



GCE

Mathematics

Unit 4722: Core Mathematics 2

Advanced Subsidiary GCE

Mark Scheme for June 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

| Annotation in scoris | Meaning |
|------------------------|----------------------------------------------------------|
| √and × | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| ٨ | Omission sign |
| MR | Misread |
| NGE | Not good enough |
| Highlighting | |
| | |
| Other abbreviations in | Meaning |
| mark scheme | |
| E1 | Mark for explaining |
| U1 | Mark for correct units |
| G1 | Mark for a correct feature on a graph |
| M1 dep* | Method mark dependent on a previous mark, indicated by * |
| сао | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |
| сwo | Correct working only |
| | |

June 2016

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

Μ

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

В

Mark for a correct result or statement independent of Method marks.

Ε

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

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| (| Juestion | Answer | Marks | Guidance | | | |
|---|----------|-----------------------------------------------------------------------------|-----------|------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|
| 1 | (i) | $\frac{1}{2} \times 8 \times AB \times \sin 30 = 20$ $AB = 10$ | M1 A1 | Equate correct attempt at area of triangle to 20 Obtain 10 | Must be using correct formula, including $\frac{1}{2}$ Allow if subsequently evaluated in radian mode (gives -3.95AB = 20) If using $\frac{1}{2} \ge b \ge h$ then must be valid use of trig to find h Must be exactly 10 | | |
| | (ii) | $BC^{2} = 8^{2} + 10^{2} - 2 \times 8 \times 10 \times \cos 30$ $BC = 5.04$ | [2] M1 | Attempt to use correct cosine rule, using their <i>AB</i> | Must be using correct cosine rule Allow M1 if not square rooted, as long as BC^2 soi Allow if subsequently evaluated in radian mode (gives 11.8), but 11.8 by itself cannot imply M1 Allow if correct formula seen but is then evaluated incorrectly (using ($8^2 + 10^2 - 2 \ge 8 \ge 10$) $\ge cos30$ gives 1.86) Allow any equiv method as long as valid use of trig | | |
| | | | A1 [2] | Obtain 5.04, or better | If > 3sf, allow answer rounding to 5.043 with no errors seen | | |

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| | Juestion | Answer | Marks | | Guidance |
|---|----------|--------------------------------------------------|--------------------|-------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2 | (i) | $54^{\circ} x \frac{\pi}{180} = \frac{8\pi}{10}$ | M1 | Attempt to use conversion factor of $\frac{\pi}{180}$ | Must use $\frac{\pi}{180}$ or $\frac{2\pi}{860}$ or equiv method such as fractions of a circle Can also use 1 rad = 57.3° or 1° = 0.0175 rad Must use fractions correct way up so multiplying by $\frac{180}{\pi}$ is M0 0.942 (or better) with no working will imply M1 |
| | | | A1 [2] | Obtain ^{BR} 10 | Allow exact simplified equiv ie 0.3π A0 if not fully simplified No ISW if decimal equiv (0.942) given as final answer However, if both decimal and exact answers seen, then allow A1 if, and only if, the exact answer is indicated as their only intended final answer (eg underlined) |
| | (ii) | $\frac{3\pi}{10}r + 2r = 60$ r = 20.4 | M1* | Attempt perimeter in terms of <i>r</i> | Must be using $r\theta$ as arc length, and also including $2r$ in the perimeter attempt Allow use of an incorrect θ from (i) Only allow incorrect θ if seen in (i), so $0.3r + 2r$ is M0, unless 0.3 was their (i) Could be using decimal equiv for θ (0.942) M0 if using 54 ⁰ , unless part of a valid attempt such as fractions of a circle M0 if using radians incorrectly eg 0.942π |
| | | | M1d* | Equate to 60, and attempt to solve | Must be a valid solution attempt, and go as far as an attempt at r M0 for $2.3\pi r = 60$, or similar Could be working exactly or in decimals |
| | | | A1 [3] | Obtain 20.4, or better | If > 3sf, allow answers in the range [20.39, 20.40] |
| | | | | | |

