

Exercise 4A

1 a $(x + y)^3$

The $(n + 1)$ th row of Pascal's triangle gives the coefficients in the expansion of $(a + b)^n$.

$$3 + 1 = 4\text{th row}$$

b $(3x - 7)^{15}$

$$15 + 1 = 16\text{th row}$$

c $\left(2x + \frac{1}{2}\right)^n$

$$n + 1 = (n + 1)\text{th row}$$

d $(y - 2x)^{n+4}$

$$n + 4 + 1 = (n + 5)\text{th row}$$

2 a $(x + y)^4$ has coefficients and terms

$$\begin{array}{cccccc} 1 & 4 & 6 & 4 & 1 \\ x^4 & x^3y & x^2y^2 & xy^3 & y^4 \end{array}$$

$$\begin{aligned} (x + y)^4 &= 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4 \\ &= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 \end{aligned}$$

b $(p + q)^5$ has coefficients and terms

$$\begin{array}{cccccc} 1 & 5 & 10 & 10 & 5 & 1 \\ p^5 & p^4q & p^3q^2 & p^2q^3 & pq^4 & q^5 \end{array}$$

$$\begin{aligned} (p + q)^5 &= 1p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + 1q^5 \\ &= p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5 \end{aligned}$$

c $(a - b)^3$ has coefficients and terms

$$\begin{array}{cccc} 1 & 3 & 3 & 1 \\ a^3 & a^2(-b) & a(-b)^2 & (-b)^3 \end{array}$$

$$\begin{aligned} (a - b)^3 &= 1a^3 - 3a^2b + 3ab^2 - 1b^3 \\ &= a^3 - 3a^2b + 3ab^2 - b^3 \end{aligned}$$

d $(x + 4)^3$ has coefficients and terms

$$\begin{array}{cccc} 1 & 3 & 3 & 1 \\ x^3 & x^2 \cdot 4 & x \cdot 4^2 & 4^3 \end{array}$$

$$\begin{aligned} (x + 4)^3 &= 1x^3 + 3x^2(4) + 3x(4)^2 + 4^3 \\ &= x^3 + 12x^2 + 48x + 64 \end{aligned}$$

e $(2x - 3)^4$ has coefficients and terms

$$\begin{array}{cccccc} 1 & 4 & 6 & 4 & 1 \\ (2x)^4 & (2x)^3(-3) & (2x)^2(-3)^2 & (2x)(-3)^3 & (-3)^4 \end{array}$$

$$\begin{aligned} (2x - 3)^4 &= 1(2x)^4 + 4(2x)^3(-3) + 6(2x)^2(-3)^2 + 4(2x)(-3)^3 + 1(-3)^4 \\ &= 16x^4 - 96x^3 + 216x^2 - 216x + 81 \end{aligned}$$

2 f $(a + 2)^5$ has coefficients and terms

$$\begin{array}{cccccc} 1 & 5 & 10 & 10 & 5 & 1 \\ a^2 & a^4 2 & a^3 2^2 & a^2 2^3 & a 2^4 & 2^5 \end{array}$$

$$\begin{aligned} (a + 2)^5 &= 1a^5 + 5a^4(2) + 10a^3(2)^2 + 10a^2(2)^3 + 5a(2)^4 + 32 \\ &= a^5 + 10a^4 + 40a^3 + 80a^2 + 80a + 32 \end{aligned}$$

g $(3x - 4)^4$ has coefficients and terms

$$\begin{array}{cccccc} 1 & 4 & 6 & 4 & 1 \\ (3x)^4 & (3x)^3(-4) & (3x)^2(-4)^2 & (3x)(-4)^3 & (-4)^4 \end{array}$$

$$\begin{aligned} (3x - 4)^4 &= 1(3x)^4 + 4(3x)^3(-4) + 6(3x)^2(-4)^2 + 4(3x)(-4)^3 + 1(-4)^4 \\ &= 81x^4 - 432x^3 + 864x^2 - 768x + 256 \end{aligned}$$

