Exercise 1B

1 a

$$x+1\overline{)x^3+6x^2+8x+3}$$

$$\begin{array}{r} x^2+5x+3 \\ \hline x^3+x^2 \\ \hline 5x^2+8x \\ \underline{5x^2+8x} \\ 3x+3 \\ \underline{3x+3} \\ 0 \end{array}$$

So $\dfrac{x^3+6x^2+8x+3}{x+1} = (x+1)(x^2+5x+3)$

b

$$x+4\overline{)x^3+10x^2+25x+4}$$

$$\begin{array}{r} x^2+6x+1 \\ \hline x^3+4x^2 \\ \hline 6x^2+25x \\ \underline{6x^2+24x} \\ x+4 \\ \underline{x+4} \\ 0 \end{array}$$

So $\dfrac{x^3+10x^2+25x+4}{x+4} = (x+4)(x^2+6x+1)$

c

$$x+2\overline{)x^3-x^2+x+14}$$

$$\begin{array}{r} x^2-3x+7 \\ \hline x^3+2x^2 \\ \hline -3x^2+x \\ \underline{-3x^2-6x} \\ 7x+14 \\ \underline{7x+14} \\ 0 \end{array}$$

So $\dfrac{x^3+x^2+x+14}{x+2} = (x+2)(x^2-3x+7)$

d

$$x-3\overline{)x^3+x^2-7x-15}$$

$$\begin{array}{r} x^2+4x+5 \\ \hline x^3-3x^2 \\ \hline 4x^2-7x \\ \underline{4x^2-12x} \\ 5x-15 \\ \underline{5x-15} \\ 0 \end{array}$$

So $\dfrac{x^3+x^2-7x-15}{x-3} = (x-3)(x^2+4x+5)$

e

$$x-5\overline{)x^3-8x^2+13x+10}$$

$$\begin{array}{r} x^2-3x-2 \\ \hline x^3-5x^2 \\ \hline -3x^2+13x \\ \underline{-3x^2+15x} \\ -2x+10 \\ \underline{-2x-10} \\ 0 \end{array}$$

So $\dfrac{x^3-8x^2+13x+10}{x-5} = (x-5)(x^2-3x-2)$

f

$$x-7\overline{)x^3-5x^2-6x-56}$$

$$\begin{array}{r} x^2+2x+8 \\ \hline x^3-7x^2 \\ \hline 2x^2-6x \\ \underline{2x^2-14x} \\ 8x-56 \\ \underline{8x-56} \\ 0 \end{array}$$

So $\dfrac{x^3-5x^2-6x-56}{x-7} = (x-7)(x^2+2x+8)$

2 a
$$\begin{array}{r} 6x^2+3x+2 \\ x+4\overline{)\,6x^3+27x^2+14+8} \\ \underline{6x^3+24x^2} \\ 3x^2+14x \\ \underline{3x^2+12x} \\ 2x+8 \\ \underline{2x+8} \\ 0 \end{array}$$

So $6x^3+27x^2+14x+8 = (x+4)(6x^2+3x+2)$

b
$$\begin{array}{r} 4x^2+x-5 \\ x+2\overline{)\,4x^3+9x^2-3x-10} \\ \underline{4x^3+8x^2} \\ x^2-3x \\ \underline{x^2+2x} \\ -5x-10 \\ \underline{-5x-10} \\ 0 \end{array}$$

So $4x^3+9x^2-3x-10 = (x+2)(4x^2+x-5)$

c
$$\begin{array}{r} 2x^2-2x-3 \\ x+3\overline{)\,2x^3+4x^2-9x-9} \\ \underline{-2x^3+6x^2} \\ -2x^2-9x \\ \underline{-2x^2-6x} \\ -3x-9 \\ \underline{-3x-9} \\ 0 \end{array}$$

So $2x^3+4x^2-9x-9 = (x+3)(2x^2-2x-3)$

d
$$\begin{array}{r} 2x^2-3x-4 \\ x-6\overline{)\,2x^3-15x^2+14x+24} \\ \underline{2x^3-12x^2} \\ -3x^2+14x \\ \underline{-3x^2+18x} \\ -4x+24 \\ \underline{-4x+24} \\ 0 \end{array}$$

So $2x^3-15x^2+14x+24 = (x-6)(2x^2-3x-4)$

e
$$\begin{array}{r} -5x^2+3x+5 \\ x+6\overline{)\,-5x^3-27x^2+23x+30} \\ \underline{-5x^3-30x^2} \\ 3x^2+23x \\ \underline{3x^2+18x} \\ 5x+30 \\ \underline{5x+30} \\ 0 \end{array}$$