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| (| Questio | on | Answer | Marks | Guidance | | |
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| 3 | (i) | | $3^{3} + (3 \times 3^{2} \times kx) + (3 \times 3 \times (kx)^{2}) + (kx)^{3}$ = 27 + 27kx + 9k ² x ² + k ³ x ³ | M1 | Attempt expansion | Must attempt at least 3 of the 4 terms Each term must be an attempt at the product of the relevant binomial coeff soi, the correct power of 3 and the correct power of kx Allow M1 if powers used incorrectly with kx ie only applied to the x and not to k as well Binomial coeff must be numerical, so ${}^{3}C_{2}$ is M0 until evaluated Allow M1 for expanding $c(1 + \frac{kx}{s})^{3}$, any c Allow M1 for reasonable attempt to expand brackets | |
| | | | | A1 | Obtain at least two correct terms | Allow 3^3 for 27 and 3^2 for 9 Allow $(kx)^2$ and/or $(kx)^3$ unless later incorrect Terms could just be listed | |
| | | | | A1 | Obtain at least one further correct term | Allow 3^3 for 27 and 3^2 for 9 Allow $(kx)^2$ and/or $(kx)^3$ unless later incorrect Terms could just be listed | |
| | | | | A1 [4] | Obtain fully correct simplified expansion | Must now be 27 and 9, not still index notation Allow $(kx)^2$ and/or $(kx)^3$ unless later incorrect Must be a correct expansion, with terms linked by '+' rather than just a list of 4 terms No ISW if correct final answer is subsequently spoiled by attempt to 'simplify' eg dividing by 27 | |
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| Question | Answer | Marks | | Guidance |
|----------|---------------------------------------------|-------|---------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (ii) | $9k^{2} = 27$ $k^{2} = 3$ $k = \pm\sqrt{3}$ | M1 | Equate their coeff of x^2 to their constant term and attempt to solve for k | Must be equating coefficients not terms - allow recovery if next line is $k^2 = 3$, but M0 if x^2 still present at this stage Must attempt k, but allow if only positive square root is considered If a division attempt was made in part (i) then allow M1 for using either their original terms or their 'simplified' terms |
| | | A1 | Obtain $k = \pm \sqrt{3}$ | Must have \pm , or two roots listed separately Final answer must be given in exact form A0 for $\pm \sqrt{(^{27}/_9)}$ Must come from correct coefficients only, not from terms that were a result of a division attempt SR allow B1 if $k = \pm \sqrt{3}$ is given as final answer, but |
| | | [2] | | inconsistent use of terms / coefficients within solution |
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| (| Question | | Answer | Marks | Guidance | | |
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| 4 | (i) | | $\log_3 x^2 - \log_3 (x+4)$ | B1* | $Obtain \log_3 x^2 - \log_3(x+4)$ | Allow no base | |
| | | | $= \log_3 \frac{x^2}{x+4}$ | | | Could be implied if both log steps done together Allow equiv eg $2(\log_3 x - \log_3 (x + 4)^{0.5})$ | |
| | | | | | | Anow equives $2(\log_3 x - \log_3 (x + 4))$ | |
| | | | | B1d* | Obtain $\log_3 \frac{x^2}{x+4}$ or equiv single | CWO so B0 if $eg \frac{\log x^2}{\log (x+4)}$ seen in solution | |
| | | | | | term x_{x+4} or equily single | No ISW if subsequently incorrectly 'simplified' eg | |
| | | | | | | $\log_3(\frac{x}{4})$ | |
| | | | | [2] | | Must now have correct base in final answer - condone | |
| | | | | | | if omitted earlier | |
| | (ii) | | $\frac{x^2}{x+4} = 3^2$ x ² = 9(x + 4) | M1* | Attempt correct method to remove | Equation must be of format $\log_3 f(x) = 2$, with $f(x)$ being the result of a legitimate attempt to combine logs | |
| | | | $x^2 = 9(x+4)$ | | logs | (but condone errors such as incorrect simplification of | |
| | | | $x^2 - 9x - 36 = 0$ | | | fraction) | |
| | | | (x-12)(x+3) = 0x = 12 | | | Allow use of their (i) only if it satisfies the above | |
| | | | x 12 | | | criteria, so $x^2 - (x + 4) = 9$ is M0 whether or not in (i) | |
| | | | | A1 | Obtain any correct equation | Not involving logs | |
| | | | | M1d* | Attempt complete method to solve for <i>x</i> | Solving a 3 term quadratic - see additional guidance Must attempt at least one value of x | |
| | | | | | | - | |
| | | | | A1 | Obtain $x = 12$ as only solution | Must be from a correct solution of a correct quadratic, | |
| | | | | | | and A0 if other root (if given) is not $x = -3$ A0 if $x = -3$ still present | |
| | | | | | | Not necessary to consider $x = -3$, and then discard, but | |
| | | | | | | A0 if discarded for incorrect reason | |
| | | | | [4] | | | |
| | | | | | | NB Despite not being 'hence' allow full credit for other valid attempts, such as combining $log_3(x + 4)$ | |
| | | | | | | with $\log_3 9$ on right-hand side before removing $\log_3(x + 4)$ | |
| | | | | | | starting with $\log_3 x - \frac{1}{2}\log_3(x+4) = 1$ | |
| | | | | | | - | |
| | | | | | | SR in (i) $\frac{\log x^2}{\log(x+4)}$ becoming $\log_3 \frac{x^2}{x+4}$ was penalised as an | |
| | | | | | | error in notation, but is eligible for full credit in (ii) | |

