Exercise 6E

1 a $\sin 4\theta = 0 \quad 0° \le \theta \le 360°$

Let $X = 4\theta$ so $0° \le X \le 1440°$

Solve $\sin X = 0$ in the interval $0° \le X \le 1440°$

From the graph of $y = \sin X$, $\sin X = 0$ where

$X = 0°, 180°, 360°, 540°, 720°, 900°, 1080°, 1260°, 1440°$

$\theta = \frac{X}{4}$

$= 0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°, 360°$

b $\cos 3\theta = -1 \quad 0° \le \theta \le 360°$

Let $X = 3\theta$ so $0° \le X \le 1080°$

Solve $\cos X = 0$ in the interval $0° \le X \le 1080°$

From the graph of $y = \cos X$, $\cos X = -1$ where

$X = 180°, 540°, 900°,$

$\theta = \frac{X}{3}$

$= 60°, 180°, 300°$

c $\tan 2\theta = 1 \quad 0° \le \theta \le 360°$

Let $X = 2\theta$

Solve $\tan X = 1$ in the interval $0° \le X \le 720°$.

A solution is $X = \tan^{-1}(1) \quad = 45°$

As $\tan X$ is +ve, X is in the first and third quadrants.

So $X = 45°, 225°, 405°, 585°$

$\theta = \frac{X}{2}$

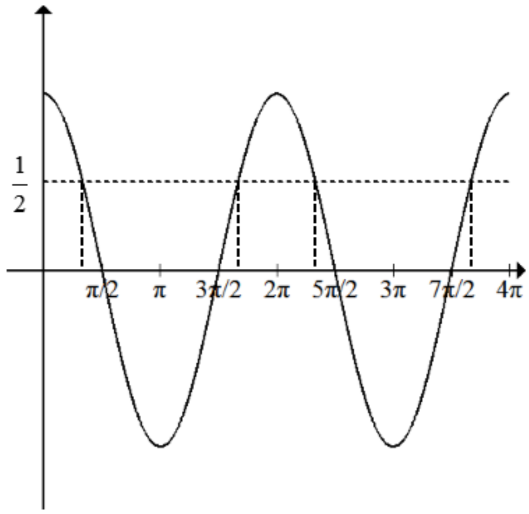
$= 22\tfrac{1}{2}°, 112\tfrac{1}{2}°, 202\tfrac{1}{2}°, 292\tfrac{1}{2}°$

2 a $\cos 2\theta = \frac{1}{2}, \quad 0 \le \theta \le 2\pi$

Let $X = 2\theta$

$\cos X = \frac{1}{2}$

As $X = 2\theta$ and $0 \le \theta \le 2\pi$, the interval for X is $0 \le X \le 4\pi$



The principal solution for X is $\frac{\pi}{3}$

The other solutions for X are $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$,

$2\pi + \frac{\pi}{3} = \frac{7\pi}{3}$ and $4\pi - \frac{\pi}{3} = \frac{11\pi}{3}$

Since $X = 2\theta$

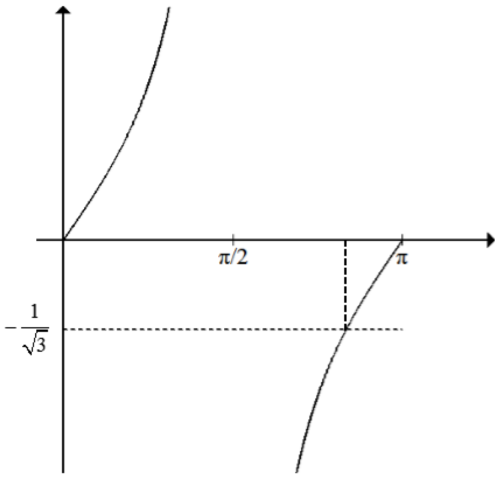
$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

2 b $\tan\left(\frac{\theta}{2}\right) = -\frac{1}{\sqrt{3}}, \quad 0 \le \theta \le 2\pi$

