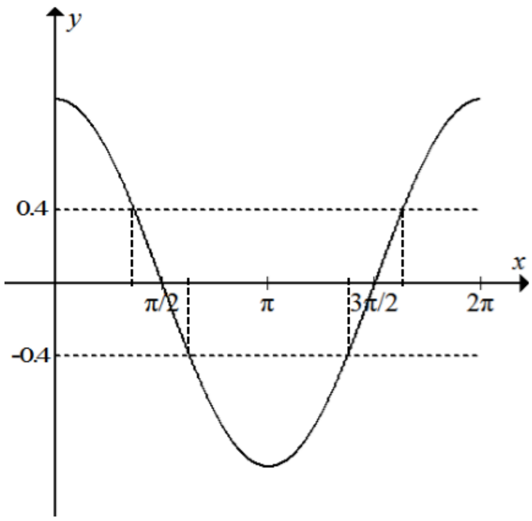
Exercise 6D

1 a Consider $\tan x = 2$
$x = \tan^{-1}(2)$
$= 63.4°$ (3 s.f.) in the first quadrant
The principal solution marked by A in the diagram is $180° - 63.4° = 116.6°$

b The other solution between 0° and 360° is $116.6° + 180° = 296.6°$
$x = 116.6°, 296.6°$ when $0° \leq x \leq 360°$

2 a $\cos x = 0.4$
$x = \cos^{-1}(0.4)$
$= 66.4$ (3 s.f.)

b $\cos x \pm 0.4$



$\cos x = 0.4$
$x = 1.16$ and $x = 2\pi - 1.16 = 5.12$
$\cos x = -0.4$
$x = 1.98$ and $x = 2\pi - 1.98 = 4.30$

3 a Using the graph of $y = \sin\theta$
$\sin\theta = -1$ when $\theta = 270°$

b $\tan\theta = \sqrt{3}$
The calculator solution is 60° $\left(\tan^{-1}\sqrt{3}\right)$
and, as $\tan\theta$ is +ve, θ lies in the first and third quadrants.
$\theta = 60°$ and $(180° + 60°) = 60°, 240°$

3 c $\cos\theta = \frac{1}{2}$
The calculator solution is 60° and as $\cos\theta$ is +ve, θ lies in the first and fourth quadrants.
$\theta = 60°$ and $(360° - 60°) = 60°, 300°$

d $\sin\theta = \sin 15°$
The acute angle satisfying the equation is $\theta = 15°$.
As $\sin\theta$ is +ve, θ lies in the 1st and 2nd quadrants, so
$\theta = 15°$ and $(180° - 15°) = 15°, 165°$

e A first solution is $\cos^{-1}(-\cos 40°) = 140°$
A second solution of $\cos\theta = k$ is
$360° -$ 1st solution.
So second solution is 220°.
(Use the quadrant diagram as a check.)

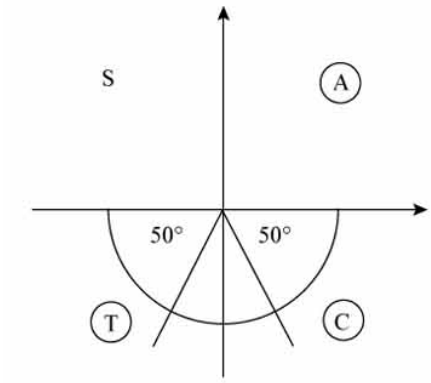
f A first solution is $\tan^{-1}(-1) = -45°$
Use the quadrant diagram, noting that as tan is $-$ve, solutions are in the 2nd and 4th quadrants.
($-45°$ is not in the given interval.)
So solutions are 135° and 315°.

g From the graph of $y = \cos\theta$
$\cos\theta = 0$ when $\theta = 90°, 270°$

3 h $\sin\theta = -0.766$

$\sin^{-1}(-0.766) = -50°$

$360° - 50° = 310°$

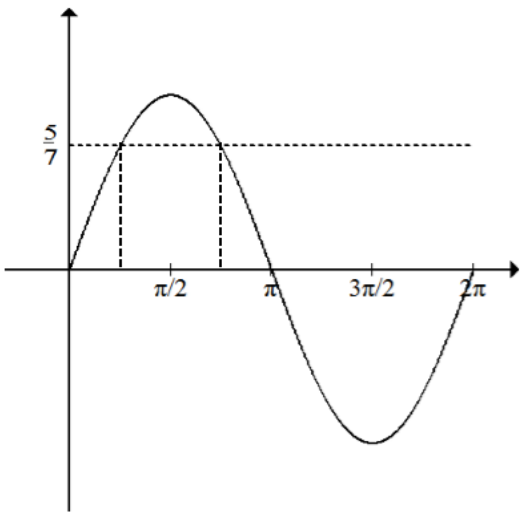


From the diagram, the second solution is $180° + 50° = 230°$.

