Exercise 1C

1 a $f(x) = 4x^3 - 3x^2 - 1$
$f(1) = 4(1)^3 - 3(1)^2 - 1$
$= 4 - 3 - 1$
$= 0$
So $(x - 1)$ is a factor of $4x^3 - 3x^2 - 1$.

b $f(x) = 5x^4 - 45x^2 - 6x - 18$
$f(-3) = 5(-3)^4 - 45(-3)^2 - 6(-3) - 18$
$= 5(81) - 45(9) + 18 - 18$
$= 405 - 405$
$= 0$
So $(x + 3)$ is a factor of
$5x^4 - 45x^2 - 6x - 18$.

c $f(x) = -3x^3 + 13x^2 - 6x + 8$
$f(4) = -3(4)^3 + 13(4)^2 - 6(4) + 8$
$= -192 + 208 - 24 + 8$
$= 0$
So $(x - 4)$ is a factor of
$-3x^3 + 13x^2 - 6x + 8$.

2 $f(x) = x^3 + 6x^2 + 5x - 12$
$f(1) = (1)^3 + 6(1)^2 + 5(1) - 12$
$= 1 + 6 + 5 - 12$
$= 0$
So $(x - 1)$ is a factor of $x^3 + 6x^2 + 5x - 12$.

$$\begin{array}{r|l} & x^2+7x+12 \\ \hline x-1 & x^3+6x^2+5x-12 \\ & \underline{x^3-x^2} \\ & \quad 7x^2+5x \\ & \quad \underline{7x^2-7x} \\ & \qquad 12x-12 \\ & \qquad \underline{12x-12} \\ & \qquad\qquad 0 \end{array}$$

$x^3 + 6x^2 + 5x - 12 = (x - 1)(x^2 + 7x + 12)$
$= (x - 1)(x + 3)(x + 4)$

3 $f(x) = x^3 + 3x^2 - 33x - 35$
$f(-1) = (-1)^3 + 3(-1)^2 - 33(-1) - 35$
$= -1 + 3 + 33 - 35$
$= 0$
So $(x + 1)$ is a factor of
$x^3 + 3x^2 - 33x - 35$.

$$\begin{array}{r|l} & x^2+2x-35 \\ \hline x+1 & x^3+3x^2-33x-35 \\ & \underline{x^3+x^2} \\ & \quad 2x^2-33x \\ & \quad \underline{2x^2+2x} \\ & \qquad -35x-35 \\ & \qquad \underline{-35x-35} \\ & \qquad\qquad 0 \end{array}$$

$x^3 + 3x^2 - 33x - 35 = (x + 1)(x^2 + 2x - 35)$
$= (x + 1)(x + 7)(x - 5)$

4 $f(x) = x^3 + 7x^2 + 2x + 40$
$f(5) = (5)^3 + 7(5)^2 + 2(5) + 40$
$= 125 - 175 + 10 + 40$
$= 0$
So $(x - 5)$ is a factor of $x^3 + 7x^2 + 2x + 40$.

$$\begin{array}{r|l} & x^2-2x-8 \\ \hline x-5 & x^3-7x^2+2x+40 \\ & \underline{x^3-5x^2} \\ & \quad -2x^2+2x \\ & \quad \underline{-2x^2+10x} \\ & \qquad -8x+40 \\ & \qquad \underline{-8x+40} \\ & \qquad\qquad 0 \end{array}$$

$x^3 - 7x^2 + 2x + 40 = (x - 5)(x^2 - 2x - 8)$
$= (x - 5)(x - 4)(x + 2)$

5 $f(x) = 2x^3 + 3x^2 - 18x + 8$

$f(2) = 2(2)^3 + 3(2)^2 - 18(2) + 8$
$= 16 + 12 - 36 + 8$
$= 0$

So $(x - 2)$ is a factor of $2x^3 + 3x^2 - 18x + 8$.

