C2 Paper J – Marking Guide

1. *(i)* $r = \frac{-15}{75} = -\frac{1}{5}$ — M1 A1

(ii) $= \frac{75}{1-(-\frac{1}{5})} = 62\frac{1}{2}$ — M1 A1 **(4)**

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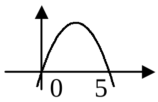
2. $5x - x^2 = 0$

$x(5 - x) = 0$

crosses x-axis at (0, 0) and (5, 0) — B1



area $= \int_0^5 (5x - x^2)\ dx$

$= [\frac{5}{2}x^2 - \frac{1}{3}x^3]_0^5$ — M1 A2

$= (\frac{125}{2} - \frac{125}{3}) - (0)$ — M1

$= 20\frac{5}{6}$ — A1 **(6)**

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3. *(i)* 11 a.m. $\therefore t = 3$

$N = 20\,000 \times (1.06)^3 = 23820$ (nearest unit) — M1 A1

(ii) $40\,000 = 20\,000 \times (1.06)^t$

$(1.06)^t = 2$ — M1

$t = \frac{\lg 2}{\lg 1.06} = 11.8957$ — M1 A1

11.8957 hours = 11 hours 54 mins $\therefore$ 7.54 p.m. — A1 **(6)**

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4. *(i)*

x	2	3	4	5	6
y	2.89	6.36	11.55	18.50	27.27

B2

(ii) area $\approx \frac{1}{2} \times 1 \times [2.89 + 27.27 + 2(6.36 + 11.55 + 18.50)]$ — B1 M1

$= 51.5$ (3sf) — A1

(iii) over-estimate — B1

the curve passes below the top edge of each trapezium — B1 **(7)**

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5. *(i)* $\sin^2\theta = (2 - \sqrt{2})^2 = 4 - 4\sqrt{2} + 2 = 6 - 4\sqrt{2}$ — M1

$\cos^2\theta = 1 - (6 - 4\sqrt{2}) = -5 + 4\sqrt{2}$ — M1 A1

(ii) $3x = \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}$ — B1 M1

$3x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}$ — A1

$x = \frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}$ — M1 A1 **(8)**

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6. *(i)* isosceles $\therefore \angle AMB = 90°$

$BM = 4 \tan 30° = \frac{4}{\sqrt{3}}$ — M1 A1

area $= \frac{1}{2} \times 8 \times \frac{4}{\sqrt{3}} = \frac{16}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{16}{3}\sqrt{3}\ \text{cm}^2$ — M1 A1

(ii) area of sector $= \frac{1}{2} \times 4^2 \times \frac{\pi}{6} = \frac{4}{3}\pi$ — B1 M1

shaded area $= \frac{16}{3}\sqrt{3} - (2 \times \frac{4}{3}\pi)$ — M1

$= \frac{16}{3}\sqrt{3} - \frac{8}{3}\pi = \frac{8}{3}(2\sqrt{3} - \pi)\ \text{cm}^2$ — A1 **(8)**

7.	*(i)*	$= 2^4 + 4(2^3)(x) + 6(2^2)(x^2) + 4(2)(x^3) + x^4$	M1 A1	
		$= 16 + 32x + 24x^2 + 8x^3 + x^4$	B1 A1	
	(ii)	$(2 - x)^4 = 16 - 32x + 24x^2 - 8x^3 + x^4$	M1	
		$(2 + x)^4 + (2 - x)^4 = 32 + 48x^2 + 2x^4$, $A = 32, B = 48, C = 2$	A1	
	(iii)	$32 + 48x^2 + 2x^4 = 136$		
		$x^4 + 24x^2 - 52 = 0$		
		$(x^2 + 26)(x^2 - 2) = 0$	M1	
		$x^2 = -26$ (no real solutions) or 2	A1	
		$x = \pm\sqrt{2}$	A1	**(9)**
8.	*(i)*	$y = \int (3 - \frac{2}{x^2})\ dx$		
		$y = 3x + 2x^{-1} + c$	M1 A2	
		$(2, 6)\ \therefore\ 6 = 6 + 1 + c$	M1	
		$c = -1$	A1	
		$\therefore\ y = 3x + 2x^{-1} - 1$	A1	
	(ii)	$\int_2^3 (6\sqrt{x} - \frac{4}{\sqrt{x}})\ dx = [\,4x^{\frac{3}{2}} - 8x^{\frac{1}{2}}\,]_2^3$	M1 A2	
		$= [4(3\sqrt{3}) - 8\sqrt{3}] - [4(2\sqrt{2}) - 8\sqrt{2}]$	M1 B1	
		$= (12\sqrt{3} - 8\sqrt{3}) - (8\sqrt{2} - 8\sqrt{2})$		
		$= 4\sqrt{3}$ $[\,k = 4\,]$	A1	**(12)**
9.	*(i)*	$f(-1) = r\ \therefore\ -1 + k + 7 - 15 = r$	M1	
		$k = r + 9$	A1	
		$f(3) = 3r\ \therefore\ 27 + 9k - 21 - 15 = 3r$	M1	
		$3k = r + 3$		
		subtracting, $2k = -6$	M1	
		$k = -3$	A1	
	(ii)	$r = -3 - 9 = -12$	B1	
	(iii)	$f(x) = x^3 - 3x^2 - 7x - 15$		
		$f(5) = 125 - 75 - 35 - 15 = 0\ \therefore\ (x - 5)$ is a factor	M1 A1	
	(iv)	$$\begin{array}{r|l} & x^2 + 2x + 3 \\ \hline x - 5 & x^3 - 3x^2 - 7x - 15 \\ & \underline{x^3 - 5x^2} \\ & \quad 2x^2 - 7x \\ & \quad \underline{2x^2 - 10x} \\ & \qquad 3x - 15 \\ & \qquad \underline{3x - 15} \end{array}$$	M1 A1	
		$\therefore\ (x - 5)(x^2 + 2x + 3) = 0$		
		$x = 5$ or $x^2 + 2x + 3 = 0$		
		$b^2 - 4ac = 2^2 - (4 \times 1 \times 3) = -8$	M1	
		$b^2 - 4ac < 0\ \therefore$ no real solutions to quadratic		
		$\therefore$ only one real solution	A1	**(12)**
			Total	**(72)**