$\theta = 230°, 310°$

4 a $7\sin\theta = 5,\ 0 \le \theta \le 2\pi$

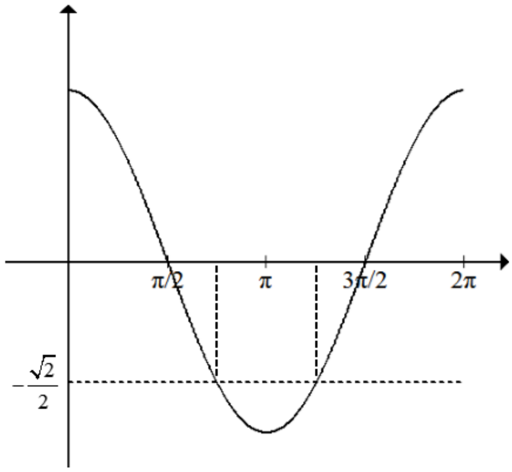
$\sin\theta = \dfrac{5}{7}$



$\theta = 0.796$ and $\theta = \pi - 0.796 = 2.35$

b $2\cos\theta = -\sqrt{2},\ 0 \le \theta \le 2\pi$

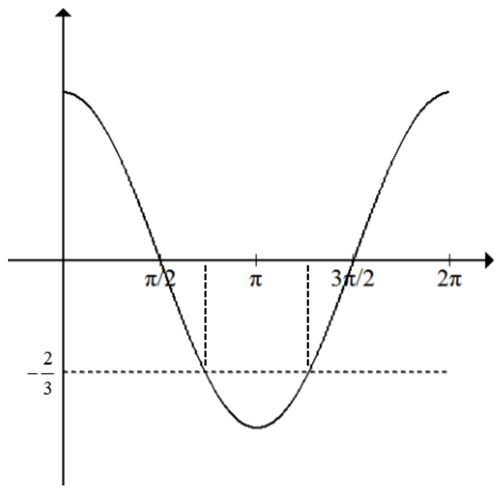
$\cos\theta = -\dfrac{\sqrt{2}}{2}$



$\theta = \dfrac{3\pi}{4}$ and $\theta = 2\pi - \dfrac{3\pi}{4} = \dfrac{5\pi}{4}$