$$\begin{array}{r} 2x^2 + 7x - 4 \\ x-2\overline{)\,2x^3 + 3x^2 - 18x + 8} \\ \underline{2x^3 - 4x^2} \qquad\qquad \\ 7x^2 - 18x \qquad \\ \underline{7x^2 - 14x} \qquad \\ -4x + 8 \\ \underline{-4x + 8} \\ 0 \end{array}$$

$2x^3 + 3x^2 - 18x + 8$
$= (x - 2)(2x^2 + 7x - 4)$
$= (x - 2)(2x - 1)(x + 4)$

6 $f(x) = 2x^3 + 17x^2 + 31x - 20$

By the factor theorem, if $(2x - 1)$ is a factor of $2x^3 + 17x^2 + 31x - 20$ then $f\left(\frac{1}{2}\right) = 0$.

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + 17\left(\frac{1}{2}\right)^2 + 31\left(\frac{1}{2}\right) - 20$$
$$= 2\left(\frac{1}{8}\right) + 17\left(\frac{1}{4}\right) + 31\left(\frac{1}{2}\right) - 20$$
$$= \frac{2}{8} + \frac{17}{4} + \frac{31}{2} - 20$$
$$= \frac{1}{4} + \frac{17}{4} + \frac{62}{4} - \frac{80}{4}$$
$$= 0$$

Therefore $(2x - 1)$ is a factor of $2x^3 + 17x^2 + 31x - 20$

7 a $f(x) = x^3 - 10x^2 + 19x + 30$

$f(-1) = (-1)^3 - 10(-1)^2 + 19(-1) + 30$
$= -1 - 10 - 19 + 30$

So $(x + 1)$ is a factor of $x^3 - 10x^2 + 19x + 30$.

$$\begin{array}{r} x^2 - 11x + 30 \\ x+1\overline{)\,x^3 - 10x^2 + 19x + 30} \\ \underline{x^3 + x^2} \qquad\qquad\qquad \\ -11x^2 + 19x \qquad \\ \underline{-11x^2 - 11x} \qquad \\ 30x + 30 \\ \underline{30x + 30} \\ 0 \end{array}$$

$x^3 - 10x^2 + 19x + 30$
$= (x + 1)(x^2 - 11x + 30)$
$= (x + 1)(x - 5)(x - 6)$

b $f(x) = x^3 + x^2 - 4x - 4$

$f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4$
$= -1 + 1 + 4 - 4$
$= 0$

So $(x + 1)$ is a factor of $x^3 + x^2 - 4x - 4$.

$$\begin{array}{r} x^2 - 4 \qquad\qquad \\ x+1\overline{)\,x^3 + x^2 - 4x - 4} \\ \underline{x^3 + x^2} \qquad\qquad \\ 0 \quad -4x - 4 \\ \underline{-4x - 4} \\ 0 \end{array}$$

$x^3 + x^2 - 4x - 4 = (x + 1)(x^2 - 4)$
$= (x + 1)(x - 2)(x + 2)$

7 c $f(x) = x^3 - 4x^2 - 11x + 30$

$f(2) = (2)^3 - 4(2)^2 - 11(2) + 30$

$= 8 - 16 - 22 + 30$

$= 0$

So $(x - 2)$ is a factor of $x^3 - 4x^2 - 11x + 30$.

$$
\begin{array}{r l}
 & x^2 - 2x - 15 \\
x - 2 \big) & x^3 - 4x^2 - 11x + 30 \\
 & \underline{x^3 - 2x^2} \\
 & \quad -2x^2 - 11x \\
 & \quad \underline{-2x^2 + 4x} \\
 & \qquad -15x + 30 \\
 & \qquad \underline{-15x + 30} \\
 & \qquad\qquad 0
\end{array}
$$

$x^3 - 4x^2 - 11x + 30 = (x - 2)(x^2 - 2x - 15)$

$= (x - 2)(x + 3)(x - 5)$

8 a i $f(x) = 2x^3 + 5x^2 - 4x - 3$

$f(1) = 2(1)^3 + 5(1)^2 - 4(1) - 3$

$= 2 + 5 - 4 - 3$

$= 0$

So $(x - 1)$ is a factor of $2x^3 + 5x^2 - 4x - 3$.