Let $X = \frac{\theta}{2}$

$\tan X = -\frac{1}{\sqrt{3}}$

As $X = \frac{\theta}{2}$ and $0 \le \theta \le 2\pi$, the interval for X is $0 \le X \le \pi$



The principal solution for X is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

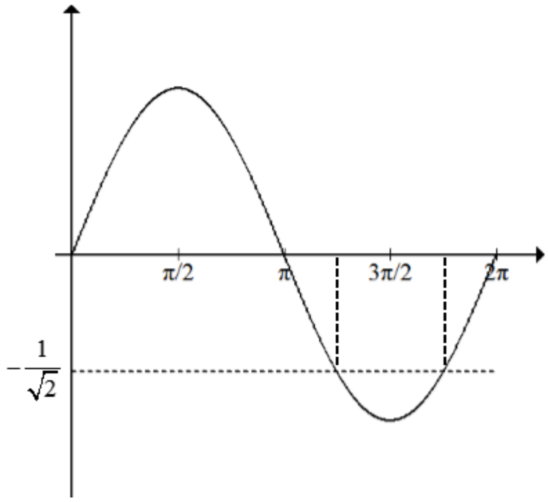
Since $X = \frac{\theta}{2}$

$\theta = \frac{5\pi}{3}$

2 c $\sin(-\theta) = \frac{1}{\sqrt{2}}, \quad 0 \le \theta \le 2\pi$

$-\sin\theta = \frac{1}{\sqrt{2}}$

$\sin\theta = -\frac{1}{\sqrt{2}}$



$\theta = \frac{5\pi}{4}$ and $\theta = \frac{7\pi}{4}$

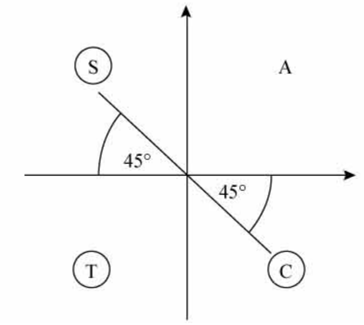
3 a $\tan(45° - \theta) = -1 \quad 0° \leq \theta \leq 360°$

Let $X = 45° - \theta$ so $0° \geq -\theta \geq -360°$

Solve $\tan X = -1$ in the interval
$45° \geq X \geq -315°$

A solution is $X = \tan^{-1}(-1) = -45°$

As $\tan X$ is −ve, X is in the second and fourth quadrants.



$X = -225°, -45°$

So $\theta = 45° - X = 90°, 270°$

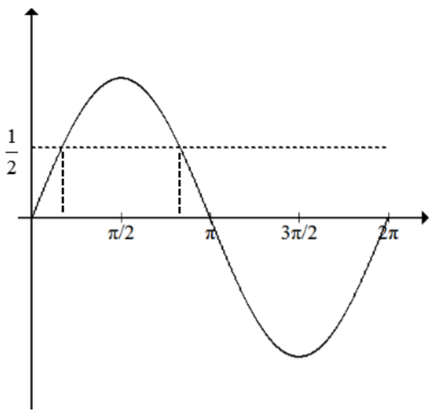
b $2\sin\left(\theta - \frac{\pi}{9}\right) = 1,\ 0 \leq \theta \leq 2\pi$

let $X = \theta - \frac{\pi}{9}$

$\sin\left(\theta - \frac{\pi}{9}\right) = \frac{1}{2}$

The interval for X is $-\frac{\pi}{9} \leq X \leq \frac{17\pi}{9}$

$\sin X = \frac{1}{2}$



The principal value of X is $\frac{\pi}{6}$

The other value of X is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Since $X = \theta - \frac{\pi}{9}$

$\theta = \frac{5\pi}{18}$ and $\theta = \frac{17\pi}{18}$

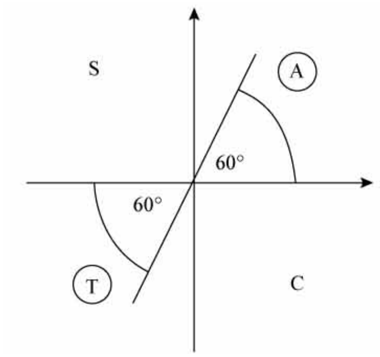
3 c Solve $\tan X = \sqrt{3}$ where $X = (\theta + 75°)$.

