

Exercise 5B

1 a $3 + 7 + 11 + 14 + \dots$ (for 20 terms)

Substitute $a = 3$, $d = 4$, $n = 20$ into

$$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{20}{2}(6 + 19 \times 4) \\ = 10 \times 82 = 820$$

b $2 + 6 + 10 + 14 + \dots$ (for 15 terms)

Substitute $a = 2$, $d = 4$, $n = 15$ into

$$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{15}{2}(4 + 14 \times 4) \\ = \frac{15}{2} \times 60 = 450$$

c $30 + 27 + 24 + 21 + \dots$ (40 terms)

Substitute $a = 30$, $d = -3$, $n = 40$ into

$$S_n = \frac{n}{2}(2a + (n-1)d) \\ = \frac{40}{2}(60 + 39 \times (-3)) \\ = 20 \times (-57) = -1140$$

d $5 + 1 + -3 + -7 + \dots$ (14 terms)

Substitute $a = 5$, $d = -4$, $n = 14$ into

$$S_n = \frac{n}{2}(2a + (n-1)d) \\ = \frac{14}{2}(10 + 13 \times (-4)) \\ = 7 \times (-42) = -294$$

1 e $5 + 7 + 9 + \dots + 75$

Here, $a = 5$, $d = 2$ and $L = 75$.

Use $L = a + (n-1)d$ to find n :

$$75 = 5 + (n-1) \times 2$$

$$70 = (n-1) \times 2$$

$$35 = n-1$$

$$n = 36 \text{ (36 terms)}$$

Substitute $a = 5$, $d = 2$, $n = 36$ and $L = 75$ into

$$S_n = \frac{n}{2}(a + L) = \frac{36}{2}(5 + 75) \\ = 18 \times 80 = 1440$$

f $4 + 7 + 10 + \dots + 91$

Here, $a = 4$, $d = 3$ and $L = 91$.

Use $L = a + (n-1)d$ to find n :

$$91 = 4 + (n-1) \times 3$$

$$87 = (n-1) \times 3$$

$$29 = (n-1)$$

$$n = 30 \text{ (30 terms)}$$

Substitute $a = 4$, $d = 3$, $L = 91$ and $n = 30$ into

$$S_n = \frac{n}{2}(a + L) = \frac{30}{2}(4 + 91) \\ = 15 \times 95 = 1425$$

g $34 + 29 + 24 + 19 + \dots + -111$

Here, $a = 34$, $d = -5$ and $L = -111$.

Use $L = a + (n-1)d$ to find n :

$$-111 = 34 + (n-1) \times (-5)$$

$$-145 = (n-1) \times (-5)$$

$$29 = (n-1)$$

$$n = 30 \text{ (30 terms)}$$

Substitute $a = 34$, $d = -5$, $L = -111$ and $n = 30$ into

$$S_n = \frac{n}{2}(a + L) = \frac{30}{2}(34 + (-111)) \\ = 15 \times (-77) = -1155$$

1 h $(x + 1) + (2x + 1) + (3x + 1) + \dots$
 $+ (21x + 1)$

Here, $a = x + 1$, $d = x$ and
 $L = 21x + 1$.

Use $L = a + (n - 1)d$ to find n :

$$21x + 1 = x + 1 + (n - 1) \times x$$

$$20x = (n - 1) \times x$$

$$20 = (n - 1)$$

$$n = 21 \text{ (21 terms)}$$

Substitute $a = x + 1$, $d = x$,
 $L = 21x + 1$ and $n = 21$ into

$$S_n = \frac{n}{2}(a + L) = \frac{21}{2}(x + 1 + 21x + 1)$$

$$= \frac{21}{2} \times (22x + 2) = 21(11x + 1)$$

2 a $5 + 8 + 11 + 14 + \dots = 670$
 Substitute $a = 5$, $d = 3$, $S_n = 670$ into

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$670 = \frac{n}{2}(10 + (n - 1) \times 3)$$

$$670 = \frac{n}{2}(3n + 7)$$

$$1340 = n(3n + 7)$$

$$0 = 3n^2 + 7n - 1340$$

$$0 = (n - 20)(3n + 67)$$

$$n = 20 \text{ or } -\frac{67}{3}$$

Number of terms is 20.