$$
\begin{array}{r l}
 & 2x^2 + 7x + 3 \\
x - 1 \big) & 2x^3 + 5x^2 - 4x - 3 \\
 & \underline{2x^3 - 2x^2} \\
 & \quad 7x^2 - 4x \\
 & \quad \underline{7x^2 - 7x} \\
 & \qquad 3x - 3 \\
 & \qquad \underline{3x - 3} \\
 & \qquad\quad 0
\end{array}
$$

$y = 2x^3 + 5x^2 - 4x - 3$

$= (x - 1)(2x^2 + 7x + 3)$

$= (x - 1)(2x + 1)(x + 3)$

8 a ii $0 = (x - 1)(2x + 1)(x + 3)$

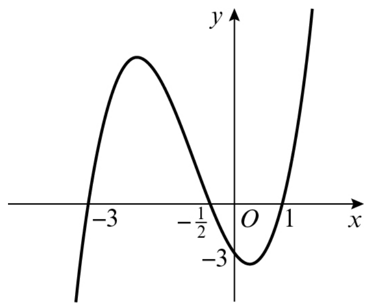
So the curve crosses the x-axis at $(1, 0)$, $(-\frac{1}{2}, 0)$ and $(-3, 0)$.

When $x = 0$, $y = (-1)(1)(3) = -3$

The curve crosses the y-axis at $(0, -3)$.

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



8 b i $f(x) = 2x^3 - 17x^2 + 38x - 15$

$f(3) = 2(3)^3 - 17(3)^2 + 38(3) - 15$

$= 54 - 153 + 114 - 15$

$= 0$

So $(x - 3)$ is a factor of $2x^3 - 17x^2 + 38x - 15$.

$$
\begin{array}{r l}
 & 2x^2 - 11x + 5 \\
x - 3 \big) & 2x^3 - 17x^2 + 38x - 15 \\
 & \underline{2x^3 - 6x^2} \\
 & \quad -11x^2 + 38x \\
 & \quad \underline{-11x^2 + 33x} \\
 & \qquad\quad 5x - 15 \\
 & \qquad\quad \underline{5x - 15} \\
 & \qquad\qquad 0
\end{array}
$$

$y = 2x^3 - 17x^2 + 38x - 15$

$= (x - 3)(2x^2 - 11x + 5)$

$= (x - 3)(2x - 1)(x - 5)$

8 b ii $0 = (x - 3)(2x - 1)(x - 5)$

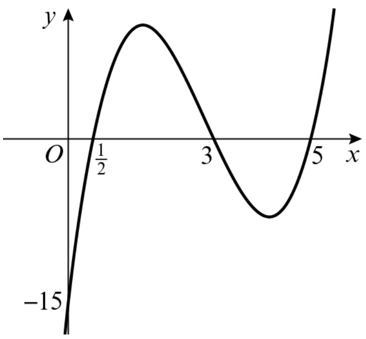
So the curve crosses the x-axis at $(3, 0)$, $(\frac{1}{2}, 0)$ and $(5, 0)$.

When $x = 0$, $y = (-3)(-1)(-5) = -15$

The curve crosses the y-axis at $(0, -15)$.