The interval for X is $75° \leq X \leq 435°$

One solution is $\tan^{-1}\left(\sqrt{3}\right) = 60°$

(This is not in the interval)

As $\tan X$ is +ve, solutions are in the first and third quadrants.



$X = 240°, 420°$

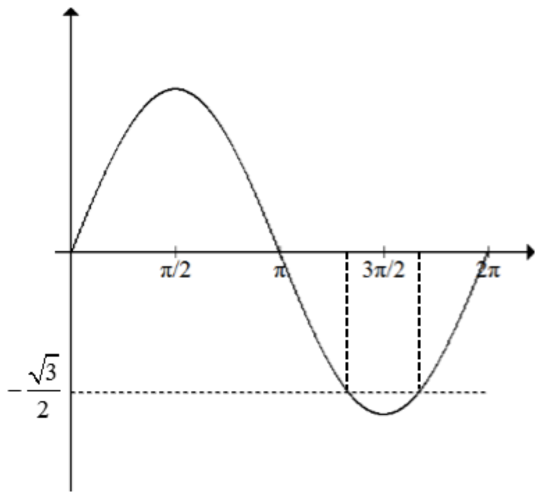
So $\theta = X - 75°$
$= 165°, 345°$

d $\sin\left(\theta - \frac{\pi}{18}\right) = -\frac{\sqrt{3}}{2},\ 0 \leq \theta \leq 2\pi$

let $X = \theta - \frac{\pi}{18}$

$\sin X = -\frac{\sqrt{3}}{2}$

The interval for X is $-\frac{\pi}{18} \leq X \leq \frac{35\pi}{18}$



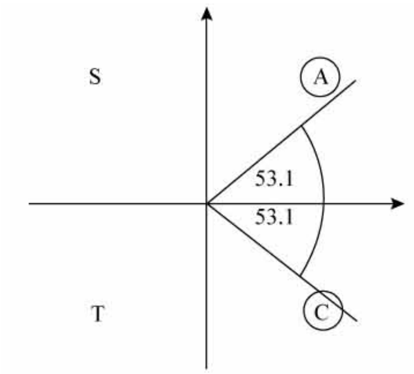
So $X = \frac{4\pi}{3}$ and $X = \frac{5\pi}{3}$

Since $X = \theta - \frac{\pi}{18}$

$\theta = \frac{25\pi}{18}$ and $\theta = \frac{31\pi}{18}$

3 e Solve $\cos X° = 0.6$ where $X = (70° - x)$.

The interval for X is
$180 + 70 \geq X \geq -180 + 70$
i.e. $-110 \leq X \leq 250$.
First solution is $\cos^{-1}(0.6) = 53.1°$
As $\cos X°$ is + ve, X is in the first and fourth quadrants.



$X = -53.1°, +53.1°$
So $\theta = 70° - X$
$= 16.9°, 123°$ (3 s.f.)

4 a Let $X = 3\theta$
So $3 \sin X = 2 \cos X$
$\frac{\sin X}{\cos X} = \frac{2}{3}$
$\tan X = \frac{2}{3}$
As $X = 3\theta$, then as $0° \leq \theta \leq 180°$
So $3 \times 0° \leq X \leq 3 \times 180°$
So the interval for X is $0° \leq X \leq 540°$.
$X = 33.7°, 213.7°, 393.7°$
i.e. $3\theta = 33.7°, 213.7°, 393.7°$
So $\theta = 11.2°, 71.2°, 131.2°$

