Exercise 2B

1 a $A(-5, 8)$ and $B(7, 2)$

Midpoint $= \left(\frac{-5+7}{2}, \frac{8+2}{2}\right)$

$= (1, 5)$

The gradient of the line segment $AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-8}{7-(-5)} = -\frac{1}{2}$

So the gradient of the line perpendicular to AB is 2.

The equation of the perpendicular line is

$y - y_1 = m(x - x_1)$

$m = 2$ and $(x_1, y_1) = (1, 5)$

So $y - 5 = 2(x - 1)$

$y = 2x + 3$

b $C(-4, 7)$ and $D(2, 25)$

Midpoint $= \left(\frac{-4+2}{2}, \frac{7+25}{2}\right) = (-1, 16)$

The gradient of the line segment $CD = \frac{y_2 - y_1}{x_2 - x_1} = \frac{25-7}{2-(-4)} = 3$

So the gradient of the line perpendicular to CD is $-\frac{1}{3}$.

The equation of the perpendicular line is

$y - y_1 = m(x - x_1)$

$m = -\frac{1}{3}$ and $(x_1, y_1) = (-1, 16)$

So $y - 16 = -\frac{1}{3}(x - (-1))$

$y = -\frac{1}{3}x + \frac{47}{3}$

1 c $E(3, -3)$ and $F(13, -7)$

Midpoint $= \left(\frac{3+13}{2}, \frac{(-3)+(-7)}{2}\right) = (8, -5)$

The gradient of the line segment $EF = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-(-3)}{13-3} = -\frac{2}{5}$

So the gradient of the line perpendicular to EF is $\frac{5}{2}$.

The equation of the perpendicular line is

$y - y_1 = m(x - x_1)$

$m = \frac{5}{2}$ and $(x_1, y_1) = (8, -5)$

So $y - (-5) = \frac{5}{2}(x - 8)$

$y + 5 = \frac{5}{2}x - 20$

$y = \frac{5}{2}x - 25$

d $P(-4, 7)$ and $Q(-4, -1)$

Midpoint $= \left(\frac{-4+(-4)}{2}, \frac{7+(-1)}{2}\right) = (-4, 3)$

P and Q both have x-coordinates of -4, so this is the line $x = -4$. So the perpendicular to PQ is a horizontal line with gradient 0.

The equation of the perpendicular line is

$y - y_1 = m(x - x_1)$

$m = 0$ and $(x_1, y_1) = (-4, 3)$

So $y - 3 = 0(x - (-4))$

$y = 3$

e $S(4, 11)$ and $T(-5, -1)$

Midpoint $= \left(\frac{4+(-5)}{2}, \frac{11+(-1)}{2}\right) = \left(-\frac{1}{2}, 5\right)$

The gradient of the line segment $ST = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-11}{-5-4} = \frac{4}{3}$

So the gradient of the line perpendicular to ST is $-\frac{3}{4}$.

The equation of the perpendicular line is

$y - y_1 = m(x - x_1)$

$m = -\frac{3}{4}$ and $(x_1, y_1) = \left(-\frac{1}{2}, 5\right)$

So $y - 5 = -\frac{3}{4}\left(x - \left(-\frac{1}{2}\right)\right)$

$y - 5 = -\frac{3}{4}x - \frac{3}{8}$

$y = -\frac{3}{4}x + \frac{37}{8}$

1 f $X(13, 11)$ and $Y(5, 11)$

Midpoint $= \left(\frac{13+5}{2}, \frac{11+11}{2}\right) = (9, 11)$

The y-coordinates of points X and Y are both 11, so this is the line $y = 11$.
So the equation of the perpendicular line is $x = a$.
The line passes through the point (9, 11) so a = 9.
$x = 9$

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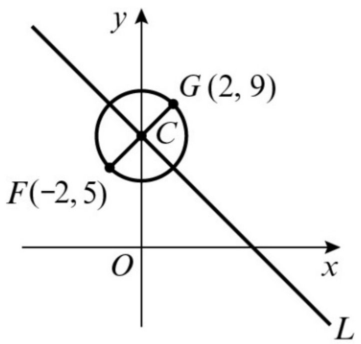

The gradient of FG is
$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9-5}{2-(-2)} = \frac{4}{4} = 1$

The gradient of a line perpendicular to FG is
$\frac{-1}{(1)} = -1$.