So $-5x^3-27x^2+23x+30 = (x+6)(-5x^2+3x+5)$

f
$$\begin{array}{r} -4x^2+x-1 \\ x-2\overline{)\,-4x^3+9x^2-3x+2} \\ \underline{-4x^3+8x^2} \\ x^2-3x \\ \underline{x^2-2x} \\ -x+2 \\ \underline{-x+2} \\ 0 \end{array}$$

So $-4x^3+9x^2-3x+2 = (x-2)(-4x^2+x-1)$

3 a
$$x+2\overline{)x^4+5x^3+2x^2-7x+2}$$
$$\begin{array}{r} x^3+3x^2-4x+1 \\ \underline{x^4+2x^3} \\ 3x^3+2x^2 \\ \underline{3x^2+6x^2} \\ -4x^2-7x \\ \underline{-4x^2-8x} \\ x+2 \\ \underline{x+2} \\ 0 \end{array}$$
So $\dfrac{x^4+5x^3+2x^2-7x+2}{x+2} = x^3+3x^2-4x+1$

b
$$x+3\overline{)4x^4+14x^3+3x^2-14x-15}$$
$$\begin{array}{r} 4x^3+2x^2-3x-5 \\ \underline{4x^4+12x^3} \\ 2x^3+3x^2 \\ \underline{2x^3+6x^2} \\ -3x^2-14x \\ \underline{-3x^2-9x} \\ -5x-15 \\ \underline{-5x-15} \\ 0 \end{array}$$
So $\dfrac{4x^4+14x^3+3x^2-14x-15}{x+3} = 4x^3+2x^2-3x-5$

c
$$x-2\overline{)-3x^4+9x^3-10x^2+x+14}$$
$$\begin{array}{r} -3x^3+3x^2-4x-7 \\ \underline{-3x^4+6x^3} \\ 3x^3-10x^2 \\ \underline{3x^3-6x^2} \\ -4x^2+x \\ \underline{-4x^2+8x} \\ -7x+14 \\ \underline{-7x+14} \\ 0 \end{array}$$
So $\dfrac{-3x^4+9x^3-10x^2+x+14}{x-2} = -3x^3+3x^2-4x-7$

d
$$x-1\overline{)-5x^2+7x^4+2x^3-7x^2+10x-7}$$
$$\begin{array}{r} -5x^4+2x^3+4x^2-3x+7 \\ \underline{-5x^2+5x^4} \\ 2x^4+2x^3 \\ \underline{2x^4-2x^3} \\ 4x^3-7x^2 \\ \underline{4x^3-4x^2} \\ -3x^2+10x \\ \underline{-3x^2+3x} \\ 7x-7 \\ \underline{7x-7} \\ 0 \end{array}$$
So $\dfrac{-5x^2+7x^4+2x^3-7x^2+10x-7}{x-1} = -5x^2+2x^3+4x^2-3x+7$

4 a
$$3x+2\overline{)3x^4+8x^3-11x^2+2x+8}$$
$$\begin{array}{r} x^3+2x^2-5x+4 \\ \hline 3x^4+2x^3 \\ \quad 6x^3-11x^2 \\ \quad \underline{6x^3+4x^2} \\ \quad\quad -15^2+2x \\ \quad\quad \underline{-15x^2-10x} \\ \quad\quad\quad 12x+8 \\ \quad\quad\quad \underline{12x+8} \\ \quad\quad\quad\quad 0 \end{array}$$
So $\dfrac{3x^4+8x^3-11x^2+2x+8}{3x+2} = x^3+2x^2-5x+4$

b
$$4x+1\overline{)4x^4-3x^3+11x^2-x-1}$$
$$\begin{array}{r} x^3-x^2+3x-1 \\ \hline 4x^4+x^3 \\ \quad -4x^3+11x^2 \\ \quad \underline{-4x^3-x^2} \\ \quad\quad 12x^2-x \\ \quad\quad \underline{12x^2+3x} \\ \quad\quad\quad -4x-1 \\ \quad\quad\quad \underline{-4x-1} \\ \quad\quad\quad\quad 0 \end{array}$$
So $\dfrac{4x^4-3x^3+11x^2-x-1}{4x+1} = x^3-x^2+3x-1$

c
$$2x-3\overline{)4x^4-6x^3+10x^2-11x-6}$$
$$\begin{array}{r} 2x^3+5x+2 \\ \hline 4x^4-6x^3 \\ \quad 0+10x^2-11x \\ \quad \underline{10x^2-15x} \\ \quad\quad 4x-6 \\ \quad\quad \underline{4x-6} \\ \quad\quad\quad 0 \end{array}$$
So $\dfrac{4x^4-6x^3+10x^2-11x-6}{2x-3} = 2x^3+5x+2$