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



8 c i $f(x) = 3x^3 + 8x^2 + 3x - 2$

$$\begin{aligned} f(-1) &= 3(-1)^3 + 8(-1)^2 + 3(-1) - 2 \\ &= -3 + 8 - 3 - 2 \\ &= 0 \end{aligned}$$

So $(x + 1)$ is a factor of $3x^3 + 8x^2 + 3x - 2$.

$$\begin{array}{r l} & 3x^2 + 5x - 2 \\ x+1 \,) & 3x^3 + 8x^2 + 3x - 2 \\ & \underline{3x^3 + 3x^2} \\ & 5x^2 + 3x \\ & \underline{5x^2 + 5x} \\ & -2x - 2 \\ & \underline{-2x - 2} \\ & 0 \end{array}$$

$$\begin{aligned} y &= 3x^3 + 8x^2 + 3x - 2 \\ &= (x + 1)(3x^2 + 5x - 2) \\ &= (x + 1)(3x - 1)(x + 2) \end{aligned}$$

8 c ii $0 = (x + 1)(3x - 1)(x + 2)$

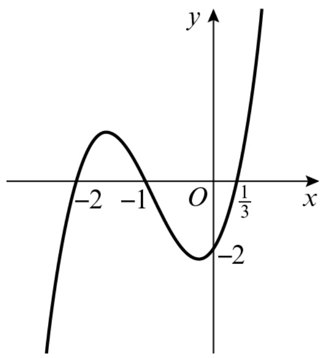
So the curve crosses the x-axis at $(-1, 0)$, $(\frac{1}{3}, 0)$ and $(-2, 0)$.

When $x = 0$, $y = (1)(-1)(2) = -2$

The curve crosses the y-axis at $(0, -2)$.

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



8 d i $f(x) = 6x^3 + 11x^2 - 3x - 2$

$$\begin{aligned} f(-2) &= 6(-2)^3 + 11(-2)^2 - 3(-2) - 2 \\ &= -48 + 44 + 6 - 2 \\ &= 0 \end{aligned}$$

So $(x + 2)$ is a factor of $6x^3 + 11x^2 - 3x - 2$.

$$\begin{array}{r l} & 6x^2 - x - 1 \\ x+2 \,) & 6x^3 + 11x^2 - 3x - 2 \\ & \underline{6x^3 + 12x^2} \\ & -x^2 - 3x \\ & \underline{-x^2 - 2x} \\ & -x - 2 \\ & \underline{-x - 2} \\ & 0 \end{array}$$

$$\begin{aligned} y &= 6x^3 + 11x^2 - 3x - 2 \\ &= (x + 2)(6x^2 - x - 1) \\ &= (x + 2)(3x + 1)(2x - 1) \end{aligned}$$

8 d ii $0 = (x + 2)(3x + 1)(2x - 1)$

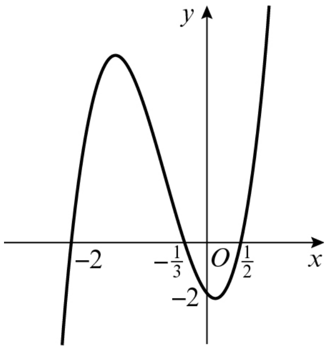
So the curve crosses the x-axis at $(-2, 0)$, $(-\frac{1}{3}, 0)$ and $(\frac{1}{2}, 0)$.

When $x = 0$, $y = (2)(1)(-1) = -2$

The curve crosses the y-axis at $(0, -2)$.

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



8 e i $f(x) = 4x^3 - 12x^2 - 7x + 30$

$f(2) = 4(2)^3 - 12(2)^2 - 7(2) + 30$

$= 32 - 48 - 14 + 30$

$= 0$

So $(x - 2)$ is a factor of $4x^3 - 12x^2 - 7x + 30$.

$$\begin{array}{r} 4x^2 - 4x - 15 \\ x-2 \overline{\smash{)}\, 4x^3 - 12x^2 - 7x + 30} \\ \underline{4x^3 - 8x^2} \qquad\qquad \\ -4x^2 - 7x \qquad \\ \underline{-4x^2 + 8x} \qquad \\ -15x + 30 \\ \underline{-15x + 30} \\ 0 \end{array}$$

$y = 4x^3 - 12x^2 - 7x + 30$

$= (x - 2)(4x^2 - 4x - 15)$

$= (x - 2)(2x + 3)(2x - 5)$

8 e ii $0 = (x - 2)(2x + 3)(2x - 5)$

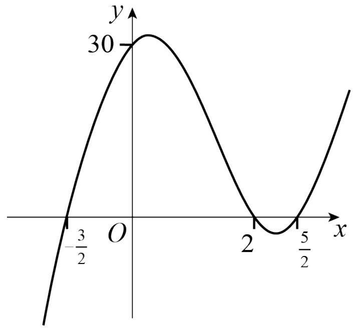
So the curve crosses the x-axis at $(2, 0)$, $(-\frac{3}{2}, 0)$ and $(\frac{5}{2}, 0)$.

