

$$A=\int_0^2\left(2x^2+2x\right)\mathrm{d}x-\int_0^2\left(x^3+x^2\right)\mathrm{d}x$$

$$=\left[\frac{2}{3}x^3+x^2\right]_0^2-\left[\frac{1}{4}x^4+\frac{1}{3}x^3\right]_0^2$$

$$=\left[\left(\frac{2}{3}(2)^3+(2)^2\right)-\left(\frac{2}{3}(0)^3+(0)^2\right)\right]-\left[\left(\frac{1}{4}(2)^4+\frac{1}{3}(2)^3\right)-\left(\frac{1}{4}(0)^4+\frac{1}{3}(0)^3\right)\right]$$

$$=\left[\left(\frac{16}{3}+4\right)-0\right]-\left[\left(4+\frac{8}{3}\right)-0\right]$$

$$=\frac{28}{3}-\frac{20}{3}$$

$$=\frac{8}{3}$$

Alternatively, since the limits are the same:

$$A=\int_0^2\left(2x^2+2x\right)\mathrm{d}x-\int_0^2\left(x^3+x^2\right)\mathrm{d}x=\int_0^2\left(-x^3+x^2+2x\right)\mathrm{d}x$$

$$=\left[-\frac{1}{4}x^4+\frac{1}{3}x^3+x^2\right]_0^2$$

$$=\left[\left(-\frac{1}{4}(2)^4+\frac{1}{3}(2)^3+(2)^2\right)-\left(-\frac{1}{4}(0)^4+\frac{1}{3}(0)^3+(0)^2\right)\right]$$

$$=\left[\left(\frac{8}{3}\right)-0\right]$$

$$=\frac{8}{3}$$

5 $y=\frac{1}{2}x^2-\frac{5}{2}x+7$ and $y=x^2-5x+7$

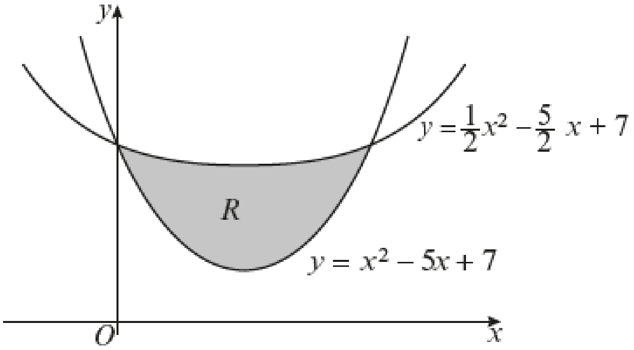
$\frac{1}{2}x^2-\frac{5}{2}x+7=x^2-5x+7$

$\frac{1}{2}x^2-\frac{5}{2}x=0$

$x(x-5)=0$

$x=0$ or $x=5$

When $x = 0$, $y = 7$
When $x = 5$, $y = 7$
So the curves intersect at (0, 7) and (5, 7)



The shaded area is given by

$$A=\int_0^5\left(\frac{1}{2}x^2-\frac{5}{2}x+7\right)dx-\int_0^5\left(x^2-5x+7\right)dx$$

$$=\left[\frac{1}{6}x^3-\frac{5}{4}x^2+7x\right]_0^5-\left[\frac{1}{3}x^3-\frac{5}{2}x^2+7x\right]_0^5$$

$$=\left[\left(\frac{1}{6}(5)^3-\frac{5}{4}(5)^2+7(5)\right)-\left(\frac{1}{6}(0)^3-\frac{5}{4}(0)^2+7(0)\right)\right]-\left[\left(\frac{1}{3}(5)^3-\frac{5}{2}(5)^2+7(5)\right)-\left(\frac{1}{3}(0)^3-\frac{5}{2}(0)^2+7(0)\right)\right]$$

$$=\left[\left(\frac{125}{6}-\frac{125}{4}+35\right)-0\right]-\left[\left(\frac{125}{3}-\frac{125}{2}+35\right)-0\right]$$

$$=\frac{295}{12}-\frac{85}{6}$$

$$=\frac{125}{12}$$

Alternatively, since the limits are the same:

$$A=\int_0^5\left(\frac{1}{2}x^2-\frac{5}{2}x+7\right)dx-\int_0^5\left(x^2-5x+7\right)dx$$

$$=\int_0^5\left(-\frac{1}{2}x^2+\frac{5}{2}x\right)dx$$

$$=\left[-\frac{1}{6}x^3+\frac{5}{4}x^2\right]_0^5$$

$$=\left[\left(-\frac{1}{6}(5)^3+\frac{5}{4}(5)^2\right)-\left(-\frac{1}{6}(0)^3+\frac{5}{4}(0)^2\right)\right]$$

$$=\left[\left(\frac{125}{12}\right)-(0)\right]=\frac{125}{12}$$