d
$$2x+3\overline{)6x^5+13x^4-4x^3-9x^2+21x+18}$$
$$\begin{array}{r} 3x^4+2x^3-5x^2+3x+6 \\ \hline 6x^5+9x^4 \\ \quad 4x^4-4x^3 \\ \quad \underline{4x^4+6x^3} \\ \quad\quad -10x^3-9x^2 \\ \quad\quad \underline{10x^3-15x^2} \\ \quad\quad\quad 6x^2+21x \\ \quad\quad\quad \underline{6x^2+9x} \\ \quad\quad\quad\quad 12x+18 \\ \quad\quad\quad\quad \underline{12x+18} \\ \quad\quad\quad\quad\quad 0 \end{array}$$
So $\dfrac{6x^5+13x^4-4x^3-9x^2+21x+18}{2x+3} = 3x^4+2x^3-5x^2+3x+6$

e
$$3x-1\overline{)6x^5-8x^4+11x^3+9x^2-25x+7}$$
$$\begin{array}{r} 2x^4-2x^3+3x^2+4x-7 \\ \hline 6x^5-2x^4 \\ \quad -6x^4+11x^3 \\ \quad \underline{-6x^4+2x^3} \\ \quad\quad 9x^3+9x^2 \\ \quad\quad \underline{9x^3-3x^2} \\ \quad\quad\quad 12x^2-25x \\ \quad\quad\quad \underline{12x^2-4x} \\ \quad\quad\quad\quad 12x^2-25x \\ \quad\quad\quad\quad \underline{12x^2-4x} \\ \quad\quad\quad\quad\quad -21x+7 \\ \quad\quad\quad\quad\quad \underline{-21x+7} \\ \quad\quad\quad\quad\quad\quad 0 \end{array}$$
So $\dfrac{6x^5-8x^4+11x^3+9x^2-25x+7}{3x-1} = 2x^4-2x^3+3x^2+4x-7$

4 f
$$\begin{array}{r} 4x^4-3x^3-2x^2+6x-5 \\ 2x-5\overline{)\,8x^5-26x^4+11x^3+22x^2-40x+25} \\ \underline{8x^5-20x^4} \\ -6x^4+11x^3 \\ \underline{-6x^4+15x^3} \\ -4x^3+22x^2 \\ \underline{-4x^3+10x^2} \\ 12x^2-40x \\ \underline{12x^2-30x} \\ -10x+25 \\ \underline{-10x+25} \\ 0 \end{array}$$

So $\dfrac{8x^5-26x^4+11x^3+22x^2-40x+25}{2x-5} = 4x^4-3x^3-2x^2+6x-5$

g
$$\begin{array}{r} 5x^3+12x^2-6x-2 \\ 5x+3\overline{)\,25x^4+75x^3+6x^2-28x-6} \\ \underline{25x^4+15x^3} \\ 60x^3+6x^2 \\ \underline{60x^3+36x^2} \\ -30x^2-28x \\ \underline{-30x^2-18x} \\ -10x-6 \\ \underline{-10x-6} \\ 0 \end{array}$$

So $\dfrac{25x^4+75x^3+6x^2-28x-6}{5x+3} = 5x^3+12x^2-6x-2$

h
$$\begin{array}{r} 3x^4+5x^3+6 \\ 7x-2\overline{)\,21x^5+29x^4-10x^3+42x-12} \\ \underline{21x^5-6x^4} \\ 35x^4-10x^3 \\ \underline{35x^4-10x^3} \\ 0+42x-12 \\ \underline{42x-12} \\ 0 \end{array}$$

So $\dfrac{21x^5+29x^4-10x^3+42x-12}{7x-2} = 3x^4+5x^3+6$

5 a
$$\begin{array}{r} x^2-2x+5 \\ x+2\overline{)\,x^3+0x^2+x+10} \\ \underline{x^3+2x^2} \\ -2x^2+x \\ \underline{-2x^2-4x} \\ 5x+10 \\ \underline{5x+10} \\ 0 \end{array}$$

So $\dfrac{x^3+x+10}{x+2} = x^2-2x+5$

b
$$\begin{array}{r} 2x^2-6x+1 \\ x+3\overline{)\,2x^3+0x^2-17x+3} \\ \underline{2x^3+6x^2} \\ -6x^2-17x \\ \underline{-6x^2-18x} \\ x+3 \\ \underline{x+3} \\ 0 \end{array}$$

