

Exercise 3B

1 a $\log_4 256 = 4$

b $\log_3 \left(\frac{1}{9}\right) = -2$

c $\log_{10} 1\,000\,000 = 6$

d $\log_{11} 11 = 1$

e $\log_{0.2} 0.008 = 3$

2 a $2^4 = 16$

b $5^2 = 25$

c $9^{\frac{1}{2}} = 3$

d $5^{-1} = 0.2$

e $10^5 = 100\,000$

3 a If $\log_2 8 = x$ then $2^x = 8$, so $x = 3$

b If $\log_5 25 = x$ then $5^x = 25$, so $x = 2$

c If $\log_{10} 10\,000\,000 = x$
then $10^x = 10\,000\,000$, so $x = 7$

d If $\log_{12} 12 = x$ then $12^x = 12$, so $x = 1$

e If $\log_3 729 = x$ then $3^x = 729$, so $x = 6$

f If $\log_{10} \sqrt{10} = x$
then $10^x = \sqrt{10}$, so $x = \frac{1}{2}$
(Power $\frac{1}{2}$ means 'square root'.)

3 g If $\log_4 (0.25) = x$ then $4^x = 0.25 = \frac{1}{4}$,
so $x = -1$
(Negative power means 'reciprocal'.)

h $\log_{0.25} 16 = x$
 $\Rightarrow 0.25^x = 16$
 $\Rightarrow \left(\frac{1}{4}\right)^x = 16$, so $x = -2$
 $\left(\left(\frac{1}{4}\right)^{-2} = \frac{1}{\left(\frac{1}{4}\right)^2} = \frac{1}{\left(\frac{1}{16}\right)} = 16\right)$

i $\log_a (a^{10}) = x$
 $\Rightarrow a^x = a^{10}$, so $x = 10$

j $\log_{\left(\frac{2}{3}\right)} \left(\frac{9}{4}\right) = x$
 $\Rightarrow \left(\frac{2}{3}\right)^x = \frac{9}{4} = \frac{1}{\left(\frac{2}{3}\right)^2} = \frac{1}{\left(\frac{4}{9}\right)} = \frac{9}{4}$
 $\Rightarrow x = -2$

4 a Using a power, $5^4 = x$
So $x = 625$

b Using a power, $x^2 = 81$
So $x = 9$
(The base of a logarithm cannot be negative, so $x = -9$ is not possible.)

c Using a power, $7^1 = x$
So $x = 7$

d $2^3 = x - 1$
 $x = 2^3 + 1$
 $= 9$

e $3^4 = 4x + 1$
 $4x = 3^4 - 1$
 $x = \frac{1}{4}(3^4 - 1)$
 $= 20$

4 f Using a power,

$$x^2 = 2x$$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$$x = 2$$

(The base of a logarithm cannot be 0,
so $x = 0$ is not possible)

5 a $\log_9 230 = 2.475$

b $\log_5 33 = 2.173$

c $\log_{10} 1020 = 3.009$

d $\log_e 3 = 1.099$

6 a Let $\log_2 50 = x$

$$2^x = 50$$

As $2^5 = 32$ and $2^6 = 64$,

$$32 < 2^x < 64$$

$$2^5 < 2^x < 2^6$$

$$\text{So } 5 < x < 6$$

b $\log_2 50 = 5.644$

7 a i $\log_2 2 = 1$

ii $\log_3 3 = 1$

iii $\log_{17} 17 = 1$

b Let $\log_a a = x$

$$a^x = a$$

$$x = 1$$

$$\therefore \log_a a = 1$$

8 a i $\log_2 1 = 0$

ii $\log_3 1 = 0$

iii $\log_{17} 1 = 0$

b Let $\log_a 1 = x$

$$a^x = 1$$

$$x = 0$$

$$\therefore \log_a 1 = 0$$