Core Mathematics C2 Paper I

1. The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_n = 2^n + kn,$$

where k is a constant.

Given that $u_1 = u_3$,

(i) find the value of k, [3]

(ii) find the value of u_5. [2]

2. Given that

$$y = 2x^{\frac{3}{2}} - 1,$$

find

$$\int y^2 \, dx.$$ [6]

3. *(i)* Sketch the curve $y = \sin x°$ for x in the interval $-180 \le x \le 180$. [2]

(ii) Sketch on the same diagram the curve $y = \sin (x - 30)°$ for x in the interval $-180 \le x \le 180$. [2]

(iii) Use your diagram to solve the equation

$$\sin x° = \sin (x - 30)°$$

for x in the interval $-180 \le x \le 180$. [2]

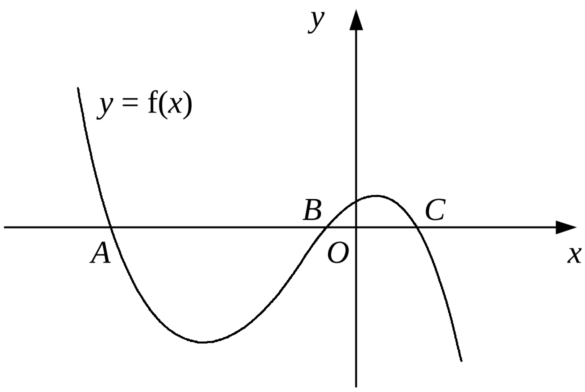
4. *(i)* Solve the inequality

$$x^2 - 13x + 30 < 0.$$ [3]

(ii) Hence find the set of values of y such that

$$2^{2y} - 13(2^y) + 30 < 0.$$ [3]

5.



The diagram shows the curve $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 4 + 5x + kx^2 - 2x^3,$$

and k is a constant.

The curve crosses the x-axis at the points A, B and C.

Given that A has coordinates $(-4, 0)$,

(i) show that $k = -7$, [2]

(ii) find the coordinates of B and C. [5]

6. Given that

$$\mathrm{f}'(x) = 5 + \frac{4}{x^2}, \quad x \neq 0,$$

(i) find an expression for $\mathrm{f}(x)$. [3]

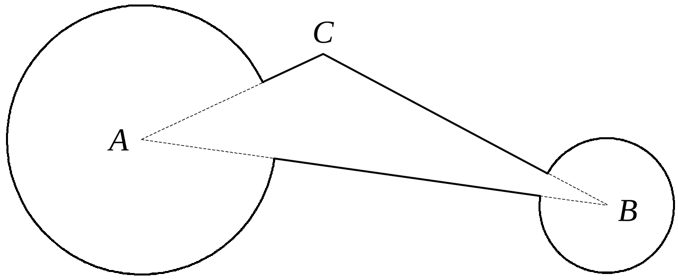
Given also that

$$\mathrm{f}(2) = 2\mathrm{f}(1),$$

(ii) find $\mathrm{f}(4)$. [5]

Turn over

7.



The diagram shows a design painted on the wall at a karting track. The sign consists of triangle ABC and two circular sectors of radius 2 metres and 1 metre with centres A and B respectively.

Given that $AB = 7$ m, $AC = 3$ m and $\angle ACB = 2.2$ radians,

(i) find the size of $\angle ABC$ in radians to 3 significant figures, [2]

(ii) show that $\angle BAC = 0.588$ radians to 3 significant figures, [2]

(iii) find the area of triangle ABC, [2]

(iv) find the area of the wall covered by the design. [4]

8. The finite region R is bounded by the curve $y = 1 + 3\sqrt{x}$, the x-axis and the lines $x = 2$ and $x = 8$.

(i) Use the trapezium rule with three intervals, each of width 2, to estimate to 3 significant figures the area of R. [5]

(ii) Use integration to find the exact area of R in the form $a + b\sqrt{2}$. [5]

(iii) Find the percentage error in the estimate made in part *(a)*. [2]

9. The first two terms of a geometric progression are 2 and x respectively, where $x \neq 2$.

(i) Find an expression for the third term in terms of x. [3]

The first and third terms of arithmetic progression are 2 and x respectively.

(ii) Find an expression for the 11th term in terms of x. [3]

Given that the third term of the geometric progression and the 11th term of the arithmetic progression have the same value,

(iii) find the value of x, [3]

(iv) find the sum of the first 50 terms of the arithmetic progression. [3]