C is the mid-point of FG, so the coordinates of C are
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+2}{2}, \frac{5+9}{2}\right) = \left(\frac{0}{2}, \frac{14}{2}\right) = (0, 7)$

The equation of l is
$$y - y_1 = m(x - x_1)$$
$$y - 7 = -1(x - 0)$$
$$y - 7 = -x$$
$$y = -x + 7$$

Or we could have recognised immediately that (0, 7) is the y-intercept.

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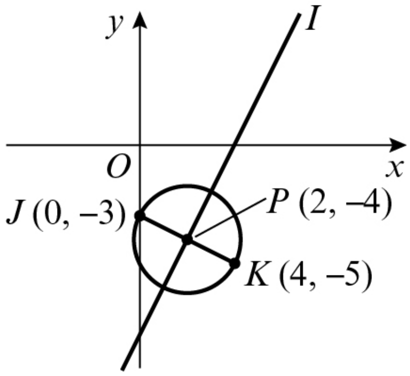

The gradient of *JK* is

$$\frac{y_2-y_1}{x_2-x_1}=\frac{-5-(-3)}{4-0}=\frac{-5+3}{4}=-\frac{2}{4}=-\frac{1}{2}$$

The gradient of a line perpendicular to *JK* is

$$\frac{-1}{\left(-\frac{1}{2}\right)}=2$$

P is the mid-point of *JK*, so the coordinates of *P* are

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{0+4}{2},\frac{-3+(-5)}{2}\right)=\left(\frac{4}{2},-\frac{8}{2}\right)=(2,-4)$$

The equation of *l* is

$$y-y_1=m(x-x_1)$$
$$y-(-4)=2(x-2)$$
$$y+4=2x-4$$
$$0=2x-y-4-4$$
$$2x-y-8=0$$

4 a $A(-4,-9)$ and $B(6,-3)$

Midpoint $=\left(\frac{-4+6}{2},\frac{-9+(-3)}{2}\right)=(1,-6)$

The gradient of the line segment $AB=\frac{y_2-y_1}{x_2-x_1}=\frac{-3-(-9)}{6-(-4)}=\frac{3}{5}$

So the gradient of the line perpendicular to *AB* is $-\frac{5}{3}$.

The equation of the perpendicular line is

$y-y_1=m(x-x_1)$

$m=-\frac{5}{3}$ and $(x_1,y_1)=(1,-6)$

So $y-(-6)=-\frac{5}{3}(x-1)$

$$y+6=-\frac{5}{3}x+\frac{5}{3}$$
$$y=-\frac{5}{3}x-\frac{13}{3}$$

4 b $C(11, 5)$ and $D(-1, 9)$

Midpoint $= \left(\frac{11+(-1)}{2}, \frac{5+9}{2}\right) = (5, 7)$

The gradient of the line segment $CD = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-5}{-1-11} = -\frac{1}{3}$

So the gradient of the line perpendicular to CD is 3.
The equation of the perpendicular line is

$y - y_1 = m(x - x_1)$

$m = 3$ and $(x_1, y_1) = (5, 7)$

So $y - 7 = 3(x - 5)$

$y = 3x - 8$

c Solve the two perpendicular bisectors simultaneously

$-\frac{5}{3}x - \frac{13}{3} = 3x - 8$

$-5x - 13 = 9x - 24$

$14x = 11$

$x = \frac{11}{14}$, so $y = 3\left(\frac{11}{14}\right) - 8 = -\frac{79}{14}$

$\left(\frac{11}{14}, -\frac{79}{14}\right)$

5 $X(7, -2)$ and $Y(4, q)$

The gradient of the line segment $XY = \frac{y_2 - y_1}{x_2 - x_1} = \frac{q-(-2)}{4-7} = \frac{q+2}{-3}$