h $(2x - 3y)^4$ has coefficients and terms

$$\begin{array}{cccccc} 1 & 4 & 6 & 4 & 1 \\ (2x)^4 & (2x)^3(-3y) & (2x)^2(-3y)^2 & (2x)(-3y)^3 & (-3y)^4 \end{array}$$

$$\begin{aligned} (2x - 3y)^4 &= 1(2x)^4 + 4(2x)^3(-3y) + 6(2x)^2(-3y)^2 + 4(2x)(-3y)^3 + 1(-3y)^4 \\ &= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4 \end{aligned}$$

3 a $(4 + x)^4$ has coefficients 1 4 6 4 1

The bold number is the coefficient of the term $4x^3$.

Term is $4 \times 4x^3 = 16x^3$.

Coefficient = 16

b $(1 - x)^5$ has coefficients 1 5 10 **10** 5 1

The bold number is the coefficient of the term $1^2(-x)^3$.

Term is $10 \times 1^2(-x)^3 = -10x^3$.

Coefficient = -10

c $(3 + 2x)^3$ has coefficients 1 3 3 **1**

The bold number is the coefficient of the term $(2x)^3$.

Term is $1 \times (2x)^3 = 8x^3$.

Coefficient = 8

d $(4 + 2x)^5$ has coefficients 1 5 10 **10** 5 1

The bold number is the coefficient of the term $4^2(2x)^3$.

Term is $10 \times 4^2(2x)^3 = 1280x^3$.

Coefficient = 1280

e $(2 + x)^6$ has coefficients 1 6 15 **20** 15 6 1

The bold number is the coefficient of the term 2^3x^3 .

Term is $20 \times 2^3x^3 = 160x^3$.

Coefficient = 160

f $\left(4 - \frac{1}{2}x\right)^4$ has coefficients 1 4 6 4 1

The bold number is the coefficient of the term $4\left(-\frac{1}{2}x\right)^3$

3 f Term is $4 \times 4 \left(-\frac{1}{2}x\right)^3 = -2x^3$

Coefficient = -2

g $(x + 2)^5$ has coefficients 1 5 **10** 10 5 1
 The bold number is the coefficient of the term $x^3 2^2$.
 Term is $10 \times x^3 2^2 = 40x^3$.
 Coefficient = 40

h $(3 - 2x)^4$ has coefficients 1 4 **6** 4 1
 The bold number is the coefficient of the term $3(-2x)^3$.
 Term is $4 \times 3(-2x)^3 = -96x^3$.
 Coefficient = -96

4 $(1 + 2x)^3$ has coefficients and terms

$$\begin{array}{cccc} 1 & 3 & 3 & 1 \\ 1^3 & 1^2(2x) & 1(2x)^2 & (2x)^3 \end{array}$$

Hence $(1 + 2x)^3 = 1 + 6x + 12x^2 + 8x^3$

$$\begin{aligned} & (1 + 3x)(1 + 2x)^3 \\ &= (1 + 3x)(1 + 6x + 12x^2 + 8x^3) \\ &= 1(1 + 6x + 12x^2 + 8x^3) + 3x(1 + 6x + 12x^2 + 8x^3) \\ &= 1 + 6x + 12x^2 + 8x^3 + 3x + 18x^2 + 36x^3 + 24x^4 \\ &= 1 + 9x + 30x^2 + 44x^3 + 24x^4 \end{aligned}$$

5 $(2 + y)^3$ has coefficients and terms

$$\begin{array}{cccc} 1 & 3 & 3 & 1 \\ 2^3 & 2^2y & 2y^2 & y^3 \end{array}$$

Hence $(2 + y)^3 = 8 + 12y + 6y^2 + y^3$

Substitute $y = x - x^2$
 $(2 + x - x^2)^3 = 8 + 12(x - x^2) + 6(x - x^2)^2 + (x - x^2)^3$
 $= 8 + 12x(1 - x) + 6x^2(1 - x^2)^2 + x^3(1 - x)^3$