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| 5 | (a) | | $\int (2x^3 - 3x^2 + 4x - 6) dx$ $= \frac{1}{2}x^4 - x^3 + 2x^2 - 6x + c$ | M1 | Expand brackets and attempt integration | Must be reasonable attempt to expand brackets, resulting in at least 3 terms, but allow slip(s) Integration attempt must have an increase in power by 1 for at least 3 of their terms | |
| | | | | A1FT | Obtain at least three correct (algebraic) terms | Following their expansion Allow unsimplified coefficients | |
| | | | | A1 | Obtain fully correct expression, including $+c$ | Coefficients must now be fully simplified A0 if integral sign or dx still present in final answer, but allow $\int =$ | |
| | (b) | (i) | $\left[-6x^{-1}+2x^{-2}\right]^{a}$ | M1 | Attempt integration | Integral must be of the form $k_1x^{-1} + k_2x^{-2}$, any k_1 and k_2 as long as numerical | |
| | | | $\begin{bmatrix} -6x^{-1} + 2x^{-2} \end{bmatrix}^{a}$ = (-6a^{-1} + 2a^{-2}) - (-6+2) = 4 - 6a^{-1} + 2a^{-2} | A1 | Obtain fully correct expression | Allow unsimplified coefficients Allow presence of $+ c$ | |
| | | | | M1 | Attempt correct use of limits | Must be $F(a) - F(1)$ is correct order and subtraction Allow $F(x)$ to be any function with indices changed from the original, even if differentiation appears to have been attempted | |
| | | | | A1 | Obtain $4 - 6a^{-1} + 2a^{-2}$ aef | Coefficients should now be simplified, and constant terms combined Could use negative indices, or write as fractions A0 if + c present in final answer A0 if integral sign or dx still present in final answer, but condone presence for first 3 marks ISW any subsequent work, such as further attempts at simplification, multiplying by a^2 , equating to a constant, or writing as an inequality | |
| | | | | [4] | | constant, or writing as an inequality | |

