Exercise 4B

1 a $4! = 4 \times 3 \times 2 \times 1$
$= 24$

b $9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 362\,880$

c $\dfrac{10!}{7!} = \dfrac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= \dfrac{3\,628\,800}{5040}$ or $10 \times 9 \times 8$ since the 7! on numerator and denominator cancel

$= 720$

d $\dfrac{15!}{13!} = \dfrac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= \dfrac{1\,307\,674\,368\,000}{6\,227\,020\,800}$ or 15×14 because the 13! cancels

$= 210$

2 a $\dbinom{4}{2} = \dfrac{4!}{2!2!}$

$= \dfrac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1}$

$= \dfrac{4 \times 3}{2 \times 1}$

$= 6$

b $\dbinom{6}{4} = \dfrac{6!}{4!2!}$

$= \dfrac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 2 \times 1}$

$= \dfrac{6 \times 5}{2 \times 1}$

$= 15$

c ${}^6C_3 = \dfrac{6!}{3!3!}$

$= \dfrac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 3 \times 2 \times 1}$

$= \dfrac{6 \times 5 \times 4}{3 \times 2 \times 1}$

$= 20$

2 d $\binom{5}{4} = \frac{5!}{4!1!}$
$= \frac{5\times4\times3\times2\times1}{4\times3\times2\times1\times1}$
$= \frac{5}{1}$
$= 5$

e ${}^{10}C_8 = \frac{10!}{8!2!}$
$= \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{8\times7\times6\times5\times4\times3\times2\times1\times2\times1}$
$= \frac{10\times9}{2\times1}$
$= 45$

f $\binom{9}{5} = \frac{9!}{5!4!}$
$= \frac{9\times8\times7\times6\times5\times4\times3\times2\times1}{5\times4\times3\times2\times1\times4\times3\times2\times1}$
$= \frac{9\times8\times7\times6}{4\times3\times2\times1}$
$= 126$

3 a $\binom{15}{6} = 5005$

b ${}^{10}C_7 = 120$

c $\binom{20}{10} = 184\,756$

d $\binom{20}{17} = 1140$

e ${}^{14}C_9 = 2002$

f ${}^{18}C_5 = 8568$

4 The rth entry in the nth row of Pascal's triangle is given by ${}^{n-1}C_{r-1}$.

a ${}^{5-1}C_{2-1} = {}^{4}C_1$

b ${}^{6-1}C_{3-1} = {}^{5}C_2$

4 c $= {}^{6}C_2$

d ${}^{7-1}C_{4-1} = {}^{6}C_3$

5 5th number on the 12th row = ${}^{12-1}C_{5-1} = {}^{11}C_4 = 330$

6 a ${}^{11-1}C_{4-1} = {}^{10}C_3 = 120$
${}^{11-1}C_{5-1} = {}^{10}C_4 = 210$

b The coefficients are 1, 10, 45, 120, 210, ...
The term in x^3 is $120(1)^7(2x)^3 = 960x^3$.
Coefficient = 960

7 a ${}^{14-1}C_{4-1} = {}^{13}C_3 = 286$
${}^{14-1}C_{5-1} = {}^{13}C_4 = 715$

b The coefficients are 1, 13, 78, 286, 715, ...
The term in x^4 is $715(1)^9(3x)^4 = 57\,915x^4$.
Coefficient = 57 915

8 $\binom{20}{10}0.5^{20} = {}^{20}C_{10}0.5^{20}$
$= 184\,756 \times 0.5^{20}$
$= 0.1762$ (to 4 s.f.)
Whilst this seems a low probability, there is more chance of the coin landing on 10 heads than any other number of heads.

9 a $${}^{n}C_1 = \frac{n!}{1!(n-1)!} = \frac{n\times(n-1)\times(n-2)\times\ldots\times2\times1}{1\times(n-1)\times(n-2)\times(n-3)\times\ldots\times2\times1} = n$$

b $${}^{n}C_2 = \frac{n!}{2!(n-2)!} = \frac{n\times(n-1)\times(n-2)\times\ldots\times2\times1}{1\times2\times(n-2)\times(n-3)\times\ldots\times2\times1} = \frac{n(n-1)}{2}$$

10 $\binom{50}{13} = \frac{50!}{13!a!}$
$\binom{50}{13} = \frac{50!}{13!37!}$
$a = 37$

11 $\binom{35}{p} = \frac{35!}{p!18!}$
$\binom{35}{17} = \frac{35!}{17!18!}$
$p = 17$

Challenge

a $^{10}C_3 = \dfrac{10!}{3!7!} = 120$

$^{10}C_7 = \dfrac{10!}{7!3!} = 120$

b $^{14}C_5 = \dfrac{14!}{5!9!} = 2002$

$^{14}C_9 = \dfrac{14!}{9!5!} = 2002$

c The two answers for part **a** are the same and the two answers for part **b** are the same.

d $^{n}C_r = \dfrac{n!}{r!(n-r)!}$ and $^{n}C_{n-r} = \dfrac{n!}{(n-r)!r!}$ because $\dfrac{n!}{(n-r)!\big(n-(n-r)\big)!}$ and $(n-(n-r)) = r$

As $\dfrac{n!}{r!(n-r)!} = \dfrac{n!}{(n-r)!r!}$, $^{n}C_r = {}^{n}C_{n-r}$