Exercise 6C

1 a As $\sin^2\frac{1}{2}\theta+\cos^2\frac{1}{2}\theta\equiv 1$

So $1-\cos^2\frac{1}{2}\theta=\sin^2\frac{1}{2}\theta$

b As $\sin^2 3\theta+\cos^2 3\theta\equiv 1$

So:

$$5\sin^2 3\theta+5\cos^2 3\theta=5\left(\sin^2 3\theta+\cos^2 3\theta\right)$$
$$=5$$

c As $\sin^2 A+\cos^2 A\equiv 1$

So $\sin^2 A-1\equiv-\cos^2 A$

d $$\frac{\sin\theta}{\tan\theta}=\frac{\sin\theta}{\sin\theta/\cos\theta}$$
$$=\sin\theta\times\frac{\cos\theta}{\sin\theta}$$
$$=\cos\theta$$

e $$\frac{\sqrt{1-\cos^2 x}}{\cos x}=\frac{\sqrt{\sin^2 x}}{\cos x}$$
$$=\frac{\sin x}{\cos x}$$
$$=\tan x$$

f $$\frac{\sqrt{1-\cos^2 3A}}{\sqrt{1-\sin^2 3A}}=\frac{\sqrt{\sin^2 3A}}{\sqrt{\cos^2 3A}}$$
$$=\frac{\sin 3A}{\cos 3A}$$
$$=\tan 3A$$

g $(1+\sin x)^2+(1-\sin x)^2+2\cos^2 x$

$$=1+2\sin x+\sin^2 x+1-2\sin x+\sin^2 x+2\cos^2 x$$
$$=2+2\sin^2 x+2\cos^2 x$$
$$=2+2\left(\sin^2 x+\cos^2 x\right)$$
$$=2+2$$
$$=4$$

h $\sin^4\theta+\sin^2\theta\cos^2\theta$

$$=\sin^2\theta\left(\sin^2\theta+\cos^2\theta\right)$$
$$=\sin^2\theta$$

1 i $\sin^4\theta+2\sin^2\theta\cos^2+\cos^4\theta$

$$=\left(\sin^2\theta+\cos^2\theta\right)^2$$
$$=1^2$$
$$=1$$

2 Given $2\sin\theta=3\cos\theta$

So $\frac{\sin\theta}{\cos\theta}=\frac{3}{2}$

(divide both side by $2\cos\theta$)

So $\tan\theta=\frac{3}{2}$

3 As $\sin x\cos y=3\cos x\sin y$

So $\frac{\sin x\cos y}{\cos x\cos y}=3\frac{\cos x\sin y}{\cos x\cos y}$

So $\tan x=3\tan y$

4 a As $\sin^2\theta+\cos^2\theta\equiv 1$

So $\cos^2\theta\equiv 1-\sin^2\theta$

b $\tan^2\theta\equiv\frac{\sin^2\theta}{\cos^2\theta}\equiv\frac{\sin^2\theta}{1-\sin^2\theta}$

c $$\cos\theta\tan\theta=\cos\theta\times\frac{\sin\theta}{\cos\theta}$$
$$=\sin\theta$$

d $$\frac{\cos\theta}{\tan\theta}=\frac{\cos\theta}{\sin\theta/\cos\theta}$$
$$=\cos\theta\times\frac{\cos\theta}{\sin\theta}$$
$$=\frac{\cos^2\theta}{\sin\theta}$$

So $\frac{\cos\theta}{\tan\theta}=\frac{1-\sin^2\theta}{\sin\theta}$ or $\frac{1}{\sin\theta}-\sin\theta$

e $(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)$

$$=\cos^2\theta-\sin^2\theta$$
$$=\left(1-\sin^2\theta\right)-\sin^2\theta$$
$$=1-2\sin^2\theta$$

5 a LHS $= (\sin\theta + \cos\theta)^2$

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$= (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta$

$= 1 + 2\sin\theta\cos\theta$

$=$ RHS

b LHS $= \dfrac{1}{\cos\theta} - \cos\theta$

$= \dfrac{1-\cos^2\theta}{\cos\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta}$

