**Pure Mathematics 2**  Solution Bank  <span style="float:right">P Pearson</span>

## Exercise 3E

**1 a** $\log_7 120 = \dfrac{\log_{10} 120}{\log_{10} 7}$
$= 2.460$ (3 d.p.)

**b** $\log_3 45 = \dfrac{\log_{10} 45}{\log_{10} 3}$
$= 3.465$ (3 d.p.)

**c** $\log_2 19 = \dfrac{\log_{10} 19}{\log_{10} 2}$
$= 4.248$ (3 d.p.)

**d** $\log_{11} 3 = \dfrac{\log_{10} 3}{\log_{10} 11}$
$= 0.458$ (3 d.p.)

**2 a** $8^x = 14$
Take the logarithm of both sides and simplify.
$x \log 8 = \log 14$
$x = \dfrac{\log 14}{\log 8}$
$= 1.27$ (3 s.f.)

**b** $9^x = 99$
Take the logarithm of both sides and simplify.
$x \log 9 = \log 99$
$x = \dfrac{\log 99}{\log 9}$
$= 2.09$ (3 s.f.)

**c** $12^x = 6$
Take the logarithm of both sides and simplify.
$x \log 12 = \log 6$
$x = \dfrac{\log 6}{\log 12}$
$= 0.721$ (3 s.f.)

**3 a** $\log_2 x = 8 + 9 \log_x 2$
Use the change of base rule.
$$\left( \log_a b = \dfrac{1}{\log_b a} \right)$$
$\log_2 x = 8 + \dfrac{9}{\log_2 x}$
Let $\log_2 x = y$
$y = 8 + \dfrac{9}{y}$
$y^2 = 8y + 9$
$y^2 - 8y - 9 = 0$
$(y+1)(y-9) = 0$
$y = -1$ or $y = 9$
When $y = -1$
$\log_2 x = -1$
$x = 2^{-1}$
$= \dfrac{1}{2}$
When $y = 9$
$\log_2 x = 9$
$x = 2^9$
$= 512$

**3 b** $\log_4 x + 2\log_x 4 + 3 = 0$

Use the change of base rule.

$$\left(\log_a b = \frac{1}{\log_b a}\right)$$

$$\log_4 x + \frac{2}{\log_4 x} + 3 = 0$$

Let $\log_4 x = y$

$$y + \frac{2}{y} + 3 = 0$$

$$y^2 + 3y + 2 = 0$$

$$(y+1)(y+2) = 0$$

$y = -1$ or $y - 2$

When $y = -1$

$$\log_4 x = -1$$

$$x = 4^{-1}$$

$$= \frac{1}{4}$$

When $y = -2$

$$\log_4 x = -2$$

$$x = 4^{-2}$$

$$= \frac{1}{16}$$

**3 c** $\log_2 x + \log_4 x = 2$

Use the change of base rule

$$\left(\log_a x = \frac{\log_b x}{\log_b a}\right)$$

$$\log_4 x = \frac{\log_2 x}{\log_2 4}$$

So $\log_2 x + \dfrac{\log_2 x}{\log_2 4} = 2$

and since $\log_2 4 = 2$

$$\log_2 x + \frac{1}{2}\log_2 x = 2$$

$$\frac{3}{2}\log_2 x = 2$$

$$\log_2 x = \frac{4}{3}$$

$$x = 2^{\frac{4}{3}}$$

$$= 2.52 \text{ (3 s.f.)}$$