So $\dfrac{2x^3-17x+3}{x+3} = 2x^2-6x+1$

5 c
$$\begin{array}{r} -3x^2-12x+2 \\ x-4\overline{)-3x^3+0x^2+50x-8} \\ \underline{-3x^3+12x^2} \\ -12x^2+50x \\ \underline{-12x^2+48x} \\ 2x-8 \\ \underline{2x-8} \\ 0 \end{array}$$

So $\dfrac{-3x^3+50x-8}{x-4} = -3x^2-12x+2$

6 a
$$\begin{array}{r} x^2+4x+12 \\ x-3\overline{)x^3+x^2+0x-36} \\ \underline{x^3-3x^2} \\ 4x^2+0x \\ \underline{4x^2-12x} \\ 12x-36 \\ \underline{12x-36} \\ 0 \end{array}$$

So $\dfrac{x^3+x^2-36}{x-3} = x^2+4x+12$

b
$$\begin{array}{r} 2x^2-x+5 \\ x+5\overline{)2x^3+9x^2+0x+25} \\ \underline{2x^3+10x^2} \\ -x^2+0x \\ \underline{-x^2-5x} \\ 5x+25 \\ \underline{5x+25} \\ 0 \end{array}$$

So $\dfrac{2x^3+9x^2+25}{x+5} = 2x^2-x+5$

c
$$\begin{array}{r} -3x^2+5x+10 \\ x-2\overline{)-3x^3+11x^2+0x-20} \\ \underline{-3x^3+6x^2} \\ 5x^2+0x \\ \underline{5x^2-10x} \\ 10x-20 \\ \underline{10x-20} \\ 0 \end{array}$$

So $\dfrac{-3x^3+11x^2-20}{x-2} = -3x^2+5x+10$

7 $RHS = (x+2)(x^2-5)$
$= x^3+2x^2-5x-10$
$= LHS$

8 a
$$\begin{array}{r} x^2-x+2 \\ x+5\overline{)x^3+4x^2-3x+2} \\ \underline{x^3+5x} \\ -x^2-3x \\ \underline{-x^2-5x} \\ 2x+2 \\ \underline{2x+10} \\ -8 \end{array}$$

So the remainder is -8.

b
$$\begin{array}{r} 3x^2-2x-2 \\ x-6\overline{)3x^3-20x^2+10x+5} \\ \underline{3x^3-18x^2} \\ -2x^2+10x \\ \underline{-2x^2+12x} \\ -2x+5 \\ \underline{-2x+12} \\ -7 \end{array}$$

So the remainder is -7.

8 c

$$\begin{array}{r} -2x^2-5x-8 \\ x-4\overline{)\,-2x^3+3x^2+12x+20} \\ \underline{-2x^3+8x^2} \\ -5x^2+12x \\ \underline{-5x^2+20x} \\ -8x+20 \\ \underline{-8x+32} \\ -12 \end{array}$$

So the remainder is -12.

9

$$\begin{array}{r} 3x^2+x+1 \\ x-1\overline{)\,3x^3-2x^2+0x+4} \\ \underline{3x^3-3x^2} \\ x^2+0x \\ \underline{x^2-x} \\ x+4 \\ \underline{x-1} \\ 5 \end{array}$$

So the remainder is 5.

10

$$\begin{array}{r} 3x^3-11x^2+21x-24 \\ x+1\overline{)\,3x^4-8x^3+10x^2-3x-25} \\ \underline{3x^4+3x^3} \\ -11x^3+10x^2 \\ \underline{-11x^3-11x^2} \\ 21x^2-3x \\ \underline{21x^2+21x} \\ -24x-25 \\ \underline{-24x-24} \\ -1 \end{array}$$

So the remainder is -1.

11

$$\begin{array}{r} 5x^2-20x+7 \\ x+4\overline{)\,5x^3+0x^2-73x+28} \\ \underline{5x^3+20x} \\ -20x^2-73x \\ \underline{-20x^2-80x} \\ 7x+28 \\ \underline{7x+28} \\ 0 \end{array}$$

The remainder is 0, so $(x+4)$ is a factor of $5x^3-73x+28$.