2 b $3 + 8 + 13 + 18 + \dots = 1575$
 Substitute $a = 3$, $d = 5$, $S_n = 1575$ into

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$1575 = \frac{n}{2}(6 + (n - 1) \times 5)$$

$$1575 = \frac{n}{2}(5n + 1)$$

$$3150 = n(5n + 1)$$

$$0 = 5n^2 + n - 3150$$

$$0 = (5n + 126)(n - 25)$$

$$n = -\frac{126}{5}, 25$$

Number of terms is 25.

c $64 + 62 + 60 + \dots = 0$
 Substitute $a = 64$, $d = -2$ and
 $S_n = 0$ into

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$0 = \frac{n}{2}(128 + (n - 1) \times (-2))$$

$$0 = \frac{n}{2}(130 - 2n)$$

$$0 = n(65 - n)$$

$$n = 0 \text{ or } 65$$

Number of terms is 65.

d $34 + 30 + 26 + 22 + \dots = 112$
 Substitute $a = 34$, $d = -4$ and
 $S_n = 112$ into

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$112 = \frac{n}{2}(68 + (n - 1) \times (-4))$$

$$112 = \frac{n}{2}(72 - 4n)$$

$$112 = n(36 - 2n)$$

$$2n^2 - 36n + 112 = 0$$

$$n^2 - 18n + 56 = 0$$

$$(n - 4)(n - 14) = 0$$

$$n = 4 \text{ or } 14$$

Number of terms is 4 or 14

$$3 \quad S = \underbrace{2+4+6+8+\dots}_{50 \text{ terms}}$$

This is an arithmetic series with $a = 2$, $d = 2$ and $n = 50$.

$$\text{Use } S_n = \frac{n}{2}(2a + (n-1)d)$$

$$\begin{aligned} \text{So } S &= \frac{50}{2}(4 + 49 \times 2) \\ &= 25 \times 102 = 2550 \end{aligned}$$

$$4 \quad 7 + 12 + 17 + 22 + 27 + \dots > 1000$$

$$\text{Using } S_n = \frac{n}{2}(2a + (n-1)d)$$

$$1000 = \frac{n}{2}(2 \times 7 + (n-1)5)$$

$$2000 = n(14 + 5n - 5)$$

$$2000 = n(5n + 9)$$

$$5n^2 + 9n - 2000 = 0$$

$$n = \frac{-9 \pm \sqrt{9^2 - 4 \times 5 \times (-2000)}}{2 \times 5}$$

$$n = \frac{-9 \pm \sqrt{40081}}{10}$$

$$n = 19.12\dots \text{ or } n = -20.92\dots$$

So 20 terms are needed.

- 5 Let common difference = d .
Substitute $a = 4$, $n = 20$, and $S_{20} = -15$ into

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$-15 = \frac{20}{2}(8 + (20-1)d)$$

$$-15 = 10(8 + 19d)$$

$$-1.5 = 8 + 19d$$

$$19d = -9.5$$

$$d = -0.5$$

The common difference is -0.5 .

Use n th term = $a + (n-1)d$ to find

$$\begin{aligned} \text{20th term} &= a + 19d \\ &= 4 + 19 \times (-0.5) \\ &= 4 - 9.5 = -5.5 \end{aligned}$$

20th term is -5.5 .

- 6 Let the first term be a and the common difference d .

Sum of first three terms is 12, so

$$\begin{aligned} a + (a + d) + (a + 2d) &= 12 \\ 3a + 3d &= 12 \\ a + d &= 4 \end{aligned} \quad (1)$$

20th term is -32 , so

$$a + 19d = -32 \quad (2)$$

Equation (2) – Equation (1):

$$18d = -36$$

$$d = -2$$

Substitute $d = -2$ into Equation (1):

$$a + (-2) = 4$$

$$a = 6$$

Therefore, first term is 6 and common difference is -2 .

$$\begin{aligned} 7 \quad S_{50} &= 1 + 2 + 3 + \dots + 48 + 49 + 50 \quad (1) \\ S_{50} &= 50 + 49 + 48 + \dots + 3 + 2 + 1 \quad (2) \end{aligned}$$

Adding (1) and (2):

$$2 \times S_{50} = 50 \times 51$$

$$\begin{aligned} S_{50} &= \frac{50 \times 51}{2} \\ &= 1275 \end{aligned}$$

$$8 \quad \text{Sum required} = \underbrace{1 + 2 + 3 + \dots + 2n}$$

Arithmetic series with $a = 1$, $d = 1$ and $n = 2n$.

$$\begin{aligned} \text{Use } S_n &= \frac{n}{2}(2a + (n-1)d) \\ &= \frac{2n}{2}(2 \times 1 + (2n-1) \times 1) \\ &= \frac{2n}{2}(2n+1) \\ &= n(2n+1) \end{aligned}$$

$$9 \quad \text{Required sum} = \underbrace{1 + 3 + 5 + 7 + \dots}_{n \text{ terms}}$$

This is an arithmetic series with $a = 1$, $d = 2$ and $n = n$.