| Question | | on | Answer | Marks | Guidance | | | |
|----------|------|------|----------------------------------------------------------|--------------------|-----------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|
| | | (ii) | 4 | B1FT | State 4, following their (i) | Their (b)(i) must be of the form $k + k_1a^{-1} + k_2a^{-2}$, with all coefficients non-zero and numerical Do not allow $4 + 0$ or equiv Must appreciate that a limit is required, so B0 for $<, \approx,$ \rightarrow , 'tends to' etc Condone confusion over use of 0 and ∞ Final answer of 4 may result from starting again, rather than using their (b)(i) | | |
| 6 | (i) | | $u_k = 5 + 1.5(k - 1)$ 5 + 1.5(k - 1) = 140 k = 91 | M1* | Attempt <i>n</i> th term of an AP, using $a = 5$ and $d = 1.5$ | Must be using correct formula, so M0 for $5 + 1.5k$ Allow if in terms of <i>n</i> not <i>k</i> Could attempt an <i>n</i> th term definition, giving $1.5k + 3.5$ | | |
| | | | | M1d* | Equate to 140 and attempt to solve for k | Must be valid solution attempt, and go as far as an attempt at k Allow equiv informal methods | | |
| | | | | A1 [3] | Obtain 91 | Answer only gains full credit | | |
| | (ii) | | $S_{16} = \frac{120(1 - 0.9^{16})}{1 - 0.9} = 978$ | M1 | Attempt to find the sum of 16 terms of GP, with $a = 120$, $r = 0.9$ | Must be using correct formula | | |
| | | | | A1 | Obtain 978, or better | If > 3sf, allow answer rounding to 977.6 with no errors seen Answer only, or listing and summing 16 terms, gains full credit | | |
| | | | | [2] | | | | |
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| Question | Answer | Marks | | Guidance |
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| (iii) | $\frac{\frac{1}{2}N(10 + (N - 1) \times 1.5) > \frac{120}{1 - 0.9}}{N(1.5N + 8.5) > 2400}$ | B1 | Correct sum to infinity stated | Could be 1200 or unsimplified expression |
| | $3N^2 + 17N - 4800 > 0$ N = 38 | B1 | Correct S_N stated | Any correct expression, including unsimplified |
| | | M1* | Link S_N of AP to S_∞ of GP and attempt to rearrange | Must be recognisable attempt at S_N of AP and S_{∞} of GP, though not necessarily fully correct Allow any (in)equality sign, including < Must rearrange to a three term quadratic, not involving brackets |
| | | A1 | Obtain correct 3 term quadratic | aef - not necessary to have all algebraic terms on the same side of the (in)equation Allow any (in)equality sign |
| | | M1d* | Attempt to solve quadratic | See additional guidance for acceptable methods May never consider the negative root M1 could be implied by sight of 37.3, as long as from correct quadratic |
| | | A1 | Obtain $N = 38$ (must be equality) | A0 for $N \ge 38$ or equiv in words eg 'N is at least 38' Allow A1 if 38 follows =, > or \ge being used but A0 if 38 follows < or \le being used A0 if second value of N given in final answer |
| | | [6] | | Must be from an algebraic method - at least as far as obtaining the correct quadratic |
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| (| Question | Answer | Marks | Guidance | | |
|---|----------|----------------------------|-------|--------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| 7 | (i) | $Q = x^2 - 4x + 3$ $R = 0$ | M1 | Attempt complete division by (<i>x</i> + 1), or equiv | Must be complete method to obtain at least the quotient (ie all 3 terms attempted) but can get M1A1 if remainder not considered Long division - must subtract lower line (allow one slip)Inspection - expansion must give at least three correct terms of the cubic Coefficient matching - must be valid attempt at all coeffs of the quadratic, considering all relevant terms each time Synthetic division - must be using -1 (not 1) and adding within each column (allow one slip); expect to see -1 1 -3 -1 3 -1 4 (-3) 1 -4 3 (0) The values in brackets come from attempting R and are not required for M1 | |
| | | | A1 | Obtain fully correct quotient | Quotient could be stated explicitly, seen in division attempt or in a factorised expression for $f(x)$. Do not ISW if their explicitly stated quotient contradicts earlier working (eg correct in division but then stated as 'quotient = 3') If using coefficient matching then $A = 1$, $B = -4$, $C = 3$ is not sufficient for A1. | |

