

GCE Examinations  
Advanced Subsidiary

## Core Mathematics C2

Paper J

Time: 1 hour 30 minutes

### *Instructions and Information*

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Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has nine questions.

### *Advice to Candidates*

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You must show sufficient working to make your methods clear to an examiner.  
Answers without working may gain no credit.



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1. During one day, a biological culture is allowed to grow under controlled conditions.

At 8 a.m. the culture is estimated to contain 20 000 bacteria. A model of the growth of the culture assumes that  $t$  hours after 8 a.m., the number of bacteria present,  $N$ , is given by

$$N = 20\,000 \times (1.06)^t.$$

Using this model,

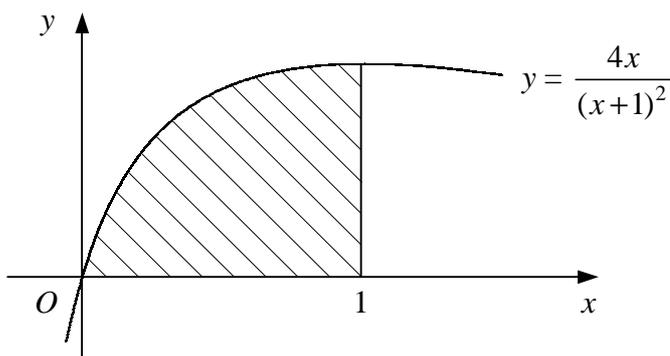
- (a) find the number of bacteria present at 11 a.m., (2)

- (b) find, to the nearest minute, the time when the initial number of bacteria will have doubled. (4)

2. The sides of a triangle have lengths of 7 cm, 8 cm and 10 cm.

Find the area of the triangle correct to 3 significant figures. (6)

- 3.



**Figure 1**

Figure 1 shows the curve with equation  $y = \frac{4x}{(x+1)^2}$ .

The shaded region is bounded by the curve, the  $x$ -axis and the line  $x = 1$ .

- (a) Use the trapezium rule with four intervals of equal width to find an estimate for the area of the shaded region. (5)

- (b) State, with a reason, whether your answer to part (a) is an under-estimate or an over-estimate of the true area. (2)

4. The first three terms in the expansion in descending powers of  $x$  of

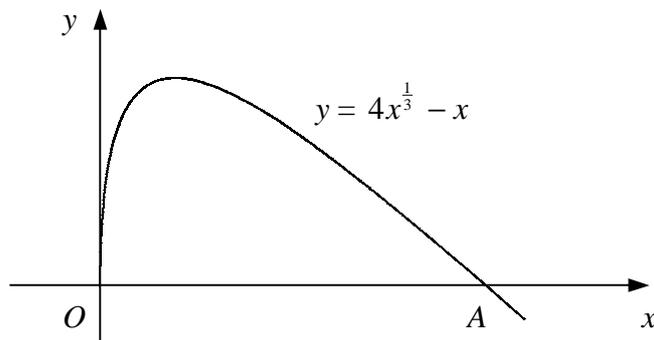
$$\left(x + \frac{k}{x^2}\right)^{15},$$

where  $k$  is a constant, are

$$x^{15} + 30x^{12} + Ax^9.$$

- (a) Find the values of  $k$  and  $A$ . (5)
- (b) Find the value of the term independent of  $x$  in the expansion. (3)

- 5.



**Figure 2**

Figure 2 shows the curve with equation  $y = 4x^{\frac{1}{3}} - x$ ,  $x \geq 0$ .

The curve meets the  $x$ -axis at the origin and at the point  $A$  with coordinates  $(a, 0)$ .

- (a) Show that  $a = 8$ . (3)
- (b) Find the area of the finite region bounded by the curve and the positive  $x$ -axis. (5)

- 6.

$$f(x) = \cos 2x, \quad 0 \leq x \leq \pi.$$

- (a) Sketch the curve  $y = f(x)$ . (2)
- (b) Write down the coordinates of any points where the curve  $y = f(x)$  meets the coordinate axes. (3)
- (c) Solve the equation  $f(x) = 0.5$ , giving your answers in terms of  $\pi$ . (4)

*Turn over*

7. The points  $P$  and  $Q$  have coordinates  $(-2, 6)$  and  $(4, -1)$  respectively.

Given that  $PQ$  is a diameter of circle  $C$ ,

- (a) find the coordinates of the centre of  $C$ , (2)

- (b) show that  $C$  has the equation

$$x^2 + y^2 - 2x - 5y - 14 = 0. \quad (5)$$

The point  $R$  has coordinates  $(2, 7)$ .

- (c) Show that  $R$  lies on  $C$  and hence, state the size of  $\angle PRQ$  in degrees. (2)
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8. The second and third terms of a geometric series are  $\log_3 4$  and  $\log_3 16$  respectively.

- (a) Find the common ratio of the series. (3)

- (b) Show that the first term of the series is  $\log_3 2$ . (2)

- (c) Find, to 3 significant figures, the sum of the first six terms of the series. (4)
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9.  $f(x) = x^3 - 4x^2 - 3x + 18.$

- (a) Show that  $(x - 3)$  is a factor of  $f(x)$ . (2)

- (b) Fully factorise  $f(x)$ . (4)

- (c) Using your answer to part (b), write down the coordinates of one of the turning points of the curve  $y = f(x)$  and give a reason for your answer. (2)

- (d) Using differentiation, find the  $x$ -coordinate of the other turning point of the curve  $y = f(x)$ . (5)
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**END**