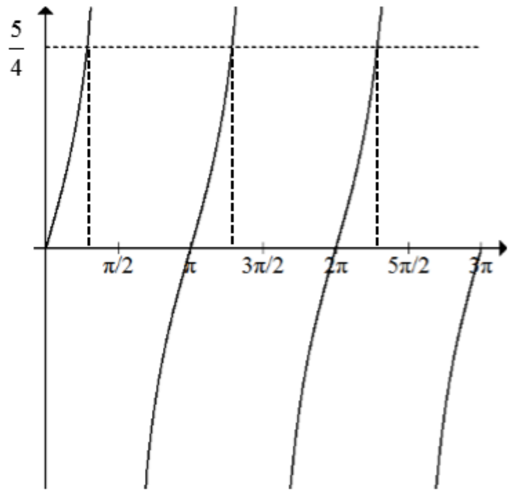
4 b $4\sin\left(\theta + \frac{\pi}{4}\right) = 5\cos\left(\theta + \frac{\pi}{4}\right),\ 0 \leq \theta \leq \frac{5\pi}{2}$

Let $X = \theta + \frac{\pi}{4}$

$4\sin X = 5\cos X,\ \frac{\pi}{4} \leq X \leq \frac{11\pi}{4}$

$\frac{\sin X}{\cos X} = \frac{5}{4}$

$\tan X = \frac{5}{4}$



$X = 0.896,\ X = \pi + 0.896 = 4.04$ and
$X = 2\pi + 0.896 = 7.18$

Since $X = \theta + \frac{\pi}{4}$

$\theta = 0.111,\ \theta = 3.25$ and $\theta = 6.39$

c Let $X = 2x$
$2 \sin X - 7 \cos X = 0$
$2 \sin X = 7 \cos X$
$\frac{\sin X}{\cos X} = \frac{7}{2}$
$\tan X = \frac{7}{2}$
As $X = 2x$, then as $0° \leq x \leq 180°$
So $2 \times 0° \leq X \leq 2 \times 180°$
So the interval for X is $0° \leq X \leq 360°$.
$X = 74.05°, 254.05°$
i.e. $2x = 74.05°, 254.05°$
So $x = 37.0°, 127.0°$

4 d $\sqrt{3}\sin\left(\theta+\frac{\pi}{4}\right)+\cos\left(\theta+\frac{\pi}{4}\right)=0,\ 0\leq\theta\leq\pi$

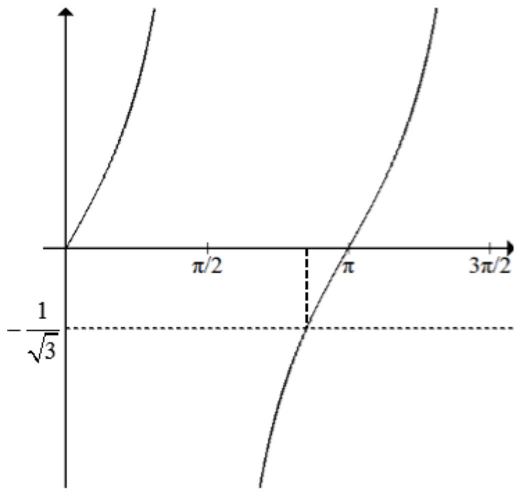
Let $X=\theta+\frac{\pi}{4}$

$\sqrt{3}\sin X+\cos X=0,\ \frac{\pi}{4}\leq\theta\leq\frac{5\pi}{4}$

$\sqrt{3}\sin X=-\cos X$

$\frac{\sin X}{\cos X}=-\frac{1}{\sqrt{3}}$

$\tan X=-\frac{1}{\sqrt{3}}$



$X=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$

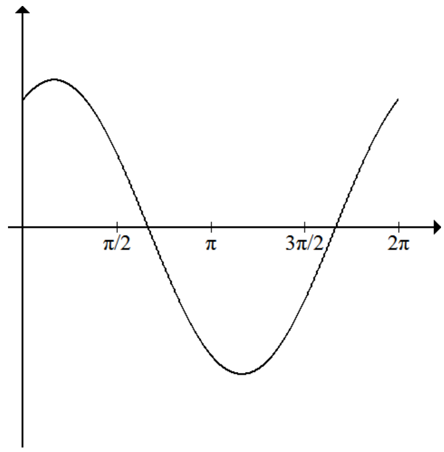
Since $X=\theta+\frac{\pi}{4}$

$\theta=\frac{7\pi}{12}$

5 a Let $X = x + 20°$
So $\sin X = \frac{1}{2}$
As $X = x + 20°$, then as $0 \leq x \leq 180°$
So $0 + 20 \leq x \leq 180° + 20$
So the interval for X is $20° \leq X \leq 200°$.
$X = 30°, 150°$
i.e. $x + 20° = 30°, 150°$
So $x = 10°, 130°$