$$\begin{aligned} \text{Use } S_n &= \frac{n}{2}(2a + (n-1)d) \\ &= \frac{n}{2}(2 \times 1 + (n-1) \times 2) \\ &= \frac{n}{2}(2 + 2n - 2) \\ &= \frac{n \times 2n}{2} \\ &= n \times n \\ &= n^2 \end{aligned}$$

$$\begin{aligned} 10 \text{ a } \quad u_5 &= 33, \text{ so } a + 4d = 33 \quad (1) \\ u_{10} &= 68, \text{ so } a + 9d = 68 \quad (2) \\ (2) - (1) &\text{ gives:} \\ 5d &= 35 \\ d &= 7 \\ a &= 5 \end{aligned}$$

$$\begin{aligned} 2225 &= \frac{n}{2}(2 \times 5 + (n-1)7) \\ 4450 &= n(7n + 3) \\ 7n^2 + 3n - 4450 &= 0 \end{aligned}$$

b

$$\begin{aligned} n &= \frac{-3 \pm \sqrt{3^2 - 4 \times 7 \times (-4450)}}{2 \times 7} \\ n &= \frac{-3 \pm \sqrt{124\,609}}{14} \\ n &= \frac{-3 \pm 353}{14} \\ n &= 25 \text{ or } -25.42 \\ \text{So } n &= 25 \end{aligned}$$

$$\begin{aligned} 11 \text{ a } \quad u_n &= a + (n-1)d \\ 303 &= k + 1 + (n-1)(k+2) \\ 303 &= k + 1 + nk + 2n - k - 2 \\ 303 &= nk + 2n - 1 \\ 304 &= n(k+2) \\ n &= \frac{304}{k+2} \end{aligned}$$

$$11 \text{ b } \quad S_n = \frac{\left(\frac{304}{k+2}\right)}{2}(k+1+303)$$

$$S_n = \frac{152}{k+2}(k+304)$$

$$S_n = \frac{152k + 46\,208}{k+2}$$

$$\begin{aligned} \text{c } \quad 2568 &= \frac{152k + 46\,208}{k+2} \\ 2568(k+2) &= 152k + 46\,208 \\ 2416k &= 41\,072 \\ k &= 17 \end{aligned}$$

$$\begin{aligned} 12 \text{ a } \quad S_n &= \frac{33}{2}(3+99) \\ &= 1683 \end{aligned}$$

$$\begin{aligned} \text{b i } \quad 4p + (n-1)4p &= 400 \\ 4pn &= 400 \\ n &= \frac{100}{p} \end{aligned}$$

$$\text{ii } \quad S_n = \frac{\left(\frac{100}{p}\right)}{2}(4p+400)$$

$$S_n = \frac{50}{p}(4p+400)$$

$$S_n = 200 + \frac{20\,000}{p}$$

$$\begin{aligned} \text{c } \quad u_{80} &= 3p + 2 + (80-1)(2p+1) \\ &= 3p + 2 + 158p + 79 \\ &= 161p + 81 \end{aligned}$$

$$\begin{aligned} 13 \text{ a } \quad u_n &= a + (n-1)d \\ &= 6 + (n-1)5 \\ &= 6 + 5n - 5 \\ &= 5n + 1 \end{aligned}$$

$$\begin{aligned} \text{b } \quad u_{10} &= 5 \times 10 + 1 = 51 \\ S_{10} &= \frac{10}{2}(6+51) \\ &= 5 \times 57 \\ &= 285 \end{aligned}$$

$$\begin{aligned}
 13 \text{ c } S_k &= \frac{k}{2}(2 \times 6 + (k-1)5) \\
 &= \frac{k}{2}(12 + 5k - 5) \\
 &= \frac{k}{2}(5k + 7)
 \end{aligned}$$

$$\frac{k}{2}(5k + 7) \leq 1029$$

$$5k^2 + 7k \leq 2058$$

$$5k^2 + 7k - 2058 \leq 0$$

$$(5k - 98)(k + 21) \leq 0$$

- d** As $k > 0$, $5k - 98 = 0$, $k = 19.6$
So $k = 19$

Challenge

$$S_n = 3n(n+2) \text{ and } 4T_{n+4} + 16 = S_{n-4}$$

The sum of the first n terms of an arithmetic

sequence is given by $S_n = \frac{n}{2}(2a + (n-1)d)$, so

$$\frac{n}{2}(2a + (n-1)d) = 3n(n+2)$$

$$2a + (n-1)d = 6(n+2)$$

When $n = 1$

$$2a + (1-1)d = 6(1+2)$$

$$a = 9$$

when $n = 2$

$$2(9) + (2-1)d = 6(2+2)$$

$$18 + d = 24$$

$$d = 6$$

So the n^{th} term of the sequence is given by

$$T_n = 2 + 6n$$

$$\text{So } T_{n+4} = 2 + 6(n+4) = 26 + 6n$$

and since

$$4T_{n+4} + 16 = S_{n-4}$$

$$4(26 + 6n) + 16 = S_{n-4}$$

$$120 + 24n = S_{n-4}$$

$$S_n = 3n(n+2)$$

so

$$S_{n-4} = 3(n-4)(n-4+2)$$

$$= 3(n-4)(n-2)$$

$$= 3(n^2 - 6n + 8)$$

$$120 + 24n = 3n^2 - 18n + 24$$

$$3n^2 - 42n - 96 = 0$$

$$n^2 - 14n - 32 = 0$$

$$(n+2)(n-16) = 0$$

$$n = -2 \text{ or } n = 16$$

Since n cannot be negative, $n = 16$.