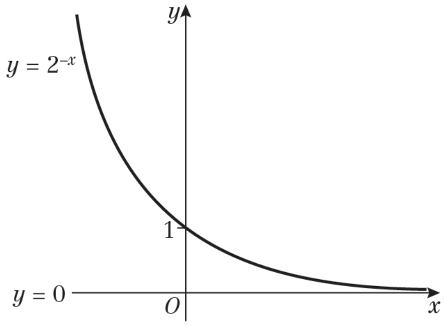
Chapter review 3

1 $y = 2^{-x} = (2^{-1})^x = (\frac{1}{2})^x$



2 a $\log_a(p^2q) = \log_a(p^2) + \log_a q$
$= 2\log_a p + \log_a q$

b $\log_a(pq) = \log_a p + \log_a q$
So
$\log_a p + \log_a q = 5$ **(1)**
$2\log_a p + \log_a q = 9$ **(2)**
Subtract **(1)** from **(2)**:
$\log_a p = 4$
So $\log_a q = 1$

3 a $p = \log_q 16$
$= \log_q(2^4)$
$= 4\log_q 2$
$\log_q 2 = \frac{p}{4}$

b $\log_q(8q) = \log_q 8 + \log_q q$
$= \log_q(2^3) + \log_q q$
$= 3\log_q 2 + \log_q q$
$= 3 \times \frac{p}{4} + 1$
$= \frac{3p}{4} + 1$

4 a $4^x = 23$
$\log_4 23 = x$
$x = 2.26$

4 b $7^{(2x+1)} = 1000$
$\log_7 1000 = 2x + 1$
$2x = \log_7 1000 - 1$
$x = \frac{1}{2}\log_7 1000 - \frac{1}{2}$
$= 1.27$

c $10^x = 6^{x+2}$
$\log(10^x) = \log(6^{x+2})$
$x\log 10 = (x + 2)\log 6$
$x\log 10 - x\log 6 = 2\log 6$
$x(\log 10 - \log 6) = 2\log 6$
$x = \frac{2\log 6}{\log 10 - \log 6}$
$= 7.02$

5 a $4^x - 2^{x+1} - 15 = 0$
$2^{2x} - 2 \times 2^x - 15 = 0$
$(2^x)^2 - 2 \times 2^x - 15 = 0$
Let $u = 2^x$
$u^2 - 2u - 15 = 0$

b $(u + 3)(u - 5) = 0$
So $u = -3$ or $u = 5$
If $u = -3$, $2^x = -3$. No solution.
If $u = 5$, $2^x = 5$
$\log 2^x = \log 5$
$x\log 2 = \log 5$
$x = \frac{\log 5}{\log 2}$
$= 2.32$ (2 d.p.)

6 $\log_2(x + 10) - \log_2(x - 5) = 4$
$\log_2\left(\frac{x+10}{x-5}\right) = 4$
$\frac{x+10}{x-5} = 2^4$
$16x - 80 = x + 10$
$15x = 90$
$x = 6$

7 a $y = 3x^2$

Take logarithms of both sides and simplify.

$\log_3 y = \log_3 3x^2$

$= \log_3 3 + \log_3 x^2$

$= 1 + 2\log_3 x$

As required.

b $1 + 2\log_3 x = \log_3(28x - 9)$

$\log_3 3x^2 = \log_3(28x - 9)$

$3x^2 = 28x - 9$

$3x^2 - 28x + 9 = 0$

$(3x - 1)(x - 9) = 0$

$x = \frac{1}{3}$ or $x = 9$

8 $2\log_3 x - \log_3(x - 2) = 2$

Rearrange the equation and solve for x.

$\log_3 \frac{x^2}{x-2} = 2$

$\frac{x^2}{x-2} = 3^2$

$x^2 = 9x - 18$

$x^2 - 9x + 18 = 0$

$(x - 3)(x - 6) = 0$

$x = 3$ or $x = 6$

9 a $5^x = 10$

Take the logarithms of both sides and solve for x.

$x \log 5 = \log 10$

$x = \frac{\log 10}{\log 5}$

$= 1.43$ (3 s.f.)

b $\log_9(x - 2) = -1$

$x - 2 = 9^{-1}$

$x = \frac{1}{9} + 2$

$= \frac{19}{9}$

10 $\log_5(4 - x) - 2\log_5 x = 1$

Rearrange the equation and solve for x.

$\log_5(4 - x) - \log_5 x^2 = 1$

$\log_5 \frac{(4-x)}{x^2} = 1$

$\frac{(4-x)}{x^2} = 5^1$

$4 - x = 5x^2$

$5x^2 + x - 4 = 0$

$(5x - 4)(x + 1) = 0$

$x = \frac{4}{5}$ or $x = -1$

Since $0 < x < 4$, $x = \frac{4}{5}$.

11 a $\log_x 64 = 2$

$2^x = 64$

$x = 6$

b $\log_2(11 - 6x) = 2\log_2(x - 1) + 3$

Rearrange the equation and solve for x.

$\log_2(11 - 6x) - \log_2(x - 1)^2 = 3$

$\log_2 \frac{(11-6x)}{(x-1)^2} = 3$

$\frac{(11-6x)}{(x-1)^2} = 2^3$

$11 - 6x = 8(x - 1)^2$

$11 - 6x = 8(x^2 - 2x + 1)$

$8x^2 - 10x - 3 = 0$

$(2x - 3)(4x + 1) = 0$

$x = \frac{3}{2}$ or $x = -\frac{1}{4}$

Since $x = -\frac{1}{4}$ is not valid for the original equation, $x = \frac{3}{2}$ is the only solution.

12 a $\log_2 y = -3$

$y = 2^{-3}$

$= \frac{1}{8}$

12 b $\dfrac{\log_2 32+\log_2 16}{\log_2 x}=\log_2 x$

$\log_2 32+\log_2 16=\left(\log_2 x\right)^2$

$5+4=\left(\log_2 x\right)^2$

$\left(\log_2 x\right)^2=9$

$\log_2 x=3$ or $\log_2 x=-3$

$x=2^3=8$ or $x=2^{-3}=\dfrac{1}{8}$

13 a $2\log_3(x-5)-\log_3(2x-13)=1$

Rearrange the equation and simplify.

$\log_3(x-5)^2-\log_3(2x-13)=1$

$\log_3\dfrac{(x-5)^2}{(2x-13)}=1$

$\dfrac{(x-5)^2}{(2x-13)}=3^1$

$(x-5)^2=3(2x-13)$

$x^2-10x+25=6x-39$

$x^2-16x+64=0$

As required.

b $x^2-16x+64=0$

$\left(x-8\right)^2=0$

$x=8$

14 a $\log_2\left(2x\right)=\log_2\left(5x+4\right)-3$

Rearrange the equation and simplify.

$\log_2\left(2x\right)-\log_2\left(5x+4\right)=-3$

$\log_2\left(\dfrac{2x}{5x+4}\right)=-3$

$\dfrac{2x}{5x+4}=2^{-3}$

$\dfrac{2x}{5x+4}=\dfrac{1}{8}$

$16x=5x+4$

$11x=4$

$x=\dfrac{4}{11}$

14 b $\log_a y+3\log_a 2=5$

Rearrange the equation and simplify.

$\log_a y+\log_a 2^3=5$

$\log_a y+\log_a 8=5$

$\log_a 8y=5$

$8y=a^5$

$y=\dfrac{a^5}{8}$