Exercise 5D

1 a $1+2+4+8+\ldots$ (8 terms)

In this series $a=1, r=2, n=8$.

As $|r|>1$ use $S_n=\dfrac{a(r^n-1)}{r-1}$.

$S_8=\dfrac{a(r^8-1)}{r-1}=\dfrac{1\times(2^8-1)}{2-1}$

$=256-1=255$

b $32+16+8+\ldots$ (10 terms)

In this series $a=32, r=\dfrac{1}{2}, n=10$.

As $|r|<1$ use $S_n=\dfrac{a(1-r^n)}{1-r}$.

$S_{10}=\dfrac{a(1-r^{10})}{1-r}$

$=\dfrac{32\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}}=63.938$ (3 d.p.)

c $a=\dfrac{2}{3}, r=\dfrac{2}{5}, n=8$

$S_8=\dfrac{\frac{2}{3}\left(1-\left(\frac{2}{5}\right)^8\right)}{1-\frac{2}{5}}$

$=1.110$

d $4-12+36-108+\ldots$ (6 terms)

In this series $a=4, r=-3, n=6$.

As $|r|>1$ use $S_n=\dfrac{a(r^n-1)}{r-1}$.

$S_6=\dfrac{a(r^6-1)}{r-1}=\dfrac{4\left((-3)^6-1\right)}{-3-1}=-728$

1 e $729-243+81-\ \ldots\ -\dfrac{1}{3}$

Here, $a=729, r=\dfrac{-243}{729}=-\dfrac{1}{3}$

and the nth term is $-\dfrac{1}{3}$.

Using nth term $=ar^{n-1}$

$-\dfrac{1}{3}=729\times\left(-\dfrac{1}{3}\right)^{n-1}$

$-\dfrac{1}{2187}=\left(-\dfrac{1}{3}\right)^{n-1}$

$\left(-\dfrac{1}{3}\right)^7=\left(-\dfrac{1}{3}\right)^{n-1}$

So $n-1=7$

$\Rightarrow n=8$

There are 8 terms in the series.

As $|r|<1$ use $S_n=\dfrac{a(1-r^n)}{1-r}$ with

$a=729, r=-\dfrac{1}{3}$ and $n=8$.

$S_8=\dfrac{729\left(1-\left(-\frac{1}{3}\right)^8\right)}{1-\left(-\frac{1}{3}\right)}=546\dfrac{2}{3}$

f $a=-\dfrac{5}{2}, r=-\dfrac{1}{2}, n=15$

$S_{15}=\dfrac{-\frac{5}{2}\left(1-\left(-\frac{1}{2}\right)^{15}\right)}{1+\frac{1}{2}}$

$=-1.67$

2 $a=3, r=0.4, n=10$

$S_{10}=\dfrac{3\left(1-0.4^{10}\right)}{1-0.4}$

$=4.9995$

3 $a = 5, r = \frac{2}{3}, n = 8$

$$S_8 = \frac{5\left(1-\left(\frac{2}{3}\right)^8\right)}{1-\frac{2}{3}}$$

$= 14.4147$

4 Let the common ratio be r.
The first three terms are 8, $8r$ and $8r^2$.
Given that the first three terms add up to 30.5,

$$8+8r+8r^2 = 30.5 \quad (\times 2)$$
$$16+16r+16r^2 = 61$$
$$16r^2+16r-45 = 0$$
$$(4r-5)(4r+9) = 0$$
$$r = \frac{5}{4}, \frac{-9}{4}$$

Possible values of r are $\frac{5}{4}$ and $\frac{-9}{4}$.

5 3 + 6 + 12 + 24 + … is a geometric series with $a = 3$, $r = 2$.

So $S_n = \frac{a(r^n-1)}{r-1} = \frac{3(2^n-1)}{2-1} = 3(2^n-1)$

We want $S_n > 1.5\,\text{million}$

$$S_n > 1500000$$
$$3(2^n-1) > 1500000$$
$$2^n-1 > 500000$$
$$2^n > 500001$$
$$\log 2^n > \log 500001$$
$$n \log 2 > \log 500001$$
$$n > \frac{\log 500001}{\log 2}$$
$$n > 18.9$$

Least value of n is 19.

6 5 + 4.5 + 4.05 + … is a geometric series with $a = 5$ and $r = \frac{4.5}{5} = 0.9$.

Using $S_n = \frac{a(1-r^n)}{1-r} = \frac{5(1-0.9^n)}{1-0.9}$

$= 50(1-0.9^n)$

We want $S_n > 45$

$$50(1-0.9^n) > 45$$
$$(1-0.9^n) > \frac{45}{50}$$
$$1-0.9^n > 0.9$$
$$0.9^n < 0.1$$

$$\log(0.9)^n < \log(0.1)$$
$$n\log(0.9) < \log(0.1)$$
$$n > \frac{\log(0.1)}{\log(0.9)}$$
$$n > 21.85$$

So $n = 22$

7 a $a = 25, r = \frac{3}{5}, S_k > 61$

$$\frac{25\left(1-\left(\frac{3}{5}\right)^k\right)}{1-\frac{3}{5}} > 61$$
$$\frac{25(1-0.6^k)}{0.4} > 61$$

$25(1 - 0.6^k) > 24.4$
$1 - 0.6^k > 0.976$
$0.6^k > 0.024$
$k \log(0.6) > \log(0.024)$

$$k > \frac{\log(0.024)}{\log(0.6)}$$

b $k > 7.301$
$k = 8$

8 $S_2 = \dfrac{a(1-r^2)}{1-r} = 4.48$

$a(1-r^2) = 4.48(1-r)$

$a = \dfrac{4.48(1-r)}{1-r^2}$

$S_4 = \dfrac{a(1-r^4)}{1-r} = 5.1968$

$a(1-r^4) = 5.1968(1-r)$

$a = \dfrac{5.1968(1-r)}{1-r^4}$

$\dfrac{4.48(1-r)}{1-r^2} = \dfrac{5.1968(1-r)}{1-r^4}$

$\dfrac{1}{1-r^2} = \dfrac{1.16}{1-r^4}$

$\dfrac{1}{1-r^2} = \dfrac{1.16}{(1-r^2)(1+r^2)}$

$1 = \dfrac{1.16}{(1+r^2)}$

$1+r^2 = 1.16$

$r^2 = 0.16$

$r = \pm 0.4$

9 $a = a,\ r = \sqrt{3}$

$S_{10} = \dfrac{a\left(\sqrt{3}^{10}-1\right)}{\sqrt{3}-1}$

$= \dfrac{a(243-1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \dfrac{242a(\sqrt{3}+1)}{3-1}$

$= 121a(\sqrt{3}+1)$

10 First series:

$a = a,\ r = 2$

$S_4 = \dfrac{a(2^4-1)}{2-1}$

$S_4 = 15a$

Second series:

$a = b,\ r = 3$

$S_4 = \dfrac{b(3^4-1)}{3-1}$

$S_4 = 40b$

$15a = 40b$

$a = \dfrac{8}{3}b$

11 a $\dfrac{2k+5}{k} = \dfrac{k}{k-6}$

$(2k+5)(k-6) = k^2$

$2k^2 + 7k - 30 = k^2$

$k^2 + 7k - 30 = 0$

b $(k+3)(k-10) = 0$

$k = -3$ or $k = 10$

As $k > 0$, $k = 10$

c $r = \dfrac{10}{10-6} = \dfrac{5}{2} = 2.5$

d $S_{10} = \dfrac{4(2.5^{10}-1)}{2.5-1}$

$= 25\,429$