$= \sin\theta \times \dfrac{\sin\theta}{\cos\theta}$

$= \sin\theta\tan\theta$

$=$ RHS

c LHS $= \tan x + \dfrac{1}{\tan x}$

$= \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}$

$= \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}$

$= \dfrac{1}{\sin x\cos x}$

$=$ RHS

d LHS $= \cos^2 A - \sin^2 A$

$\equiv \cos^2 A - (1 - \cos^2 A)$

$\equiv \cos^2 A - 1 + \cos^2 A$

$\equiv 2\cos^2 A - 1$ ✓

$\equiv 2(1 - \sin^2 A) - 1$

$\equiv 2 - 2\sin^2 A - 1$

$\equiv 1 - 2\sin^2 A$ ✓

e LHS $= (2\sin\theta - \cos\theta)^2 + (\sin\theta + 2\cos\theta)^2$

$\equiv 4\sin^2\theta - 4\sin\theta\cos\theta + \cos^2\theta$

$+ \sin^2\theta + 4\sin\theta\cos\theta + 4\cos^2\theta$

$\equiv 5\sin^2\theta + 5\cos^2\theta$

$\equiv 5(\sin^2\theta + \cos^2\theta)$

$\equiv 5$

$\equiv$ RHS

5 f LHS $= 2 - (\sin\theta - \cos\theta)^2$

$= 2 - (\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta)$

$= 2 - (1 - 2\sin\theta\cos\theta)$

$= 1 + 2\sin\theta\cos\theta$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= (\sin\theta + \cos\theta)^2$

$=$ RHS

g LHS $= \sin^2 x\cos^2 y - \cos^2 x\sin^2 y$

$= \sin^2 x(1 - \sin^2 y)$

$- (1 - \sin^2 x)\sin^2 y$

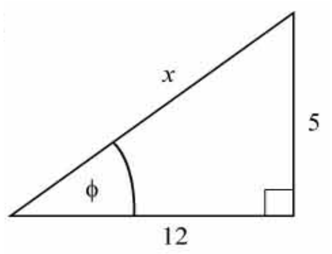
$= \sin^2 x - \sin^2 x\sin^2 y$

$- \sin^2 y + \sin^2 x\sin^2 y$

$= \sin^2 x - \sin^2 y$

$=$ RHS

6 a



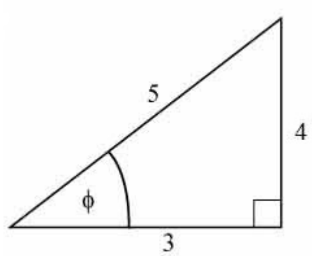
Using Pythagoras' theorem:

$x^2 = 12^2 + 5^2 = 169$

$x = 13$

So $\sin\theta = \dfrac{5}{13}$ and $\cos\theta = \dfrac{12}{13}$

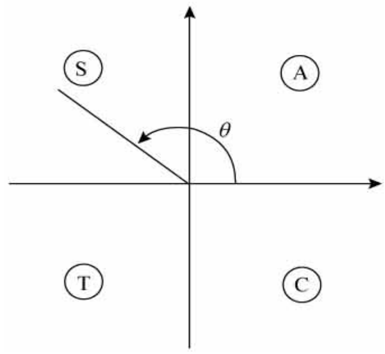
b



Using Pythagoras' theorem, $x = 4$

So $\sin\phi = \frac{4}{5}$ and $\tan\phi = -\frac{4}{3}$

6 b



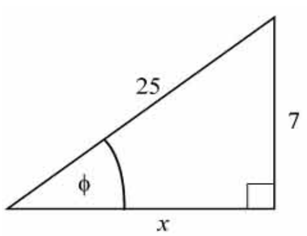
As θ is obtuse:

$\sin\theta = \sin\phi = \frac{4}{5}$

and

$\tan\theta = -\tan\phi = -\frac{4}{3}$

c



Using Pythagoras' theorem

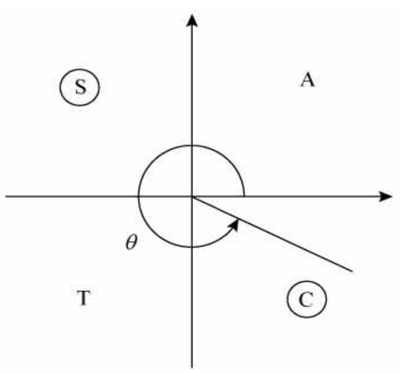
$x^2 + 7^2 = 25^2$

$x^2 = 25^2 - 7^2$

$= 576$

$x = 24$

So $\cos\phi = \frac{24}{25}$ and $\tan\phi = \frac{7}{24}$



As θ is in the fourth quadrant

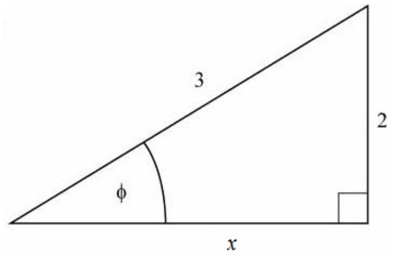
$\cos\theta = +\cos\phi$

$= \frac{24}{25}$

and $\tan\theta = -\tan\phi$

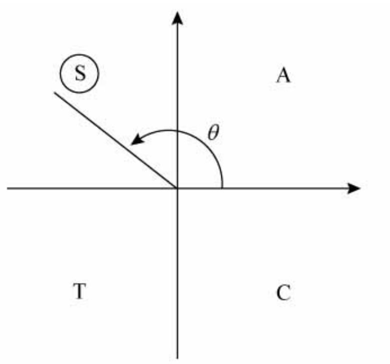
$= -\frac{7}{24}$

7 Consider the angle ϕ where $\sin\phi = \frac{2}{3}$.



Using Pythagoras' theorem, $x = \sqrt{5}$

a So $\cos\phi = \frac{\sqrt{5}}{3}$



As θ is obtuse, $\cos\theta = -\cos\phi = -\frac{\sqrt{5}}{3}$

b From the triangle

$\tan\phi = \frac{2}{\sqrt{5}}$

$= \frac{2\sqrt{5}}{5}$

Using the quadrant diagram

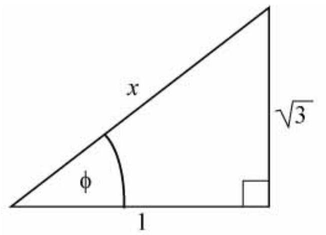
$\tan\theta = -\tan\phi$

$= -\frac{2\sqrt{5}}{5}$

8 **a** Draw a right-angled triangle with

$\tan\phi = +\sqrt{3}$

$= \dfrac{\sqrt{3}}{1}$

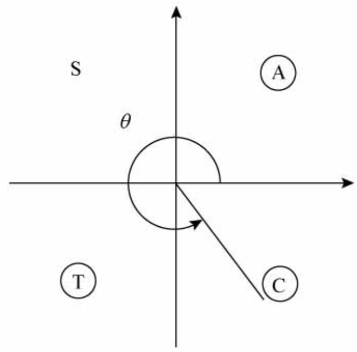


Using Pythagoras' theorem

$x^2 = \left(\sqrt{3}\right)^2 + 1^2 = 4$

So $x = 2$

$\sin\phi = \dfrac{\sqrt{3}}{2}$



As θ is reflex and $\tan\phi$ is –ve, ϕ is in the fourth quadrant.