Must equate to 0 before dividing by 4

PMT

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| Question | | Answer | Marks | Guidance | | | |
|----------|---|---------------------------------------------------------------------------------------------------------------|-----------|-----------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|
| | | | A1 | Obtain remainder as 0, must be stated explicitly | Not sufficient to just see 0 at bottom of division attempt (algebraic or synthetic) Allow 'no remainder' for 'remainder = 0' $f(-1) = 0$ is not sufficient for A1 unless identified as | | |
| (ii) | | $x^{2}-4x+3 = (x-1)(x-3)$ hence $x = -1, 1, 3$ | [3] M1 | Attempt to solve their quadratic quotient | Allow for solving any three term quadratic from their attempt at quotient, even if M0 in (i) See additional guidance for acceptable methods Could now be a different quotient if there is another division attempt with the factor as $(x - 1)$ or $(x - 3)$ | | |
| | | | A1 | Obtain $x = 1, 3$ | M1A1 if both roots just stated with no method shown (but no partial credit if only one root correct) | | |
| | | | B1 | State $x = -1$ | Independent of M mark B0 if $x = -1$ is clearly as result of solving their quadratic quotient only Must be seen in (ii) - no back credit if only seen in (i) | | |
| (iii |) | $\frac{dy}{dx} = 4x^3 - 12x^2 - 4x + 12$ $4x^3 - 12x^2 - 4x + 12 = 0$ hence $x^3 - 3x^2 - x + 3 = 0$ AG | M1 | Attempt differentiation | Decrease in power by 1 for at least 3 of the terms (could include $9 \rightarrow 0$) Not sufficient to substitute their roots to show $y = 0$ | | |

Equate to 0 and rearrange to given

A1

[2]

answer

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| (iv) | $ \begin{bmatrix} \frac{1}{5}x^5 - x^4 - \frac{2}{3}x^3 + 6x^2 + 9x \end{bmatrix}_{1}^{3} $ = $\begin{pmatrix} \frac{153}{5} \end{pmatrix} - \begin{pmatrix} -\frac{53}{15} \end{pmatrix}$ | M1* | Attempt integration | Increase in power by 1 for at least 3 of the terms Must be integrating equation of curve, not $f(x)$ |
| | $=\frac{512}{15}$ | A1 | Obtain fully correct expression | Allow unsimplified coefficients Allow presence of $+ c$ |
| | | M1d* | Attempt correct use of correct limits | No follow-through from incorrect roots in (ii) Must be $F(3) - F(-1)$ ie correct order and subtraction Could find area between 1 and 3, but must double this for M1 If final area is incorrect then must see evidence of use of limits to award M1; if all that is shown is the difference of two numerical values then both must be correct eg just $\left(\frac{153}{5}\right) - \left(-\frac{23}{15}\right) = \frac{482}{15}$ is M0 as no evidence for second term |
| | | A1 | Obtain ⁵¹² / ₁₅ , or any exact equiv | Decimal equiv must be exact ie 34.13 , so A0 for 34.13, 34.133 etc Allow A1 if exact value seen, but followed by decimal equiv Answer only is 0/4 - need to see evidence of integration, but use of limits does not need to be |
| | | [4] | | explicit |
| | | | | |

| | Questio | on Answer | Marks | Guidance | | |
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| 8 | (i) | 2 (units) in the positive <i>x</i> -direction | M1 | Correct direction | Identify that the translation is in the x-direction (either positive or negative, so M1 for eg '2 in negative x- direction') Allow any terminology as long as intention is clear, such as in/on/along the x-axis Ignore the magnitude | |
| | | | A1 | Fully correct description | Must have correct magnitude and correct direction, using precise language - such as 'in the x-direction', 'parallel to the x-axis', 'horizontally' or 'to the right' A0 for in/on/along the x-axis etc Allow M1A1 for '2 in the x-direction' as positive is implied A0 for 'factor 2' 'Units' is not required, but A0 for 'places', 'spaces', 'squares' etc | |
| | | | [2] | | Allow in vector notation as well, so M1 for $\binom{k}{0}$ and M1A1 for $\binom{2}{0}$ | |
| | (ii) | sf $\frac{1}{9}$ in the <i>y</i> -direction | M1 | Correct direction, with sf of $\frac{1}{9}$ or 9 | Identify that the stretch is in the y-direction, with a scale factor of either $\frac{1}{2}$ or 9 (or equiv in index notation) Allow just $\frac{1}{2}$ or 9, with no mention of 'scale factor' Allow exact decimal equiv for $\frac{1}{2}$ Allow any terminology as long as the intention is clear, such as in/on/along the y-axis | |
| | | | A1 | Fully correct description | Must have correct scale factor and correct direction, using precise language - such as 'in the y-direction', 'parallel to the y-axis' or 'vertically' A0 for in/on/along the y-axis etc | |
| | | | [2] | | Must now have 'scale factor' or 'factor' Allow 'positive <i>y</i> -direction' (not incorrect as graph is wholly above <i>x</i> -axis) | |