c $3\cos\theta = -2,\ 0 \le \theta \le 2\pi$

$\cos\theta = -\dfrac{2}{3}$



$\theta = 2.30$ and $\theta = 2\pi - 2.30 = 3.98$

d $4\sin\theta = -3,\ 0 \le \theta \le 2\pi$

$\sin\theta = -\dfrac{3}{4}$

$\theta = (-0.848), \pi - (-0.848), 2\pi + (-0.848)$

$\theta = 3.99, 5.44$

5 a $\tan\theta = \frac{1}{7}$

$\theta = 8.13°$ or $188°$

b $\tan\theta = \frac{15}{8}$

$\theta = 61.9°$ or $242°$

5 **c** $\tan\theta = -\frac{11}{3}$

$\theta = -74.7°$

$\theta = 105.3°$ or $285°$

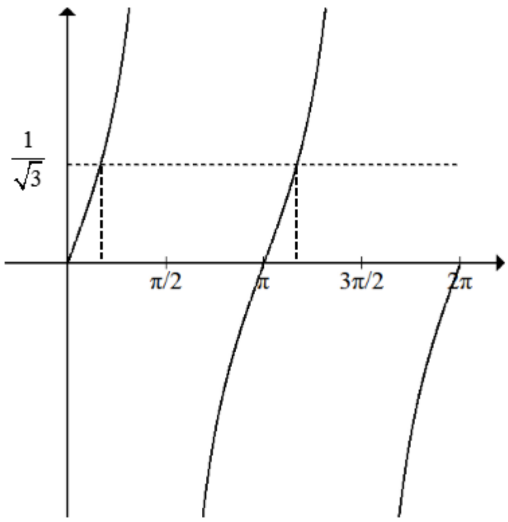
d $\cos\theta = \frac{\sqrt{5}}{3}$

$\theta = 41.8°, 318°$

6 a $\sqrt{3}\sin\theta = \cos\theta,\ 0 \le \theta \le 2\pi$

$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$



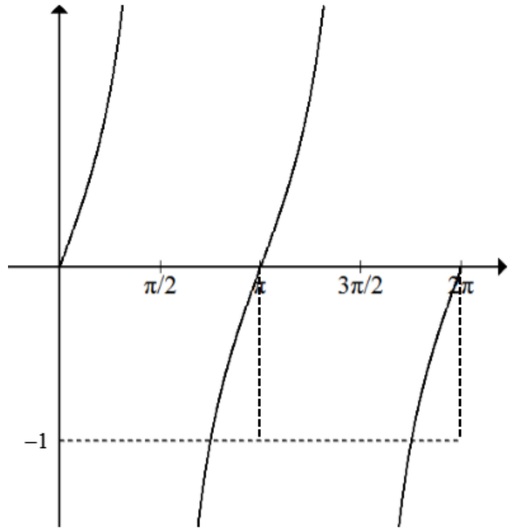
$\theta = \frac{\pi}{6}$ and $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

b $\sin\theta + \cos\theta = 0,\ 0 \le \theta \le 2\pi$

$\sin\theta = -\cos\theta$

$\frac{\sin\theta}{\cos\theta} = -1$

$\tan\theta = -1$

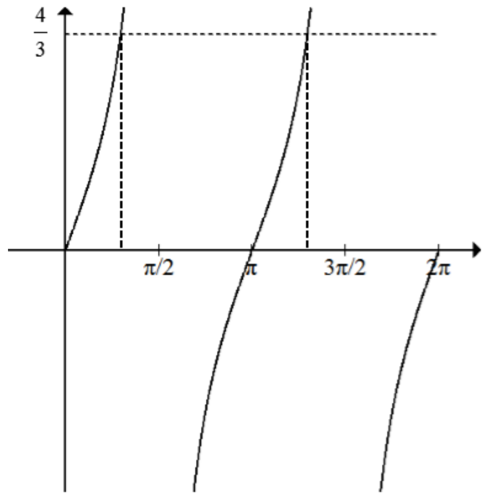


$\theta = \frac{3\pi}{4}$ and $\theta = \pi + \frac{3\pi}{4} = \frac{7\pi}{4}$

c $3\sin\theta = 4\cos\theta,\ 0 \le \theta \le 2\pi$

$\frac{\sin\theta}{\cos\theta} = \frac{4}{3}$

$\tan\theta = \frac{4}{3}$



$\theta = 0.927$ and $\theta = \pi + 0.927 = 4.07$

7 a $2\sin\theta - 3\cos\theta = 0$

$$\tan\theta = \frac{3}{2}$$

$$\theta = 56.3° \text{ or } 236°$$

b $\sqrt{2}\sin\theta = 2\cos\theta$

$$\tan\theta = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\theta = 54.7° \text{ or } 235°$$

c $\sqrt{5}\sin\theta + \sqrt{2}\cos\theta = 0$

$$\sqrt{5}\tan\theta + \sqrt{2} = 0$$

$$\tan\theta = -\frac{\sqrt{2}}{\sqrt{5}}$$

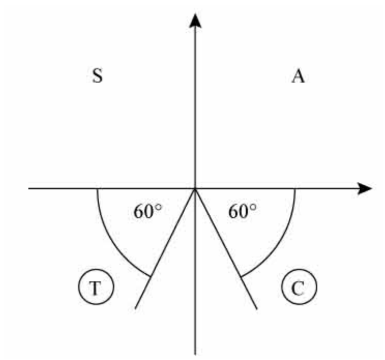
$$\theta = -32.3° \quad \theta > 0$$

$$\theta = 148° \text{ or } 328°$$

8 a Calculator solution of

$\sin x° = -\frac{\sqrt{3}}{2}$ is $x = -60°$

As $\sin x°$ is – ve, x is in the third and fourth quadrants.