Now

$$\begin{aligned} (1 - x)^3 &= (1 - x)(1 - x)^2 \\ (1 - x)^3 &= (1 - x)(1 - 2x + x^2) \\ (1 - x)^3 &= 1 - 2x + x^2 - x + 2x^2 - x^3 \\ (1 - x)^3 &= 1 - 3x + 3x^2 - x^3 \end{aligned}$$

Or, using Pascal's triangle

$$\begin{aligned} (1 - x)^3 &= 1(1)^3 + 3(1)^2(-x) + 3(1)(-x)^2 + 1(-x)^3 \\ (1 - x)^3 &= 1 - 3x + 3x^2 - x^3 \end{aligned}$$

$$\begin{aligned} \text{So } (2 + x - x^2)^3 &= 8 + 12x(1 - x) + 6x^2(1 - 2x + x^2) + x^3(1 - 3x + 3x^2 - x^3) \\ &= 8 + 12x - 12x^2 + 6x^2 - 12x^3 + 6x^4 + x^3 - 3x^4 + 3x^5 - x^6 \\ &= 8 + 12x - 6x^2 - 11x^3 + 3x^4 + 3x^5 - x^6 \end{aligned}$$

- 6 $(2 + ax)^3$ has coefficients 1 **3** 3 1
 The bold number is the coefficient of the term $2(ax)^2$
 Term in x^2 is $3 \times 2(ax)^2 = 6a^2x^2$
 Coefficient of x^2 is $6a^2$

$$\text{So } 6a^2 = 54$$

$$a^2 = 9$$

$$a = \pm 3$$

- 7 $(3 + bx)^3$ has coefficients and terms

$$\begin{array}{cccc} 1 & 3 & 3 & 1 \\ 3^3 & 3^2(bx) & 3(bx)^2 & (bx)^3 \end{array}$$

$$\begin{aligned} (3 + bx)^3 &= 1 \times 3^3 + 3 \times 3^2bx + 3 \times 3(bx)^2 + 1 \times (bx)^3 \\ &= 27 + 27bx + 9b^2x^2 + b^3x^3 \end{aligned}$$

$$\text{So } (2 - x)(3 + bx)^3 = (2 - x)(27 + 27bx + 9b^2x^2 + b^3x^3)$$

$$\text{Term in } x^2 \text{ is } 2 \times 9b^2x^2 - x \times 27bx = 18b^2x^2 - 27bx^2$$

$$\text{Coefficient of } x^2 \text{ is } 18b^2 - 27b$$

$$\text{So } 18b^2 - 27b = 45$$

$$2b^2 - 3b = 5$$

$$2b^2 - 3b - 5 = 0$$

$$(2b - 5)(b + 1) = 0$$

$$b = \frac{5}{2}, -1$$

- 8 The coefficients are 1, 3, **3**, 1
 The term in x^2 is $3(p)(-2x)^2 = 12px^2$
 So the coefficient of the term x^2 is $12p$

9 $500 \left(1 + \frac{X}{100} \right)^5$

The coefficients are 1, 5, 10, 10, 5, 1

$$500 \left(1 + \frac{X}{100} \right)^5 = 500 \left(1(1)^5 + 5(1)^4 \left(\frac{X}{100} \right) + 10(1)^3 \left(\frac{X}{100} \right)^2 + \dots \right)$$

$$= 500 \left(1 + \frac{X}{20} + \frac{X^2}{1000} + \dots \right)$$

$$\approx 500 + 25X + \frac{X^2}{2}$$

$$A = 500, B = 25, C = \frac{1}{2}$$

Challenge

$$\left(x^2 - \frac{1}{2x} \right)^3$$

Coefficients are 1 **3** 3 1

$$\text{Third term is } 3(x^2)^1 \left(-\frac{1}{2x} \right)^2 = \frac{3x^2}{4x^2} = \frac{3}{4}$$