When $x = 0$, $y = (-2)(3)(-5) = 30$

The curve crosses the y-axis at $(0, 30)$.

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



9 $f(x) = 2x^3 + 5x^2 - 4x - 3$

By the factor theorem, if $(x - p)$ is a factor of $2x^3 + 5x^2 - 4x - 3$ then $f(p) = 0$

Try some different values of x until you find $f(p) = 0$

$f(1) = 2(1)^3 + 5(1)^2 - 4(1) - 3$

$= 2 + 5 - 4 - 3$

$= 0$

Therefore $(x - 1)$ is a factor of $2x^3 + 5x^2 - 4x - 3$

Either divide or factorise out $(x - 1)$

$$\begin{array}{r} 2x^2 + 7x + 3 \\ x-1 \overline{\smash{)}\, 2x^3 + 5x^2 - 4x - 3} \\ \underline{2x^3 - 2x^2} \qquad\qquad \\ 7x^2 - 4x - 3 \\ \underline{7x^2 - 7x} \qquad \\ 3x - 3 \\ \underline{3x - 3} \\ 0 \end{array}$$

So

$2x^3 + 5x^2 - 4x - 3 = (x - 1)(2x^2 + 7x + 3)$

Now factorise the quadratic

$(2x^2 + 7x + 3) = (2x + 1)(x + 3)$

So

$2x^3 + 5x^2 - 4x - 3 = (x - 1)(2x + 1)(x + 3)$

10 $f(x) = 5x^3 - 9x^2 + 2x + a$
$f(1) = 0$
$5(1)^3 - 9(1)^2 + 2(1) + a = 0$
$5 - 9 + 2 + a = 0$
$a = 2$

11 $f(x) = 6x^3 - bx^2 + 18$
$f(-3) = 0$
$6(-3)^3 - b(-3)^2 + 18 = 0$
$-162 - 9b + 18 = 0$
$9b = -144$
$b = -16$

12 $f(x) = px^3 + qx^2 - 3x - 7$
$f(1) = 0$
$p(1)^3 + q(1)^2 - 3(1) - 7 = 0$
$p + q - 3 - 7 = 0$
$p + q = 10$ **(1)**

$f(-1) = 0$
$p(-1)^3 + q(-1)^2 - 3(-1) - 7= 0$
$-p + q + 3 - 7 = 0$
$-p + q = 4$ **(2)**

(1) + **(2)**:
$2q = 14$
$q = 7$
Substituting in (**1**):
$p + 7 = 10$
$p = 3$
So $p = 3$, $q = 7$

13 $f(x) = cx^3 + dx^2 - 9x - 10$
$f(-1) = 0$
$c(-1)^3 + d(-1)^2 - 9(-1) - 10 = 0$
$-c + d + 9 - 10 = 0$
$d = c + 1$ **(1)**

$f(2) = 0$
$c(2)^3 + d(2)^2 - 9(2) - 10 = 0$
$8c + 4d - 18 - 10 = 0$
$8c + 4d - 28 = 0$
$8c + 4d = 28$ **(2)**

Substituting **(1)** in (**2**):
$8c + 4(c + 1) = 28$
$12c + 4 = 28$
$c = 2$
Substituting in (**1**):
$d = 2 + 1 = 3$
So $c = 2$, $d = 3$