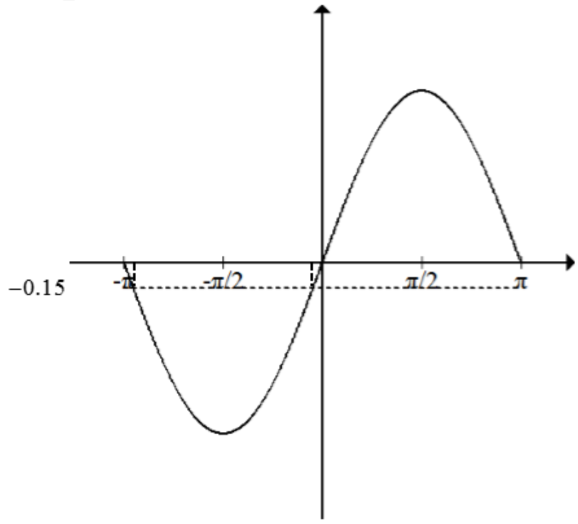


Read off all solutions in the interval $-180° \le x \le 540°$.

$x = -120°, -60°, 240°, 300°$

8 b $2\sin x = -0.3, \ -\pi \le x \le \pi$

$\sin x = -0.15$

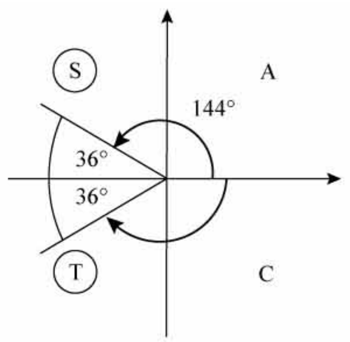


$x = -0.151$ and $x = -\pi + 0.151 = -2.99$

c $\cos x° = -0.809$

Calculator solution is $144°$ (3 s.f.)

As $\cos x°$ is – ve, x is in the second and third quadrants.

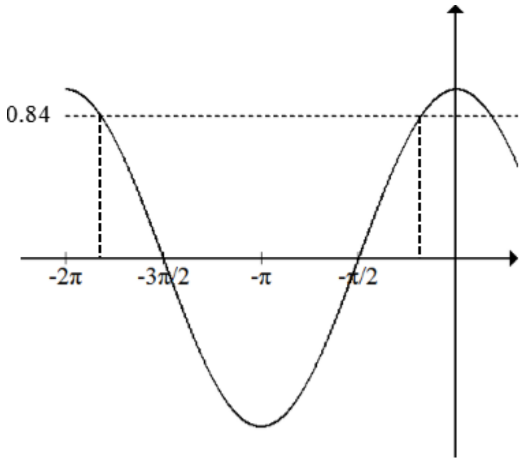


Read off all the solutions in the interval $-180° \le x \le 180°$.

$x = -144°, +144°$

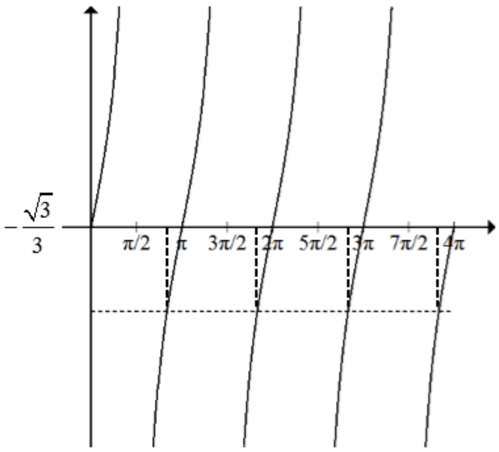
Note: Here solutions are $\cos^{-1}(-0.809)$ and $(360° - \cos^{-1}(-0.809))$.

8 d $\cos x = 0.84,\ \ -2\pi \le x \le 0$



$x = -0.574$ and $x = -2\pi + 0.574 = -5.71$

e $\tan x = -\dfrac{\sqrt{3}}{3},\ \ 0 \le x \le 4\pi$



$x = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6},\ \ x = 2\pi - \dfrac{\pi}{6} = \dfrac{11\pi}{6},$

$x = 3\pi - \dfrac{\pi}{6} = \dfrac{17\pi}{6}$ and $x = 3\pi - \dfrac{\pi}{6} = \dfrac{23\pi}{6}$

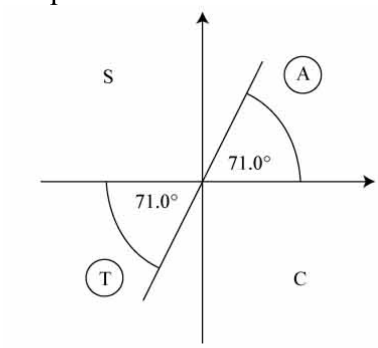
8 f $\tan x° = 2.90$

Calculator solution is

$\tan^{-1}(2.90) = 71.0°(3\text{ s.f.})$

(not in interval).