5 b Let $X = 2x$
So $\cos X = -0.8$
As $X = 2x$, then as $0 \leq x \leq 180°$
So $2 \times 0 \leq X \leq 2 \times 180°$
So the interval for X is $0 \leq X \leq 360°$
$X = 143.13°, 216.87°$
i.e. $2x = 143.13°, 216.87°$
So $x = 71.6°, 108.4°$

6 a

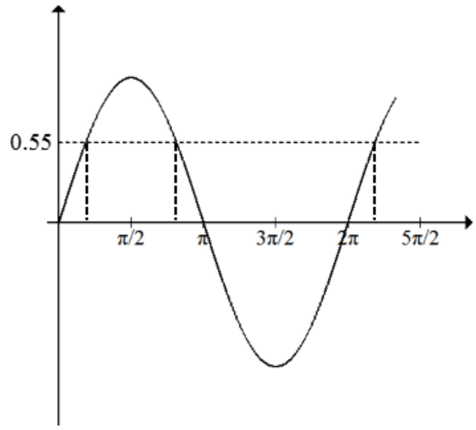


b $\left(0,\frac{\sqrt{3}}{2}\right)$, $\left(\frac{2\pi}{3},0\right)$ and $\left(\frac{5\pi}{3},0\right)$

6 c $\sin\left(x+\frac{\pi}{3}\right)=0.55,\ 0\le x\le 2\pi$

Let $X=x+\frac{\pi}{3}$

$\sin X=0.55,\ \frac{\pi}{3}\le x\le\frac{7\pi}{3}$



$X=0.582$, $X=\pi-0.582=2.56$ and $X=2\pi+0.582=6.87$

Since $X=x+\frac{\pi}{3}$

$x=-0.465$, $x=1.51$ and $x=5.82$

$x=-0.465$ lies outside the limits so $x=1.51$ and $x=5.82$

7 a $4\sin x=3\cos x$

$\frac{\sin x}{\cos x}=\frac{3}{4}$

$\tan x=\frac{3}{4}$

b Let $X=2\theta$

So $\tan X=\frac{3}{4}$

As $X=2\theta$, then as $0^\circ\le\theta\le 360^\circ$

So $2\times 0^\circ\le X\le 2\times 360^\circ$

So the interval for X is $0^\circ\le X\le 720^\circ$.

$X=36.87^\circ, 216.87^\circ, 396.87^\circ, 576.87^\circ$

i.e. $2\theta=36.87^\circ, 216.87^\circ, 396.87^\circ, 576.87^\circ$

So $\theta=18.4^\circ, 108.4^\circ, 198.4^\circ, 288.4^\circ$

8 a $\tan kx=-\frac{1}{\sqrt{3}},\ k>0$

Since $x=\frac{\pi}{3}$ is a solution

$\tan\left(\frac{\pi k}{3}\right)=-\frac{1}{\sqrt{3}}$

$\frac{\pi k}{3}=\frac{5\pi}{6}$

$k=\frac{5}{2}$

b This is not the only possible value of k as increasing k will bring another 'branch' of the tan graph into place.

Challenge

Let $X=3x-45^\circ$

So $\sin X=\frac{1}{2}$

As $X=3x-45^\circ$, then as $0^\circ\le x\le 180^\circ$

So $3\times 0^\circ-45^\circ\le x\le 3\times 180^\circ-45^\circ$

So the interval for X is $-45^\circ\le X\le 495^\circ$.

$X=30^\circ, 150^\circ, 390^\circ$

i.e. $3x-45^\circ=30^\circ, 150^\circ, 390^\circ$

So $x=25^\circ, 65^\circ, 145^\circ$