Core Mathematics C2 Paper J

1. A geometric progression has first term 75 and second term –15.

(i) Find the common ratio. [2]

(ii) Find the sum to infinity. [2]

2. Find the area of the finite region enclosed by the curve $y = 5x - x^2$ and the x-axis. [6]

3. During one day, a biological culure is allowed to grow under controlled conditions. At 8 a.m. the culture is estimated to contain 20 000 bacteria. A model of the growth of the culture assumes that t hours after 8 a.m., the number of bacteria present, N, is given by

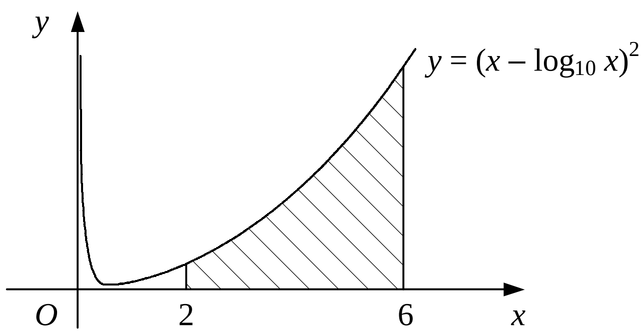
$$N = 20\,000 \times (1.06)^t.$$

Using this model,

(i) find the number of bacteria present at 11 a.m., [2]

(ii) find, to the nearest minute, the time when the initial number of bacteria will have doubled. [4]

4.



The diagram shows the curve with equation $y = (x - \log_{10} x)^2$, $x > 0$.

(i) Copy and complete the table below for points on the curve, giving the y values to 2 decimal places.

x	2	3	4	5	6
y	2.89	6.36			

[2]

The shaded region is bounded by the curve, the x-axis and the lines $x = 2$ and $x = 6$.

(ii) Use the trapezium rule with all the values in your table to estimate the area of the shaded region. [3]

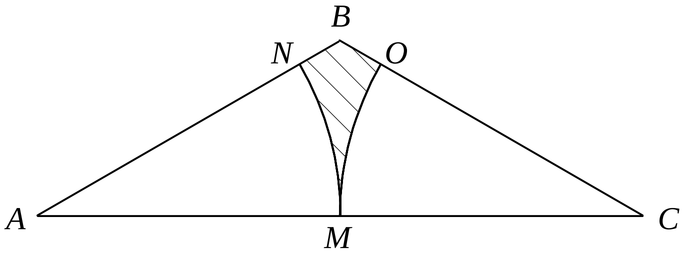
(iii) State, with a reason, whether your answer to part *(b)* is an under-estimate or an over-estimate of the true area. [2]

5. *(i)* Given that $\sin\theta = 2 - \sqrt{2}$, find the value of $\cos^2\theta$ in the form $a + b\sqrt{2}$ where a and b are integers. [3]

(ii) Find, in terms of π, all values of x in the interval $0 \leq x < \pi$ for which

$$\cos 3x = \frac{\sqrt{3}}{2}.$$ [5]

6.



The diagram shows triangle ABC in which $AC = 8$ cm and $\angle BAC = \angle BCA = 30°$.

(i) Find the area of triangle ABC in the form $k\sqrt{3}$. [4]

The point M is the mid-point of AC and the points N and O lie on AB and BC such that MN and MO are arcs of circles with centres A and C respectively.

(ii) Show that the area of the shaded region $BNMO$ is $\frac{8}{3}(2\sqrt{3} - \pi)$ cm^2. [4]

7. *(i)* Expand $(2 + x)^4$ in ascending powers of x, simplifying each coefficient. [4]

(ii) Find the integers A, B and C such that

$$(2 + x)^4 + (2 - x)^4 \equiv A + Bx^2 + Cx^4.$$ [2]

(iii) Find the real values of x for which

$$(2 + x)^4 + (2 - x)^4 = 136.$$ [3]

Turn over

8. *(i)* The gradient of a curve is given by

$$\frac{dy}{dx} = 3 - \frac{2}{x^2}, \quad x \neq 0.$$

Find an equation for the curve given that it passes through the point (2, 6). [6]

(ii) Show that

$$\int_2^3 \left(6\sqrt{x} - \frac{4}{\sqrt{x}}\right) dx = k\sqrt{3},$$

where k is an integer to be found. [6]

9. The polynomial f(x) is given by

$$f(x) = x^3 + kx^2 - 7x - 15,$$

where k is a constant.

When f(x) is divided by (x + 1) the remainder is r.

When f(x) is divided by (x – 3) the remainder is $3r$.

(i) Find the value of k. [5]

(ii) Find the value of r. [1]

(iii) Show that (x – 5) is a factor of f(x). [2]

(iv) Show that there is only one real solution to the equation f(x) = 0. [4]