| Question | Answer | Marks | Guidance | |
|----------|-------------------------------------------------------------------------------------------------------|-------|-----------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (iii) | intersect at $(0, \frac{1}{9})$ | B1* | Correct sketch, in both quadrants | Curve must tend towards the negative <i>x</i> -axis, but not touch or cross it, nor a significant flick back upwards If from plotted points then there must be enough of the graph shown to demonstrate the correct general shape, including the negative <i>x</i> -axis being an asymptote Ignore any numerical values given |
| | | B1d* | State $(0, \frac{1}{9})$ | Condone $x = 0$, $y = \frac{1}{9}$ as an alternative, but $x = 0$ must be stated explicitly rather than implied Allow no brackets around the coordinates Allow exact decimal equiv for $\frac{1}{9}$ Allow just $\frac{1}{9}$ as long as marked on the y-axis Allow BOD for $(\frac{1}{9}, 0)$ on y-axis, but not if just stated Just being seen in a table of values is not sufficient |
| (iv) | $log3^{x-2} = log180 (or \ x - 2 = log_3 180)$ $(x - 2)log \ 3 = log180$ $x - 2 = 4.7268$ $x = 6.73$ | M1* | Introduce logs and drop power | Ignore any other labelled coordinatesCan use logs to any base, as long as consistent on bothsides, and allow no explicit base as wellThe power must also be dropped for the M1Brackets must be seen around the $(x - 2)$, or implied bylater workingIf taking log ₃ then base must be explicit |
| | | M1d* | Attempt to solve for <i>x</i> | Correct order of operations, and correct operations so M0 for $\log_3 180 - 2$ M0 if logs used incorrectly eg $x - 2 = \log(\frac{180}{8})$ |
| | | A1 | Obtain 6.73, or better | If > 3sf, allow answer rounding to 6.727 with no errors seen 0/3 for answer only or T&I If rewriting eqn as $3^{x-2} = 3^{4.73}$ then 0/3 unless evidence of use of logs to find the index of 4.73 SR If using index rules first then B1 for $3^x = 1620$ |
| | | [3] | | M1 for attempting to use logs to solve $3^x = k$ A1 for 6.73 |

Mark Scheme

| June | 2016 |
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| Question | Answer | Marks | Guidance | |
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| (v) | $0.5 \times 1.5 \times \left\{ 3^{-1} + 2 \times 3^{0.5} + 3^2 \right\}$ = 9.60 | B1 | State the 3 correct <i>y</i> -values, and no others | B0 if other <i>y</i> -values also found (unless not used) Allow for unsimplified, even if subsequent error made Allow decimal equivs |
| | | M1 | Attempt use of correct trapezium rule to attempt area between $x = 1$ and $x = 4$ | Correct placing of <i>y</i> -values required <i>y</i> -values may not necessarily be correct, but must be from attempt at using correct <i>x</i> -values The 'big brackets' must be seen, or implied by later working Could be implied by stating general rule in terms of y_0 etc, as long as these have been attempted elsewhere and clearly labelled Could use other than 2 strips as long as of equal width (but M0 for just one strip) Must have <i>h</i> as 1.5, or a value consistent with the number of strips used if not 2 |
| | | A1 | Obtain 9.60, or better (allow 9.6) | Allow answers in the range [9.595, 9.600] if > 3sf Answer only is $0/3$ Using the trap. rule on the result of an integration attempt is $0/3$, even if integration is not explicit Using two separate trapezia can get full marks Using other than 2 trapezia (but not just 1) can get M1 only |
| | | [3] | | |