14 $f(x) = px^3 + qx^2 + 9x - 2$
Since $(x-1)$ and $(2x-1)$ are factors of f(x), then by the factor theorem
$f(1) = p(1)^3 + q(1)^2 + 9(1) - 2 = 0$
$p + q = -7$ **(1)**
and
$f\left(\frac{1}{2}\right) = p\left(\frac{1}{2}\right)^3 + q\left(\frac{1}{2}\right)^2 + 9\left(\frac{1}{2}\right) - 2 = 0$
$\frac{1}{8}p + \frac{1}{4}q = -\frac{5}{2}$ **(2)**
To solve the simultaneous equations in p and q, first multiply equation **(2)** by -4
$-\frac{1}{2}p - q = 10$ **(3)**
Then add equations **(1)** and **(3)**
$p + q = -7$
$-\frac{1}{2}p - q = 10$
$\frac{1}{2}p \quad = 3$
$p \quad = 6$
When $p = 6$, $q = -13$.

15 $f(x) = gx^3 + hx^2 - 14x + 24$
$f(-2) = 0$
$g(-2)^3 + h(-2)^2 - 14(-2) + 24 = 0$
$-8g + 4h + 28 + 24 = 0$
$-8g + 4h + 52 = 0$
$h = 2g - 13$ **(1)**

$f(3) = 0$
$g(3)^3 + h(3)^2 - 14(3) + 24 = 0$
$27g + 9h - 42 + 24 = 0$
$27g + 9h = 18$ **(2)**

Substituting **(1)** in **(2)**:
$27g + 9(2g - 13) = 18$
$45g = 135$
$g = 3$
Substituting in **(1)**:
$h = 2(3) - 13 = -7$
So $g = 3$, $h = -7$

16 a $f(x) = 3x^3 + bx^2 - 3x - 2$

Since $(3x+2)$ is a factor of $f(x)$, then by the factor theorem

$$f\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right)^3 + b\left(-\frac{2}{3}\right)^2 - 3\left(-\frac{2}{3}\right) - 2 = 0$$

$$3\left(-\frac{8}{27}\right) + b\left(\frac{4}{9}\right) - 3\left(-\frac{2}{3}\right) - 2 = 0$$

$$-\frac{8}{9} + \frac{4}{9}b + 2 - 2 = 0$$

$$\frac{4}{9}b = \frac{8}{9} \text{ so } b = 2$$

b $f(x) = 3x^3 + 2x^2 - 3x - 2$

$(3x+2)$ is a factor of $3x^3 + 2x^2 - 3x - 2$

Either divide or factorise out $(3x + 2)$

$$\begin{array}{r} x^2 - 1 \\ 3x+2 \overline{)\, 3x^3 + 2x^2 - 3x - 2} \\ \underline{3x^3 + 2x^2} \\ -3x - 2 \\ \underline{3x + 3} \\ 0 \end{array}$$

So

$3x^3 + 2x^2 - 3x - 2 = (3x+2)(x^2 - 1)$

Now factorise the quadratic

$(x^2 - 1) = (x+1)(x-1)$ So

$3x^3 + 2x^2 - 3x - 2 = (3x+2)(x+1)(x-1)$

17 a $f(x) = 3x^3 - 12x^2 + 6x - 24$
$f(4) = 3(4)^3 - 12(4)^2 + 6(4) - 24$
$= 192 - 192 + 24 - 24$
$= 0$
So $(x - 4)$ is a factor of $f(x)$.

b
$$\begin{array}{r} 3x^2 + 6 \\ x-4 \overline{)\, 3x^3 - 12x^2 + 6x - 24} \\ \underline{3x^3 - 12x^2} \\ 0 + 6x - 24 \\ \underline{6x - 24} \\ 0 \end{array}$$

$f(x) = (x - 4)(3x^2 + 6)$
$(x - 4)(3x^2 + 6) = 0$
Using the discriminant for $3x^2 + 6$:
$b^2 - 4ac = 0 - 4(3)(6) = -72 < 0$.
Therefore $3x^2 + 6$ has no real roots, so $f(x)$ only has one real root of $x = 4$.