12

$$\begin{array}{r} 3x^2+6x+4 \\ x-2\overline{)\,3x^3+0x^2-8x-8} \\ \underline{3x^3-6x^2} \\ 6x^2-8x \\ \underline{6x^2-12x} \\ 4x-8 \\ \underline{4x-8} \\ 0 \end{array}$$

So $\dfrac{3x^3-8x-8}{x-2}=3x^2+6x+4$

13

$$\begin{array}{r} x^2+x+1 \\ x-1\overline{)\,x^3+0x^2+0x-1} \\ \underline{x^3-x^2} \\ x^2+0x \\ \underline{x^2-x} \\ x-1 \\ \underline{x-1} \\ 0 \end{array}$$

So $\dfrac{x^3-1}{x-1}=x^2+x+1$

14 $x+2\overline{)\,x^4+0x^3+0x^2+0x-16}$

$$\begin{array}{r} x^3-2x^2+4x-8 \\ \hline x^4+2x^3 \\ -2x^3+0x^2 \\ \underline{-2x^3-4x^2} \\ 4x^2+0x \\ \underline{4x^2+8x} \\ -8x-16 \\ \underline{-8x-16} \\ 0 \end{array}$$

So $\dfrac{x^4-16}{x+2}=x^3-2x^2+4x-8$

15 $5x+4\overline{)\,10x^3+43x^2-2x-10}$

$$\begin{array}{r} 2x^2+7x-6 \\ \hline 10x^3+8x^2 \\ 35x^2-2x \\ \underline{35x^2+28x} \\ -30x-10 \\ \underline{-30x-24} \\ 14 \end{array}$$

So the remainder is 14.

16 a $x-3\overline{)\,3x^3-14x^2-47x-14}$

$$\begin{array}{r} 3x^2-5x-62 \\ \hline 3x^3-9x^2 \\ -5x^2-47x \\ \underline{-5x^2+15x} \\ -62x-14 \\ \underline{-62x+186} \\ -200 \end{array}$$

So the remainder is -200.

b $x+2\overline{)\,3x^3-14x^2-47x-14}$

$$\begin{array}{r} 3x^2-20x-7 \\ \hline 3x^3+6x^2 \\ -20x^2-47x \\ \underline{-20x^2-40x} \\ -7x-14 \\ \underline{-7x-14} \\ 0 \end{array}$$

$$\begin{aligned} f(x) &= 3x^3-14x^2-47x-14 \\ &= (x+2)(3x^2-20x-7) \\ &= (x+2)(3x+1)(x-7) \end{aligned}$$

17 a i $x-2\overline{)\,x^3+6x^2+5x-12}$

$$\begin{array}{r} x^2+8x+21 \\ \hline x^3-2x^2 \\ 8x^2+5x \\ \underline{8x^2-16x} \\ 21x-12 \\ \underline{21x-42} \\ 30 \end{array}$$

So the remainder is 30.

ii $x+3\overline{)\,x^3+6x^2+5x-12}$

$$\begin{array}{r} x^2+3x-4 \\ \hline x^3+3x^2 \\ 3x^2+5x \\ \underline{3x^2+9x} \\ -4x-12 \\ \underline{-4x-12} \\ 0 \end{array}$$

So the remainder is 0.

b $f(x)=x^3+6x^2+5x-12$

$=(x+3)(x^2+3x-4)$

$=(x+3)(x+4)(x-1)$

So $x=-3,\ x=-4, x=1$

18 a
$$2x-1\overline{)2x^3+3x^2-8x+3}\quad x^2+2x-3$$
$$\begin{array}{r} 2x^3-x^2 \\ \hline 4x^2-8x \\ 4x^2-2x \\ \hline -6x+3 \\ -6x+3 \\ \hline 0 \end{array}$$
$$f(x)=2x^3+3x^2-8x+3$$
$$=(2x-1)(x^2+2x-3)$$
$a=1,\ b=2, c=-3$

b $f(x)=2x^3+3x^2-8x+3$
$=(2x-1)(x^2+2x-3)$
$=(2x-1)(x-1)(x+3)$

c $(2x-1)(x-1)(x+3)=0$
$x=\frac{1}{2},\ x=1$ and $x=-3$

19 a
$$4x-1\overline{)12x^3+5x^2+2x-1}\quad 3x^2+2x+1$$
$$\begin{array}{r} 12x^3-3x^2 \\ \hline 8x^2+2x \\ 8x^2-2x \\ \hline 4x-1 \\ 4x-1 \\ \hline 0 \end{array}$$
$f(x)=(4x-1)(3x^2+2x+1)$
$a=3,\ b=2,\ c=1$

b $(4x-1)(3x^2+2x+1)=0$

Using the discriminant for
$3x^2+2x+1: b^2-4ac=2^2-4(3)(1)$ so
$=-8\le 0$
there are no real solutions.
So $f(x)$ has only one real solution,
$x=\frac{1}{4}$