June 2016

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|---------------------------------------|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|-------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------|
| 9 | (i) | 2 <u>π</u> a | B1 | State $\frac{2\pi}{a}$ | Any exact equiv Allow in degrees ie $\frac{360}{a}$ |
| | | | | | B0 if given as a range eg $0 \le x \le \frac{2\pi}{a}$ |
| | (ii) | $\frac{1}{5}\pi a = \pi - \frac{2}{5}\pi a$ hence $a = \frac{5}{3}$ $k = \frac{1}{2}\sqrt{3}$ | M1 | Attempt to use symmetry of sine curve, or equiv | Allow any correct relationship between the two solutions, in radians or degrees Could also identify that the period must be $\frac{6}{5}\pi$ |
| | | | A1 | Obtain $a = \frac{5}{8}$ | Any exact equiv CWO, but allow working in degrees |
| | | | A1 [3] | Obtain $k = \frac{1}{2}\sqrt{3}$ | Any exact equiv, but not involving sin CWO, but allow working in degrees A0 if from incorrect <i>a</i> |
| | | | [0] | | Ao ii nom meoriet <i>a</i> |
| | | Alternative solution $ sin(\frac{1}{5}\pi a) = sin(\frac{2}{5}\pi a) $ $ sin(\frac{1}{5}\pi a) = 2sin(\frac{1}{5}\pi a)cos(\frac{1}{5}\pi a) $ | M1 | Attempt to use correct sin2 <i>A</i> identity | As far as $2\cos(\frac{1}{5}\pi a) = 1$ |
| | | $3\operatorname{In}\left(\frac{1}{5}\pi a\right) = 2\operatorname{Sin}\left(\frac{1}{5}\pi a\right) = 0$ $2\operatorname{cos}\left(\frac{1}{5}\pi a\right) = 1, \text{ hence } \frac{1}{5}\pi a = \frac{\pi}{3}$ $a = \frac{5}{3}$ $k = \frac{1}{2}\sqrt{3}$ | A1 | Obtain $a = \frac{5}{8}$ | |
| | | $k = \frac{1}{2}\sqrt{3}$ | A1 | Obtain $k = \frac{1}{2}\sqrt{3}$ | |
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Mark Scheme

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| Question | Answer | Marks | | Guidance |
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| (iii) | $\tan(ax) = \sqrt{3}$ $ax = \frac{\pi}{3}, \frac{4\pi}{3}$ $x = \frac{\pi}{3a}, \frac{4\pi}{3a}$ | B1 | State $tan(ax) = \sqrt{3}$ | Allow B1 for correct equation even if no, or an incorrect, attempt to solve Give BOD on notation eg $\frac{\sin}{\cos}(ax)$ as long as correct equation is seen or implied at some stage Allow $\tan(ax) - \sqrt{3} = 0$, or equiv Allow B1 for identifying that $ax = \frac{\pi}{s}$ or 60° even if equation in $\tan(ax)$ not seen – M1 would then be awarded for an attempt at x |
| | | M1 | Attempt to solve $tan(ax) = c$ | Attempt $\frac{1}{a} \tan^{-1}(c)$, any (non-zero) numerical c M0 for $\tan^{-1}(\frac{c}{a})$ Allow if attempted in degrees not radians M1 could be implied rather than explicit M1 can be awarded if using a numerical value for a |
| | | A1 | Obtain $x = \frac{\pi}{aa}$ | Must be in radians not degrees Allow any exact equiv eg $\frac{\pi}{2}$ as long as intention clear - but A0 if this is then given as $\frac{a\pi}{3}$ |
| | | A1 | Obtain $x = \frac{4\pi}{3a}$ | Must be in radians not degrees Allow any exact equiv eg $\frac{4\pi}{a}$ as long as intention clear - but A0 if this is then given as $\frac{4a\pi}{a}$ Allow $\frac{\pi}{a} + \frac{\pi}{a}$, unless then incorrectly simplified If more than two solutions given, then mark the two smallest ones and ISW the rest eg $\frac{\pi}{a}$, $\frac{4\pi}{a}$, $\frac{7\pi}{a}$ would be A1A1 but $\frac{\pi}{a}$, $\frac{2\pi}{a}$, $\frac{4\pi}{a}$ would be A1A0 |
| | | [4] | | |