18 a $f(x) = 4x^3 + 4x^2 - 11x - 6$
$f(-2) = 4(-2)^3 + 4(-2)^2 - 11(-2) - 6$
$= -32 + 16 + 22 - 6$
$= 0$
So $(x + 2)$ is a factor of $f(x)$.

18 b
$$\begin{array}{r} 4x^2 - 4x - 3 \\ x+2 \overline{)\, 4x^3 + 4x^2 - 11x - 6} \\ \underline{4x^3 + 8x^2} \\ -4x^2 - 11x \\ \underline{-4x^2 - 8x} \\ -3x - 6 \\ \underline{-3x - 6} \\ 0 \end{array}$$

$f(x) = (x + 2)(4x^2 - 4x - 3)$
$= (x + 2)(2x - 3)(2x + 1)$

c $0 = (x + 2)(2x - 3)(2x + 1)$
The solutions are $x = -2$, $x = \frac{3}{2}$ and $x = -\frac{1}{2}$.

19 a $f(x) = 9x^4 - 18x^3 - x^2 + 2x$
$f(2) = 9(2)^4 - 18(2)^3 - (2)^2 + 2(2)$
$= 144 - 144 - 4 + 4$
$= 0$
So $(x - 2)$ is a factor of $9x^4 - 18x^3 - x^2 + 2x$.

19 b

$$\begin{array}{r} 9x^3 - x \\ x-2\,\overline{)\,9x^4 - 18x^3 - x^2 + 2x} \\ \underline{9x^4 - 18x^3} \\ 0 - x^2 + 2x \\ \underline{-x^2 + 2x} \\ 0 \end{array}$$

$9x^4 - 18x^3 - x^2 + 2x$
$= (x-2)(9x^3 - x)$
$= x(x-2)(9x^2 - 1)$
$= x(x-2)(3x+1)(3x-1)$
$0 = x(x-2)(3x+1)(3x-1)$

The solutions are $x = 0$, $x = 2$, $x = -\frac{1}{3}$ and $x = \frac{1}{3}$.

Challenge

a $f(x) = 2x^4 - 5x^3 - 42x^2 - 9x + 54 = 0$

$f(1) = 2(1)^4 - 5(1)^3 - 42(1)^2 - 9(1) + 54$
$= 2 - 5 - 42 - 9 + 54$
$= 0$

$f(-3) = 2(-3)^4 - 5(-3)^3 - 42(-3)^2 - 9(-3) + 54$
$= 2(81) - 5(-27) - 42(9) - 9(-3) + 54$
$= 162 + 135 - 378 + 27 + 54$
$= 0$

b $(x-1)$ and $(x+3)$ are factors of $2x^4 - 5x^3 - 42x^2 - 9x + 54$
so $(x-1)(x+3) = x^2 + 2x - 3$ must also be a factor

Either divide or factorise out $(x^2 + 2x - 3)$

$$\begin{array}{r} 2x^2 - 9x - 18 \\ x^2 + 2x - 3\,\overline{)\,2x^4 - 5x^3 - 42x^2 - 9x + 54} \\ \underline{2x^4 + 4x^3 - 6x^2} \\ -9x^3 - 36x^2 - 9x \\ \underline{9x^3 + 18x^2 - 27x} \\ 18x^2 - 36x + 54 \\ \underline{18x^2 - 36x + 54} \\ 0 \end{array}$$

So
$2x^4 - 5x^3 - 42x^2 - 9x + 54$
$= (x-1)(x+3)(2x^2 - 9x - 18)$

Now factorise the quadratic
$(2x^2 - 9x - 18) = (2x+3)(x-6)$

$2x^4 - 5x^3 - 42x^2 - 9x + 54$
$= (x-1)(x+3)(2x+3)(x-6)$

$x = 1,\ x = -3,\ x = -\dfrac{3}{2},\ x = 6$