From the equation of the perpendicular bisector of PQ, $y = 4x + b$, the gradient is 4

Therefore, the gradient of $XY = -\frac{1}{4}$, so $-\frac{1}{4} = \frac{q+2}{-3}$

$q = -\frac{5}{4}$

Midpoint of $XY = \left(\frac{7+4}{2}, \frac{-2+\left(-\frac{5}{4}\right)}{2}\right) = \left(\frac{11}{2}, -\frac{13}{8}\right)$

Substituting $x = \frac{11}{2}$ and $y = -\frac{13}{8}$ into $y = 4x + b$ gives

$-\frac{13}{8} = 4\left(\frac{11}{2}\right) + b$

$b = -\frac{189}{8}$

So $b = -\frac{189}{8}$, $q = -\frac{5}{4}$

Challenge

a $P(6, 9)$ and $Q(3, -3)$

Midpoint of $PQ = \left(\frac{6+3}{2}, \frac{9+(-3)}{2}\right) = \left(\frac{9}{2}, 3\right)$

The gradient of the line segment $PQ = \frac{-3-9}{3-6} = 4$

So the gradient of the line perpendicular to PQ is $-\frac{1}{4}$.

The equation of the perpendicular line is

$y - y_1 = m(x - x_1)$

$m = -\frac{1}{4}$ and $(x_1, y_1) = \left(\frac{9}{2}, 3\right)$

So $y - 3 = -\frac{1}{4}\left(x - \frac{9}{2}\right)$

$y = -\frac{1}{4}x + \frac{33}{8}$

$R(-9, 3)$ and $Q(3, -3)$

Midpoint of $RQ = \left(\frac{(-3)+3}{2}, \frac{3+(-9)}{2}\right) = (-3, 0)$

The gradient of the line segment $RQ = \frac{3-(-3)}{-9-3} = -\frac{1}{2}$

So the gradient of the line perpendicular to RQ is 2.

The equation of the perpendicular line is

$y - y_1 = m(x - x_1)$

$m = 2$ and $(x_1, y_1) = (-3, 0)$

So $y - 0 = 2(x - (-3))$

$y = 2x + 6$

$P(6, 9)$ and $R(-9, 3)$

Midpoint of $PR = \left(\frac{6+(-9)}{2}, \frac{9+3}{2}\right) = \left(-\frac{3}{2}, 6\right)$

The gradient of the line segment $PR = \frac{3-9}{-9-6} = \frac{2}{5}$

So the gradient of the line perpendicular to PR is $-\frac{5}{2}$.

The equation of the perpendicular line is

$y - y_1 = m(x - x_1)$

$m = -\frac{5}{2}$ and $(x_1, y_1) = \left(-\frac{3}{2}, 6\right)$

So $y - 6 = -\frac{5}{2}\left(x - \left(-\frac{3}{2}\right)\right)$

$y - 6 = -\frac{5}{2}x - \frac{15}{4}$

$y = -\frac{5}{2}x + \frac{9}{4}$

Challenge

b Solving each pair of pair of perpendicular bisectors simultaneously

PQ: $y = -\frac{1}{4}x + \frac{33}{8}$ and RQ: $y = 2x + 6$

$$-\frac{1}{4}x+\frac{33}{8}=2x+6$$
$$-2x+33=16x+48$$
$$18x=-15$$
$$x = -\frac{5}{6}, y=2\left(-\frac{5}{6}\right)+6=\frac{13}{3}$$

Lines PQ and RQ intersect at the point $\left(-\frac{5}{6},\frac{13}{3}\right)$

RQ: $y = 2x + 6$ and PR: $y = -\frac{5}{2}x + \frac{9}{4}$

$$2x+6=-\frac{5}{2}x+\frac{9}{4}$$
$$8x+24=-10x+9$$
$$18x=-15$$
$$x = -\frac{5}{6}, y=2\left(-\frac{5}{6}\right)+6=\frac{13}{3}$$

Therefore, all three perpendicular bisectors meet at the point $\left(-\frac{5}{6},\frac{13}{3}\right)$