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| Question Answer | | Marks | Marks Guidance | | |
|-----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|----------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| | Alternative solution | | | | |
| | $\sin^{2}(ax) = 3 \cos^{2}(ax)$ $4\sin^{2}(ax) = 3 \text{ or } 4\cos^{2}(ax) = 1$ $\sin(ax) = \pm \sqrt{4}\sqrt{3} \text{ or } \cos(ax) = \pm \frac{4}{3}$ | B1 | Obtain $4\sin^2(ax) = 3$ or $4\cos^2(ax) = 1$ | Any correct, simplified, equation in a single trig ratio | |
| | $\sin(ax) = \pm \frac{\pi}{2}\sqrt{3} \text{ or } \cos(ax) = \pm \frac{\pi}{2}$ $ax = \frac{\pi}{3}, \frac{4\pi}{5}$ $x = \frac{\pi}{3a}, \frac{4\pi}{3a}$ | M1 | Attempt to solve $\sin^2(ax) = c$ or $\cos^2(ax) = c$ | Allow M1 if just the positive square root used Attempt $\frac{1}{a}\sin^{-1}(\sqrt{c})$ or $\frac{1}{a}\cos^{-1}(\sqrt{c})$, any (non-zero) numerical c M0 for $\sin^{-1}(\frac{\sqrt{c}}{a})$ M0 for $\cos^{-1}(\frac{\sqrt{c}}{a})$ Allow if attempted in degrees not radians M1 could be implied rather than explicit M1 can be awarded if using a numerical value for a | |
| | | A1 | Obtain $x = \frac{\pi}{3a}$ | Must be in radians not degrees Allow any exact equive $g^{\frac{\pi}{2}}_{a}$ as long as intention clear - but A0 if this is then given as $\frac{a\pi}{s}$ Must be in radians not degrees | |
| | | A1 | Obtain $x = \frac{4\pi}{3a}$ | Allow any exact equive $g_{a}^{\frac{4\pi}{3}}$ as long as intention clear - but A0 if this is then given as $\frac{4a\pi}{3}$ Allow a correct answer still in two terms, unless then incorrectly simplified If more than two solutions given, then mark the two smallest ones and ISW the rest eg $\frac{\pi}{3a}$, $\frac{4\pi}{3a}$, $\frac{7\pi}{3a}$ would be A1A1 but $\frac{\pi}{3a}$, $\frac{2\pi}{3a}$, $\frac{4\pi}{3a}$ would be A1A0 | |

June 2016

APPENDIX 1

Guidance for marking C2

Accuracy

Allow answers to 3sf or better, unless an integer is specified or clearly required.

Answers to 2 sf are penalised, unless stated otherwise in the mark scheme.

3sf is sometimes explicitly specified in a question - this is telling candidates that a decimal is required rather than an exact answer eg in logs, and more than 3sf should not be penalised unless stated in mark scheme.

If more than 3sf is given, allow the marks for an answer that falls within the guidance given in the mark scheme, with no obvious errors.

Extra solutions

Candidates will usually be penalised if any extra, incorrect, solutions are given. However, in trigonometry questions only look at solutions in the given range and ignore any others, correct or incorrect.

Solving equations

With simultaneous equations, the method mark is given for eliminating one variable. Any valid method is allowed ie balancing or substitution for two linear equations, substitution only if at least one is non-linear.

Solving quadratic equations

Factorising - candidates must get as far as factorising into two brackets which, on expansion, would give the correct coefficient of x^2 and at least one of the other two coefficients. This method is only credited if it is possible to factorise the quadratic – if the roots are surds then candidates are expected to use either the quadratic formula or complete the square.

Completing the square - candidates must get as far as $(x + p) = \pm \sqrt{q}$, with reasonable attempts at p and q.

Using the formula - candidates need to substitute values into the formula, with some attempt at evaluation (eg calculating 4ac). The correct formula must be seen, either algebraic or numerical. If the algebraic formula is quoted then candidates are allowed to make one slip when substituting their values. The division line must extend under the entire numerator (seen or implied by later working). Condone not dividing by 2a as long as it has been seen earlier.

Solutions with no method shown

If a correct equation is seen, then the correct answers will imply that the method is correct – unless specified otherwise in the mark scheme then this must be all answers to the equation. So, if solving a quadratic, only the correct two roots will imply a correct method (NB on this paper the MS does identify two exceptions to this rule – Q4 and Q6).

If an incorrect equation is seen, and no supporting method for solving it is shown, then examiners must not try to deduce the method used from the solutions provided.

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