As $\tan x°$ is +ve, x is in the first and third quadrants.



Read off all solutions in the interval

$80° \le x \le 440°.$

$x = 251°,\ 431°$

(Note: Here solutions are

$\tan^{-1}(2.90) + 180°,\ \tan^{-1}(2.90) + 360°.$)

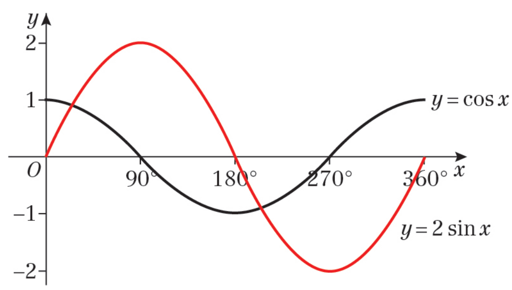
9 a It should be $\tan x = \dfrac{2}{3}$, not $\dfrac{3}{2}$.

b Squaring both sides creates extra solutions.

9 c $\tan x = \dfrac{2}{3}$

$x = 33.7°$ or $x = -146.3°$

10 a



b The graphs intersect at 2 points in the given range so there are 2 solutions.

10 c $2 \sin x = \cos x$

$\frac{\sin x}{\cos x} = \frac{1}{2}$

$\tan x = \frac{1}{2}$

$x = 26.6°$

$x = 26.6° + 180° = 206.6°$

$x = 26.6°$ or $206.6°$

11 $\tan \theta = \pm 3$

When $\tan \theta = 3$, $\theta = 71.6°$

or $\theta = 71.6° + 180° = 251.6°$

When $\tan \theta = -3$, $\theta = -71.6°$

or $\theta = -71.6° + 180° = 108.4°$ or

$\theta = 108.4° + 180° = 288.4°$

$\theta = 71.6°, 108.4°, 251.6°$ or $288.4°$

12 a $4 \sin^2 x - 3 \cos^2 x = 2$

$4 \sin^2 x - 3(1 - \sin^2 x) = 2$

$4 \sin^2 x - 3 + 3\sin^2 x = 2$

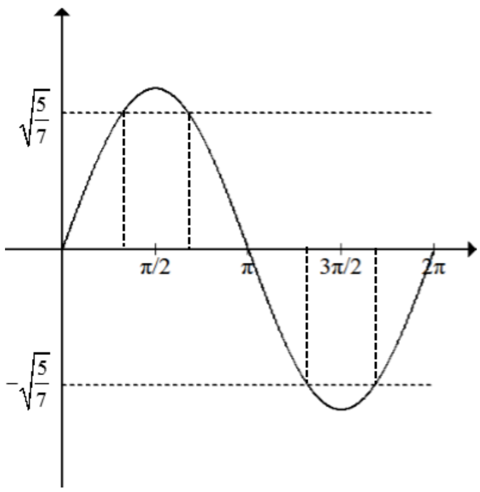
$7 \sin^2 x = 5$

12 b $4\sin 2x - 3\cos 2x = 2,\ 0 \le x \le 2\pi$

$7\sin^2 x = 5$

$\sin^2 x = \frac{5}{7}$

$\sin x = \pm\sqrt{\frac{5}{7}}$



$x = 1.0$, $x = \pi - 1.01 = 2.1$,

$x = \pi + 1.01 = 4.1$ and $x = 2\pi - 1.01 = 5.3$

13 a $2 \sin^2 x + 5 \cos^2 x = 1$

$2 \sin^2 x + 5(1 - \sin^2 x) = 1$

$2 \sin^2 x + 5 - 5 \sin^2 x = 1$

$3 \sin^2 x = 4$

b Using $3 \sin^2 x = 4$

$\sin^2 x = \frac{4}{3}$

$\sin x > 1$, therefore there are no solutions.