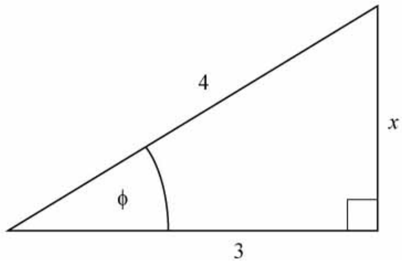
So $\sin\theta = -\sin\phi$

$= \dfrac{-\sqrt{3}}{2}$

b $\cos\phi = \frac{1}{2}$

As $\cos\theta = \cos\phi$, $\cos\theta = \frac{1}{2}$

9 Draw a right-angled triangle with $\cos\phi = \frac{3}{4}$.



Using Pythagoras' theorem

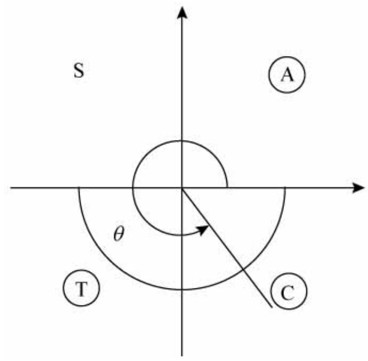
$x^2 + 3^2 = 4^2$

$x^2 = 4^2 - 3^2$

$= 7$

$x = \sqrt{7}$

So $\sin\phi = \dfrac{\sqrt{7}}{4}$ and $\tan\phi = \dfrac{\sqrt{7}}{3}$



As θ is reflex and $\cos\theta$ is +ve, θ is in the fourth quadrant.

a $\sin\theta = -\sin\phi$

$= -\dfrac{\sqrt{7}}{4}$

b $\tan\theta = -\tan\phi$

$= -\dfrac{\sqrt{7}}{3}$

10 a As $\sin^2\theta + \cos^2\theta \equiv 1$

$x^2 + y^2 = 1$

b $\sin\theta = x$ and $\cos\theta = \frac{y}{2}$

So, using $\sin^2\theta + \cos^2\theta \equiv 1$

$x^2 + \left(\frac{y}{2}\right)^2 = 1$

or $x^2 + \frac{y^2}{4} = 1$

or $4x^2 + y^2 = 4$

c As $\sin\theta = x$

$\sin^2\theta = x^2$

Using $\sin^2\theta + \cos^2\theta \equiv 1$

$x^2 + y = 1$

d As $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cos\theta = \frac{\sin\theta}{\tan\theta}$

So $\cos\theta = \frac{x}{y}$

Using $\sin^2\theta + \cos^2\theta \equiv 1$

$x^2 + \frac{x^2}{y^2} = 1$ or $x^2y^2 + x^2 = y^2$

e $\sin\theta + \cos\theta = x$

$-\sin\theta + \cos\theta = y$

Adding the two equations:

$2\cos\theta = x + y$

So $\cos\theta = \frac{x+y}{2}$

Subtracting the two equations:

$2\sin\theta = x - y$

So $\sin\theta = \frac{x-y}{2}$

Using $\sin^2\theta + \cos^2\theta \equiv 1$

$$\left(\frac{x-y}{2}\right)^2 + \left(\frac{x+y}{2}\right)^2 = 1$$

$$x^2 - 2xy + y^2 + x^2 + 2xy + y^2 = 4$$

$$2x^2 + 2y^2 = 4$$

$$x^2 + y^2 = 2$$

11 a Using the cosine rule

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \frac{8^2 + 12^2 - 10^2}{2 \times 8 \times 12}$

$\cos B = \frac{64 + 144 - 100}{192}$

$\cos B = \frac{108}{192}$

$\cos B = \frac{9}{16}$

b Since $\sin^2\theta + \cos^2\theta = 1$

$\sin^2 B + \left(\frac{9}{16}\right)^2 = 1$

$\sin^2 B = 1 - \frac{81}{256}$

$= \frac{175}{256}$

So $\sin B = \sqrt{\frac{175}{256}}$

$= \frac{5\sqrt{7}}{16}$

12 a Using the sine rule

$\frac{\sin Q}{q} = \frac{\sin P}{p}$

$\frac{\sin Q}{8} = \frac{\sin 30°}{6}$

$\sin Q = \frac{8 \sin 30°}{6}$

$= \frac{8 \times \frac{1}{2}}{6}$

$= \frac{2}{3}$

b Since $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{2}{3}\right)^2 + \cos^2 Q = 1$

$\cos^2 Q = 1 - \frac{4}{9}$

$= \frac{5}{9}$

Since Q is obtuse Q is in the second quadrant where cosine is negative.

So $\cos Q = -\frac{\sqrt{5}}{3}$